Problem Solutions for Chapter 4

4-1. From Eq. (4-1), $n_i = 2\left(\frac{2\pi k_B T}{h^2}\right)^{3/2} (m_e m_h)^{3/4} \exp\left(-\frac{E_g}{2k_B T}\right)$

$$= 2\left[\frac{2\pi(1.38\times10^{-23}\,\text{J/K})}{(6.63\times10^{-34}\,\text{J.s})^2}\right]^{3/2} T^{3/2}\left[(.068)(.56)(9.11\times10^{-31}\,\text{kg})^2\right]^{3/4}$$

$$\times \exp\left[-\frac{(1.55-4.3\times10^{-4}\,T)\text{eV}}{2(8.62\times10^{-5}\,\text{eV/K})T}\right]$$

$$= 4.15\times10^{14}\, T^{3/2} \exp\left[-\frac{1.55}{2(8.62\times10^{-5})T}\right]\exp\left[\frac{4.3\times10^{-4}}{2(8.62\times10^{-5})}\right]$$

$$= 5.03\times10^{15}\, T^{3/2} \exp\left(-\frac{8991}{T}\right)$$

4-2. The electron concentration in a p-type semiconductor is $n_P = n_i = p_i$

Since both impurity and intrinsic atoms generate conduction holes, the total conduction-hole concentration p_P is

$p_P = N_A + n_i = N_A + n_P$

From Eq. (4-2) we have that $n_P = n_i^2/p_P$. Then

$p_P = N_A + n_P = N_A + n_i^2/p_P$ or $p_P^2 - N_A p_P - n_i^2 = 0$

so that

$$p_P = \frac{N_A}{2}\left(\sqrt{1+\frac{4n_i^2}{N_A^2}} + 1\right)$$

If $n_i \ll N_A$, which is generally the case, then to a good approximation

$p_P \approx N_A$ and $n_P = n_i^2/p_P \approx n_i^2/N_A$

4-3. (a) From Eq. (4-4) we have $1.540 = 1.424 + 1.266x + 0.266x^2$ or

$x^2 + 4.759x - 0.436 = 0$. Solving this quadratic equation yields (taking the plus sign only)

$$x = \frac{1}{2}\left[- 4.759 + \sqrt{(4.759)^2 + 4(.436)}\ \right] \quad = \underline{0.090}$$

The emission wavelength is $\lambda = \dfrac{1.240}{1.540} = 805$ nm.

(b) $E_g = 1.424 + 1.266(0.15) + 0.266(0.15)^2 = 1.620$ eV, so that

$$\lambda = \frac{1.240}{1.620} = 766 \text{ nm}$$

4-4. (a) The lattice spacings are as follows:

$a(BC) = a(GaAs) = 5.6536$ Å

$a(BD) = a(GaP) = 5.4512$ Å

$a(AC) = a(InAs) = 6.0590$ Å

$a(AD) = a(InP) = 5.8696$ Å

$$a(x,y) = xy\, 5.6536 + x(1-y)\, 5.4512 + (1-x)y\, 6.0590 + (1-x)(1-y) 5.8696$$

$$= 0.1894y - 0.4184x + 0.0130xy + 5.8696$$

(b) Substituting $a(xy) = a(InP) = 5.8696$ Å into the expression for $a(xy)$ in (a), we have

$$y = \frac{0.4184x}{0.1894 - 0.0130x} \approx \frac{0.4184x}{0.1894} = 2.20x$$

(c) With $x = 0.26$ and $y = 0.56$, we have

$$E_g = 1.35 + 0.668(.26) - 1.17(.56) + 0.758(.26)^2 + 0.18(.56)^2$$

$$- .069(.26)(.56) - .322(.26)^2(.56) + 0.03(.26)(.56)^2 = 0.956 \text{ eV}$$

4-5. Differentiating the expression for E, we have

$$\Delta E = \frac{hc}{\lambda^2}\Delta\lambda \quad \text{or} \quad \Delta\lambda = \frac{\lambda^2}{hc}\Delta E$$

For the same energy difference ΔE, the spectral width $\Delta\lambda$ is proportional to the wavelength squared. Thus, for example,

$$\frac{\Delta\lambda_{1550}}{\Delta\lambda_{1310}} = \left(\frac{1550}{1310}\right)^2 = 1.40$$

4-6. (a) From Eq. (4-10), the internal quantum efficiency is

$\eta_{int} = \dfrac{1}{1+25/90} = 0.783$, and from Eq. (4-13) the internal power level is

$$P_{int} = (0.783)\frac{hc(35\ \text{mA})}{q(1310\ \text{nm})} = 26\ \text{mW}$$

