Problem Solutions for Chapter 7

7-1. We want to compare $F_1 = kM + (1 - k)\left(2 - \frac{1}{M}\right)$ and $F_2 = M^x$.

For silicon, k = 0.02 and we take x = 0.3:

M	**F_1(M)**	**F_2(M)**	**% difference**
9	2.03	1.93	0.60
25	2.42	2.63	8.7
100	3.95	3.98	0.80

For InGaAs, k = 0.35 and we take x = 0.7:

M	**F_1(M)**	**F_2(M)**	**% difference**
4	2.54	2.64	3.00
9	4.38	4.66	6.4
25	6.86	6.96	1.5
100	10.02	9.52	5.0

For germanium, k = 1.0, and if we take x = 1.0, then $F_1 = F_2$.

7-2. The Fourier transform is

$$h_B(t) = \int_{-\infty}^{\infty} H_B(f) e^{j2\pi ft}\, df = R \int_{-\infty}^{\infty} \frac{e^{j2\pi ft}}{1 + j2\pi fRC}\, df$$

Using the integral solution from Appendix B3:

$$\int_{-\infty}^{\infty} \frac{e^{jpx}}{(\beta + jx)^n}\, dx = \frac{2\pi (p)^{n-1} e^{-\beta p}}{\Gamma(n)} \qquad \text{for } p > 0\text{, we have}$$

$$h_B(t) = \frac{1}{2\pi C} \int_{-\infty}^{\infty} \frac{e^{j2\pi ft}}{\left(\frac{1}{2\pi RC} + jf\right)}\, df = \frac{1}{C} e^{-t/RC}$$

7-3. Part (a):

$$\int_{-\infty}^{\infty} h_p(t)\, dt = \frac{1}{\alpha T_b} \int_{-\alpha T_b/2}^{\alpha T_b/2} dt = \frac{1}{\alpha T_b}\left(\frac{\alpha T_b}{2} + \frac{\alpha T_b}{2}\right) = 1$$

Part (b):

$$\int_{-\infty}^{\infty} h_p(t)\ dt = \frac{1}{\sqrt{2\pi}}\ \frac{1}{\alpha T_b}\ \int_{-\infty}^{\infty} \exp\left[-\frac{t^2}{2(\alpha T_b)^2}\right] dt$$

$$= \frac{1}{\sqrt{2\pi}}\frac{1}{\alpha T_b}\sqrt{\pi}\,\alpha T_b\ \sqrt{2} = 1 \quad \text{(see Appendix B3 for integral solution)}$$

Part (c):

$$\int_{-\infty}^{\infty} h_p(t)\ dt = \frac{1}{\alpha T_b}\ \int_{0}^{\infty} \exp\left[-\frac{t}{\alpha T_b}\right] dt = -\left[e^{-\infty} - e^{-0}\right] = 1$$

7-4. The Fourier transform is

$$F[p(t)*q(t)] = \int_{-\infty}^{\infty} p(t) * q(t) e^{-j2\pi ft}\ dt = \int_{-\infty}^{\infty} q(x)\ \int_{-\infty}^{\infty} p(t-x)\ e^{-j2\pi ft}\ dt\ dx$$

$$= \int_{-\infty}^{\infty} q(x)\ e^{-j2\pi fx}\ \int_{-\infty}^{\infty} p(t-x)\ e^{-j2\pi f(t-x)}\ dt\ dx$$

$$= \int_{-\infty}^{\infty} q(x)\ e^{-j2\pi fx}\ dx \int_{-\infty}^{\infty} p(y)\ e^{-j2\pi fy}\ dy \qquad \text{where } y = t - x$$

$= F[q(t)]\ F[p(t)] = F[p(t)]\ F[q(t)] = P(f)\ Q(f)$

7-5. From Eq. (7-18) the probability for unbiased data ($a = b = 0$) is

$$P_e = \frac{1}{2}\left[P_0(v_{th}) + P_1(v_{th})\right].$$

Substituting Eq. (7-20) and (7-22) for P_0 and P_1, respectively, we have

$$P_e = \frac{1}{2}\ \frac{1}{\sqrt{2\pi\sigma^2}}\ \left[\int_{V/2}^{\infty} e^{-v^2/2\sigma^2}\ dv + \int_{-\infty}^{V/2} e^{-(v-V)^2/2\sigma^2}\ dv\right]$$

In the first integral, let $x = v/\sqrt{2\sigma^2}$ so that $dv = \sqrt{2\sigma^2}\ dx$.

In the second integral, let $q = v - V$, so that $dv = dq$. The second integral then becomes

$$\int_{-\infty}^{V/2-V} e^{-q^2/2\sigma^2} dq = \sqrt{2\sigma^2} \int_{-\infty}^{-V/2\sqrt{2\sigma^2}} e^{-x^2}\, dx \qquad \text{where } x = q/\sqrt{2\sigma^2}$$

Then

$$P_e = \frac{1}{2} \frac{\sqrt{2\sigma^2}}{\sqrt{2\pi\sigma^2}} \left[\int_{V/2\sqrt{2\sigma^2}}^{\infty} e^{-x^2}\, dx + \int_{-\infty}^{-V/2\sqrt{2\sigma^2}} e^{-x^2}\, dx \right]$$

$$= \frac{1}{2\sqrt{\pi}} \left[\int_{-\infty}^{\infty} e^{-x^2}\, dx - 2 \int_{0}^{V/2\sqrt{2\sigma^2}} e^{-x^2}\, dx \right]$$

