Problem Solutions for Chapter 5

5-3. (a) $\cos^L 30° = 0.5$

$\cos 30° = (0.5)^{1/L} = 0.8660$

$L = \log 0.5/\log 0.8660 = 4.82$

(b) $\cos^T 15° = 0.5$

$\cos 15° = (0.5)^{1/T} = 0.9659$

$T = \log 0.5/\log 0.9659 = 20.0$

5-4. The source radius is less than the fiber radius, so Eq. (5-5) holds:

$$P_{LED\text{-}step} = \pi^2 r_s^2\ B_0(NA)^2 = \pi^2(2\times10^{-3}\text{ cm})^2(100\text{ W/cm}^2)(.22)^2 = 191\ \mu W$$

From Eq. (5-9)

$$P_{LED\text{-}graded} = 2\pi^2(2\times10^{-3}\text{ cm})^2(100\text{ W/cm}^2)(1.48)^2(.01)\left[1 - \frac{1}{2}\left(\frac{2}{5}\right)^2\right] = 159\ \mu W$$

5-5. Using Eq. (5-10), we have that the reflectivity at the source-to-gel interface is

$$R_{s-g} = \left(\frac{3.600-1.305}{3.600+1.305}\right)^2 = 0.219$$

Similarly, the relfectivity at the gel-to-fiber interface is

$$R_{g-f} = \left(\frac{1.465-1.305}{1.465+1.305}\right)^2 = 3.34\times10^{-3}$$

The total reflectivity then is $R = R_{s-g}R_{g-f} = 7.30\times10^{-4}$

The power loss in decibels is (see Example 5-3)

$$L = -10\ \log\ (1-R) = -10\ \log\ (0.999) = 3.17\times10^{-3}\ \text{dB}$$

5-6. Substituting $B(\theta) = B_0 \cos^m \theta$ into Eq. (5-3) for $B(\theta,\phi)$, we have

$$P = \int_0^{r_m} \int_0^{2\pi} \left[2\pi \int_0^{\theta_{0\text{-max}}} \cos^3\theta \sin\theta \, d\theta \right] d\theta_s \, r \, dr$$

Using

$$\int_0^{\theta_0} \cos^3\theta \sin\theta \, d\theta = \int_0^{\theta_0} (1 - \sin^2\theta) \sin\theta \, d(\sin\theta) = \int_0^{\sin\theta_0} (x - x^3) \, dx$$

we have

$$P = 2\pi \int_0^{r_m} \int_0^{2\pi} \left[\frac{\sin^2\theta_{0\text{-max}}}{2} - \frac{\sin^4\theta_{0\text{-max}}}{4} \right] d\theta_s \, r \, dr$$

$$= \pi \int_0^{r_m} \int_0^{2\pi} \left[NA^2 - \frac{1}{2} NA^4 \right] d\theta_s \, r \, dr$$

$$= \frac{\pi}{2} [2NA^2 - NA^4] \int_0^{r_m} r \, dr \int_0^{2\pi} d\theta_s$$

5-7. (a) Let a = 25 μm and NA = 0.16. For $r_s \geq a(NA) = 4$ μm, Eq. (5-17) holds. For $r_s \leq 4$ μm, $\eta = 1$.

(b) With a = 50 μm and NA = 0.20, Eq. (5-17) holds for $r_s \geq 10$ μm. Otherwise, $\eta = 1$.

5-8. Using Eq. (5-10), the relfectivity at the gel-to-fiber interface is

$$R_{g-f} = \left(\frac{1.485 - 1.305}{1.485 + 1.305}\right)^2 = 4.16 \times 10^{-3}$$

The power loss is (see Example 5-3)

$$L = -10 \ \log \ (1 - R) = -10 \ \log \ (0.9958) = 0.018 \ \text{dB}$$

When there is no index-matching gel, the joint loss is

$$R_{a-f} = \left(\frac{1.485 - 1.000}{1.485 + 1.000}\right)^2 = 0.038$$

The power loss is $L = -10 \ \log \ (1 - R) = -10 \ \log \ (0.962) = 0.17 \ \text{dB}$

5-9. Shaded area = (circle segment area) - (area of triangle) = $\frac{1}{2}$ sa - $\frac{1}{2}$ cy

$$s = a\theta = a\,[2 \arccos (y/a)] = 2a \arccos \left(\frac{d}{2a}\right)$$

$$c = 2\left[a^2 - \left(\frac{d}{2}\right)^2\right]^{1/2}$$

Therefore

$$A_{common} = 2(\text{shaded area}) = sa - cy = 2a^2 \arccos \left(\frac{d}{2a}\right) - d\left[a^2 - \left(\frac{d^2}{4}\right)\right]^{1/2}$$

5-10.