(b) From Eq. (4-16),

$$P = \frac{1}{3.5(3.5+1)^2}\,26\ \text{mW} = 0.37\ \text{mW}$$

4-7. Plot of Eq. (4-18). Some representative values of P/P_0 are given in the table:

f in MHz	**P/P_0**
1	0.999
10	0.954
20	0.847
40	0.623
60	0.469
80	0.370
100	0.303

4-8. The 3-dB optical bandwidth is found from Eq. (4-21). It is the frequency f at which the expression is equal to -3; that is,

$$10\log\left(\frac{1}{\left[1+(2\pi f\tau)^2\right]^{1/2}}\right) = -3$$

With a 5-ns lifetime, we find $f = \frac{1}{2\pi(5\text{ ns})}\left(10^{0.6} - 1\right) = 9.5\text{ MHz}$

4-9. (a) Using Eq. (4-28) with $\Gamma = 1$

$$g_{th} = \frac{1}{0.05\text{ cm}} \ln\left(\frac{1}{0.32}\right)^2 + 10\text{ cm}^{-1} = 55.6\text{ cm}^{-1}$$

(b) With $R_1 = 0.9$ and $R_2 = 0.32$,

$$g_{th} = \frac{1}{0.05\text{ cm}} \ln\left[\frac{1}{0.9(0.32)}\right] + 10\text{ cm}^{-1} = 34.9\text{ cm}^{-1}$$

(c) From Eq. (4-37) $\eta_{ext} = \eta_i\,(g_{th} - \bar{\alpha})/g_{th}$;

thus for case (a): $\eta_{ext} = 0.65(55.6 - 10)/55.6 = 0.53$

For case (b): $\eta_{ext} = 0.65(34.9 - 10)/34.9 = 0.46$

4-10. Using Eq. (4-4) to find E_g and Eq. (4-3) to find λ, we have for x = 0.03,

$$\lambda = \frac{1.24}{E_g} = \frac{1.24}{1.424 + 1.266(0.3) + 0.266(0.3)^2} = 1.462\ \mu\text{m}$$

From Eq. (4-38)

$$\eta_{ext} = 0.8065\,\lambda(\mu\text{m})\,\frac{dP(\text{mW})}{dI(\text{mA})}$$

Taking dI/dP = 0.5 mW/mA, we have $\eta_{ext} = 0.8065\,(1.462)(0.5) = 0.590$

4-11. (a) From the given values, D = 0.74, so that $\Gamma_T = 0.216$

Then $n_{eff}^2 = 10.75$ and W = 3.45, yielding $\Gamma_L = 0.856$

(b) The total confinement factor then is $\Gamma = 0.185$

4-12. From Eq. (4-46) the mode spacing is

$$\Delta\lambda = \frac{\lambda^2}{2Ln} = \frac{(0.80\ \mu\text{m})^2}{2(400\ \mu\text{m})(3.6)} = 0.22\text{ nm}$$

Therefore the number of modes in the range 0.75-to-0.85 μm is

$$\frac{.85-.75}{.22\times10^{-3}} = \frac{.1}{.22} \times 10^3 = \underline{455 \text{ modes}}$$

4-13. (a) From Eq. (4-44) we have $g(\lambda) = (50 \text{ cm}^{-1}) \exp\left[-\frac{(\lambda - 850 \text{ nm})^2}{2(32 \text{ nm})^2}\right]$

$$= (50 \text{ cm}^{-1}) \exp\left[-\frac{(\lambda - 850)^2}{2048}\right]$$

(b) On the plot of $g(\lambda)$ versus λ, drawing a horizontal line at $g(\lambda) = \alpha_t$

$= 32.2 \text{ cm}^{-1}$ shows that lasing occurs in the region $820 \text{ nm} < \lambda < 880 \text{ nm}$.

(c) From Eq. (4-47) the mode spacing is

$$\Delta\lambda = \frac{\lambda^2}{2Ln} = \frac{(850)^2}{2(3.6)(400 \ \mu\text{m})} = 0.25 \text{ nm}$$

Therefore the number of modes in the range 820-to-880 nm is

$$N = \frac{880 - 820}{0.25} = \underline{240 \text{ modes}}$$

4-14. (a) Let $N_m = n/\lambda = \frac{m}{2L}$ be the wave number (reciprocal wavelength) of mode m. The difference ΔN between adjacent modes is then

$$\Delta N = N_m - N_{m-1} = \frac{1}{2L} \qquad \text{(a-1)}$$

We now want to relate ΔN to the change $\Delta\lambda$ in the free-space wavelength. First differentiate N with respect to λ:

$$\frac{dN}{d\lambda} = \frac{d}{d\lambda}\left(\frac{n}{\lambda}\right) = \frac{1}{\lambda}\frac{dn}{d\lambda} - \frac{n}{\lambda^2} = -\frac{1}{\lambda^2}\left(n - \lambda\frac{dn}{d\lambda}\right)$$

Thus for an incremental change in wavenumber ΔN, we have, in absolute values,

$$\Delta N = \frac{1}{\lambda^2}\left(n - \lambda\frac{dn}{d\lambda}\right)\Delta\lambda \qquad \text{(a-2)}$$

Equating (a-1) and (a-2) then yields $\Delta\lambda = \dfrac{\lambda^2}{2L\left(n - \lambda \dfrac{dn}{d\lambda}\right)}$

(b) The mode spacing is $\Delta\lambda = \dfrac{(.85\ \mu m)^2}{2(4.5)(400\ \mu m)} = 0.20$ nm

4-15. (a) The reflectivity at the GaAs-air interface is

$$R_1 = R_2 = \left(\frac{n-1}{n+1}\right)^2 = \left(\frac{3.6-1}{3.6+1}\right)^2 = 0.32$$

Then $J_{th} = \dfrac{1}{\beta}\left[\overline{\alpha} + \dfrac{1}{2L}\ln\dfrac{1}{R_1R_2}\right] = 2.65\times10^3\ A/cm^2$

Therefore

$I_{th} = J_{th} \times l \times w = (2.65\times10^3\ A/cm^2)(250\times10^{-4}\ cm)(100\times10^{-4}\ cm) = 663$ mA

(b) $I_{th} = (2.65\times10^3\ A/cm^2)(250\times10^{-4}\ cm)(10\times10^{-4}\ cm) = 66.3$ mA

4-16. From the given equation

$$\Delta E_{11} = 1.43\ eV + \frac{\left(6.6256\times10^{-34}\ J\cdot s\right)^2}{8(5\ nm)^2}\left(\frac{1}{6.19\times10^{-32}\ kg} + \frac{1}{5.10\times10^{-31}\ kg}\right)$$

$$= 1.43\ eV + 0.25\ eV = 1.68\ eV$$

Thus the emission wavelength is $\lambda = hc/E = 1.240/1.68 = 739$ nm.

4-17. Plots of the external quantum efficiency and power output of a MQW laser.

4-18. From Eq. (4-48a) the effective refractive index is

$$n_e = \frac{m\lambda_B}{2\Lambda} = \frac{2(1570\ nm)}{2(460\ nm)} = 3.4$$

Then, from Eq. (4-48b), for m = 0

$$\lambda = \lambda_B \pm \frac{\lambda_B^2}{2n_e L}\left(\frac{1}{2}\right) = 1570\text{ nm} \pm \frac{(1.57\ \mu\text{m})(1570\text{ nm})}{4(3.4)(300\ \mu\text{m})} = 1570\text{ nm} \pm 1.20\text{ nm}$$

Therefore for m = 1, $\lambda = \lambda_B \pm 3(1.20\text{ nm}) = 1570\text{ nm} \pm 3.60\text{ nm}$

For m = 2, $\lambda = \lambda_B \pm 5(1.20\text{ nm}) = 1570\text{ nm} \pm 6.0\text{ nm}$

4-19. (a) Integrate the carrier-pair-density versus time equation from time 0 to t_d (time for onset of stimulated emission). In this time the injected carrier pair density changes from 0 to n_{th}.

$$t_d = \int_0^{t_d} dt = \int_0^{n_{th}} \frac{1}{\frac{J}{qd} - \frac{n}{\tau}}\, dn = -\tau \left(\frac{J}{qd} - \frac{n}{\tau}\right)\Bigg|_{n=0}^{n=n_{th}} = \tau\ \ln\left(\frac{J}{J - J_{th}}\right)$$

where $J = I_p/A$ and $J_{th} = I_{th}/A$. Therefore $t_d = \tau \ln\left(\frac{I_p}{I_p - I_{th}}\right)$