Core/cladding diameters (μm)	**Coupling loss (dB) for Given axial misalignments (μm)**			
	1	**3**	**5**	**10**
50/125	0.112	0.385	0.590	1.266
62.5/125	0.089	0.274	0.465	0.985
100/140	0.056	0.169	0.286	0.590

5-11. $\arccos x = \frac{\pi}{2} - \arcsin x$

For small values of x, $\arcsin x = x + \frac{x^3}{2(3)} + \frac{x^5}{2(4)(5)} + \ldots$

Therefore, for $\frac{d}{2a} \ll 1$, we have $\quad \arccos\frac{d}{2a} \approx \frac{\pi}{2} - \frac{d}{2a}$

Thus Eq. (5-30) becomes $\quad P_T = \frac{2}{\pi} P\left(\frac{\pi}{2} - \frac{d}{2a} - \frac{5d}{6a}\right) = P\left(1 - \frac{8d}{3\pi a}\right)$

d/a	**P_T/P (Eq.5-30)**	**P_T/P (Eq.5-31)**
0.00	1.00	1.00
0.05	0.9576	0.9576
0.10	0.9152	0.9151
0.15	0.8729	0.8727
0.20	0.8309	0.8302
0.25	0.7890	0.7878
0.30	0.7475	0.7454
0.35	0.7063	0.7029
0.40	0.6656	0.6605

5-12. Plots of mechanical misalignment losses.

5-13. From Eq. (5-20) the coupling efficiency η_F is given by the ratio of the number of modes in the receiving fiber to the number of modes in the emitting fiber, where the number of modes M is found from Eq. (5-19). Therefore

$$\eta_F = \frac{M_{aR}}{M_{aE}} = \frac{k^2 NA^2(0)\left(\frac{1}{2} - \frac{1}{\alpha+2}\right) a_R^2}{k^2 NA^2(0)\left(\frac{1}{2} - \frac{1}{\alpha+2}\right) a_E^2} = \frac{a_R^2}{a_E^2}$$

Therefore from Eq. (5-21) the coupling loss for $a_R \le a_E$ is $L_F = -10 \log\left(\frac{a_R^2}{a_E^2}\right)$

5-14. For fibers with different NAs, where $NA_R < NA_E$

$$L_F = -10 \log \eta_F = -10 \log \frac{M_R}{M_E} = -10 \log \frac{k^2 NA_R^2(0)\left(\frac{\alpha}{2\alpha+4}\right) a^2}{k^2 NA_E^2(0)\left(\frac{\alpha}{2\alpha+4}\right) a^2}$$

$$= -10 \log \left[\frac{NA_R^2(0)}{NA_E^2(0)}\right]$$

5-15. For fibers with different α values, where $\alpha_R < \alpha_E$

$$L_F = -10 \log \eta_F = -10 \log \frac{k^2 NA^2(0)\left(\frac{\alpha_R}{2\alpha_R + 4}\right)a^2}{k^2 NA^2(0)\left(\frac{\alpha_E}{2\alpha_E + 4}\right)a^2} = -10 \log \left[\frac{\alpha_R(\alpha_E + 2)}{\alpha_E(\alpha_R + 2)}\right]$$

5-16. The splice losses are found from the sum of Eqs. (5-35) through (5-37). First find NA(0) from Eq. (2-80b).

For fiber 1: $NA_1(0) = n_1\sqrt{2\Delta} = 1.46\sqrt{2(0.01)} = 0.206$

For fiber 2: $NA_2(0) = n_1\sqrt{2\Delta} = 1.48\sqrt{2(0.015)} = 0.256$

(a) The only loss is that from index-profile differences. From Eq. (5-37)

$$L_{1\to 2}(\alpha) = -10 \log \frac{1.80(2.00+2)}{2.00(1.80+2)} = 0.24 \text{ dB}$$

(b) The losses result from core-size differences and NA differences.

$$L_{2\to 1}(a) = -20 \log \left(\frac{50}{62.5}\right) = 1.94 \text{ dB}$$

$$L_{2\to 1}(NA) = -20 \log \left[\frac{.206}{.256}\right] = 1.89 \text{ dB}$$

5-17. Plots of connector losses using Eq. (5-43).

5-18. When there are no losses due to extrinsic factors, Eq. (5-43) reduces to

$$L_{SM;ff} = -10 \log \left[\frac{4}{\left(\frac{W_1}{W_2} + \frac{W_2}{W_1} \right)^2} \right]$$

For $W_1 = 0.9W_2$, we then have $L_{SM;ff} = -10 \log \left[\frac{4}{4.0446} \right] = -\ 0.0482$ dB

5-19. Plot of Eq. (5-44).

5-20. Plot of the throughput loss.