(b) At time t = 0 we have $n = n_B$, and at $t = t_d$ we have $n = n_{th}$. Therefore,

$$t_d = \int_0^{t_d} dt = \int_{n_B}^{n_{th}} \frac{1}{\frac{J}{qd} - \frac{n}{\tau}}\, dn = \tau \ln\left(\frac{\frac{J}{qd} - \frac{n_B}{\tau}}{\frac{J}{qd} - \frac{n_{th}}{\tau}}\right)$$

In the steady state before a pulse is applied, $n_B = J_B\tau/qd$. When a pulse is applied, the current density becomes $I/A = J = J_B + J_p = (I_B + I_p)/A$

Therefore, $t_d = \tau \ln\left(\frac{I - I_B}{I - I_{th}}\right) = \tau \ln\left(\frac{I_p}{I_p + I_B - I_{th}}\right)$

4-20. A common-emitter transistor configuration:

4-21. Laser transmitter design.

4-22. Since the dc component of x(t) is 0.2, its range is -2.36 < x(t) < 2.76. The power has the form $P(t) = P_0[1 + mx(t)]$ where we need to find m and P_0. The average value is

$$\langle P(t) \rangle = P_0[1 + 0.2m] = 1 \text{ mW}$$

The minimum value is

$$P(t) = P_0[1 - 2.36m] \geq 0 \qquad \text{which implies } m \leq \frac{1}{2.36} = 0.42$$

Therefore for the average value we have $\langle P(t) \rangle = P_0[1 + 0.2(0.42)] \leq 1$ mW, which implies

$$P_0 = \frac{1}{1.084} = 0.92 \text{ mW} \qquad \text{so that } P(t) = 0.92[1 + 0.42x(t)] \text{ mW and}$$

$$i(t) = 10\, P(t) = 9.2[1 + 0.42x(t)] \text{ mA}$$

4-23. Substitute x(t) into y(t):

$$y(t) = a_1 b_1 \cos \omega_1 t + a_1 b_2 \cos \omega_2 t$$
$$+ a_2(b_1^2 \cos^2 \omega_1 t + 2b_1 b_2 \cos \omega_1 t \cos \omega_2 t + b_2^2 \cos^2 \omega_2 t)$$
$$+ a_3(b_1^3 \cos^3 \omega_1 t + 3b_1^2 b_2 \cos^2 \omega_1 t \cos \omega_2 t + 3b_1 b_2^2 \cos \omega_1 t \cos^2 \omega_2 t + b_2^3 \cos^3 \omega_2 t)$$
$$+ a_4(b_1^4 \cos^4 \omega_1 t + 4b_1^3 b_2 \cos^3 \omega_1 t \cos \omega_2 t + 6b_1^2 b_2^2 \cos^2 \omega_1 t \cos^2 \omega_2 t$$
$$+ 4b_1 b_2^3 \cos \omega_1 t \cos^3 \omega_2 t + b_2^4 \cos^4 \omega_2 t)$$

Use the following trigonometric relationships:

i) $\cos^2 x = \frac{1}{2}(1 + \cos 2x)$

ii) $\cos^3 x = \frac{1}{4}(\cos 3x + 3\cos x)$

iii) $\cos^4 x = \frac{1}{8}(\cos 4x + 4\cos 2x + 3)$

iv) $2\cos x \cos y = \cos(x+y) + \cos(x-y)$

v) $\cos^2 x \cos y = \frac{1}{4}[\cos(2x+y) + 2\cos y + \cos(2x-y)]$

vi) $\cos^2 x \cos^2 y = \frac{1}{4} \left[1 + \cos 2x + \cos 2y + \frac{1}{2} \cos(2x+2y) + \frac{1}{2} \cos(2x-2y)\right]$

vii) $\cos^3 x \cos y = \frac{1}{8} \left[\cos (3x+y) + \cos (3x-y) + 3\cos (x+y) + 3\cos (x-y)\right]$

then

$$y(t) = \frac{1}{2}\left[a_2 b_1^2 + a_2 b_2^2 + \frac{3}{4} a_4 b_1^4 + 3 a_4 b_1^2 b_2^2 + \frac{3}{4} a_4 b_2^4\right] \qquad \text{constant terms}$$

$$+ \frac{3}{4}\left[a_3 b_1^3 + 2 a_3 b_1 b_2^2\right] \cos \omega_1 t + \frac{3}{4} a_3\left[b_2^3 + 2 b_1^2 b_2\right] \cos \omega_2 t \qquad \text{fundamental terms}$$

$$+ \frac{b_1^2}{2}\left[a_2 + a_4 b_1^2 + 3 a_4 b_2^2\right] \cos 2\omega_1 t + \frac{b_2^2}{2}\left[a_2 + a_4 b_2^2 + 3 a_4 b_1^2\right] \cos 2\omega_2 t$$

2nd-order harmonic terms

$$+ \frac{1}{4} a_3 b_1^3 \cos 3\omega_1 t + \frac{1}{4} a_3 b_2^3 \cos 3\omega_2 t \qquad \text{3rd-order harmonic terms}$$

$$+ \frac{1}{8} a_4 b_1^4 \cos 4\omega_1 t + \frac{1}{8} a_4 b_2^4 \cos 4\omega_2 t \qquad \text{4th-order harmonic terms}$$

$$+ \left[a_2 b_1 b_2 + \frac{3}{2} a_4 b_1^3 b_2 + \frac{3}{2} a_4 b_1 b_2^3\right]\left[\cos (\omega_1+\omega_2)t + \cos (\omega_1-\omega_2)t\right]$$

2nd-order intermodulation terms

$$+ \frac{3}{4} a_3 b_1^2 b_2 \left[\cos(2\omega_1+\omega_2)t + \cos(2\omega_1-\omega_2)t\right] + \frac{3}{4} a_3 b_1 b_2^2 \left[\cos(2\omega_2+\omega_1)t + \cos(2\omega_2-\omega_1)t\right]$$

3rd-order intermodulation terms

$$+ \frac{1}{2} a_4 b_1^3 b_2 \left[\cos(3\omega_1+\omega_2)t + \cos(3\omega_1-\omega_2)t\right]$$
$$+ \frac{3}{4} a_4 b_1^2 b_2^2 \left[\cos(2\omega_1+2\omega_2)t + \cos(2\omega_1-2\omega_2)t\right]$$
$$+ \frac{1}{2} a_4 b_1 b_2^3 \left[\cos(3\omega_2+\omega_1)t + \cos(3\omega_2-\omega_1)t\right] \qquad \text{4th-order intermodulation}$$

terms

This output is of the form

$$y(t) = A_0 + A_1(\omega_1) \cos \omega_1 t + A_2(\omega_1) \cos 2\omega_1 t + A_3(\omega_1) \cos 3\omega_1 t$$
$$+ A_4(\omega_1) \cos 4\omega_1 t + A_1(\omega_2) \cos \omega_2 t + A_2(\omega_2) \cos 2\omega_2 t$$

$+ A_3(\omega_2) \cos 3\omega_2 t + A_4(\omega_2) \cos 4\omega_2 t + \sum_m \sum_n B_{mn} \cos(m\omega_1 + n\omega_2)t$

where $A_n(\omega_j)$ is the coefficient for the $\cos(n\omega_j)t$ term.

4-24. From Eq. (4-58) $P = P_0 e^{-t/\tau_m}$ where $P_0 = 1$ mW and $\tau_m = 2(5\times10^4 \text{ hrs}) = 10^5$ hrs.

(a) 1 month = 720 hours. Therefore:

P(1 month) = (1 mW) exp($-720/10^5$) = 0.99 mW

(b) 1 year = 8760 hours. Therefore

P(1 year) = (1 mW) exp($-8760/10^5$) = 0.92 mW

(c) 5 years = 5×8760 hours = 43,800 hours. Therefore

P(5 years) = (1 mW) exp($-43800/10^5$) = 0.65 mW

4-25. From Eq. (4-60) $\tau_s = K e^{E_A/k_BT}$ or $\ln \tau_s = \ln K + E_A/k_BT$

where $k_B = 1.38\times10^{-23}$ J/°K $= 8.625\times10^{-5}$ eV/°K

At T = 60°C = 333°K, we have

$\ln 4\times10^4 = \ln K + E_A/[(8.625\times10^{-5} \text{ eV})(333)]$

or $10.60 = \ln K + 34.82\, E_A$ (1)

At T = 90°C = 363°K, we have

$\ln 6500 = \ln K + E_A/[(8.625\times10^{-5} \text{ eV})(363)]$

or $8.78 = \ln K + 31.94\, E_A$ (2)

Solving (1) and (2) for E_A and K yields

$E_A = 0.63$ eV and $k = 1.11\times10^{-5}$ hrs

Thus at T = 20°C = 293°K

$$\tau_s = 1.11\times10^{-5} \exp\{0.63/[(8.625\times10^{-5})(293)]\} = 7.45\times10^{5} \text{ hrs}$$