Using the following relationships from Appendix B,

$$\int_{-\infty}^{\infty} e^{-p^2x^2}\, dx = \frac{\sqrt{\pi}}{p} \quad \text{and} \quad \frac{2}{\sqrt{\pi}} \int_{0}^{t} e^{-x^2}\, dx = \text{erf}(t), \text{ we have}$$

$$P_e = \frac{1}{2}\left[1 - \text{erf}\left(\frac{V}{2\sigma\sqrt{2}}\right)\right]$$

7-6. (a) $V = 1$ volt and $\sigma = 0.2$ volts, so that $\frac{V}{2\sigma} = 2.5$. From Fig. 7-6 for $\frac{V}{2\sigma} = 2.5$, we find $P_e \approx 7\times10^{-3}$ errors/bit. Thus there are $(2\times10^5$ bits/second$)(7\times10^{-3}$ errors/bit$) = 1400$ errors/second, so that

$$\frac{1}{1400 \text{ errors/second}} = 7\times10^{-4} \text{ seconds/error}$$

(b) If V is doubled, then $\frac{V}{2\sigma} = 5$ for which $P_e \approx 3\times10^{-7}$ errors/bit from Fig. 7-6. Thus

$$\frac{1}{(2\times10^5 \text{ bits/second})(3\times10^{-7} \text{ errors/bit})} = 16.7 \text{ seconds/error}$$

7-7. (a) From Eqs. (7-20) and (7-22) we have

$$P_0(v_{th}) = \frac{1}{\sqrt{2\pi\sigma^2}} \int_{V/2}^{\infty} e^{-v^2/2\sigma^2}\, dv = \frac{1}{2}\left[1 - \text{erf}\left(\frac{V}{2\sigma\sqrt{2}}\right)\right]$$

and

$$P_1(v_{th}) = \frac{1}{\sqrt{2\pi\sigma^2}}\int_{-\infty}^{V/2} e^{-(v-V)^2/2\sigma^2}dv = \frac{1}{2}\left[1 - \mathrm{erf}\left(\frac{V}{2\sigma\sqrt{2}}\right)\right]$$

Then for $V = V_1$ and $\sigma = 0.20V_1$

$$P_0(v_{th}) = \frac{1}{2}\left[1 - \mathrm{erf}\left(\frac{1}{2(.2)\sqrt{2}}\right)\right] \quad = \frac{1}{2}\left[1 - \mathrm{erf}\left(\frac{\sqrt{2}}{0.8}\right)\right]$$

$$= \frac{1}{2}\left[1 - \mathrm{erf}(1.768)\right] \quad = \frac{1}{2}(1 - 0.987) \quad = 0.0065$$

Likewise, for $V = V_1$ and $\sigma = 0.24V_1$

$$P_1(v_{th}) = \frac{1}{2}\left[1 - \mathrm{erf}\left(\frac{1}{2(.24)\sqrt{2}}\right)\right] \quad = \frac{1}{2}\left[1 - \mathrm{erf}\left(\frac{\sqrt{2}}{0.96}\right)\right]$$

$$= \frac{1}{2}\left[1 - \mathrm{erf}(1.473)\right] \quad = \frac{1}{2}(1 - 0.963) \quad = 0.0185$$

(b) $P_e = 0.65(0.0185) + 0.35(0.0065) = 0.0143$

(c) $P_e = 0.5(0.0185) + 0.5(0.0065) = 0.0125$

7-8. From Eq. (7-1), the average number of electron-hole pairs generated in a time t is

$$N = \frac{\eta E}{h\nu} \quad = \frac{\eta P t}{hc/\lambda} \quad = \frac{0.65(25\times10^{-10}\,\mathrm{W})(1\times10^{-9}\,\mathrm{s})(1.3\times10^{-6}\,\mathrm{m})}{(6.6256\times10^{-34}\,\mathrm{Js})(3\times10^{8}\,\mathrm{m/s})} = 10.6$$

Then, from Eq. (7-2)

$$P(n) = N^n\frac{e^{-N}}{n!} \quad = (10.6)^5\frac{e^{-10.6}}{5!} \quad = \frac{133822}{120} \quad e^{-10.6} = 0.05 = 5\%$$

7-9. $v_N = v_{out} - \langle v_{out}\rangle$

$$\langle v_N^2\rangle = \left\langle\left[v_{out} - \langle v_{out}\rangle\right]^2\right\rangle$$

$$= \langle v_{out}^2\rangle - 2\langle v_{out}\rangle^2 + \langle v_{out}\rangle^2$$

$$= \langle v_{out}^2\rangle - \langle v_{out}\rangle^2$$

7-10. (a) Letting $\phi = fT_b$ and using Eq. (7-40), Eq. (7-30) becomes

$$B_{bae} = \left|\frac{H_p(0)}{H_{out}(0)}\right|^2 \int_0^\infty T_b^2 \left|\frac{H'_{out}(\phi)}{H'_p(\phi)}\right|^2 \frac{d\phi}{T_b} = \frac{I_2}{T_b}$$

since $H_p(0) = 1$ and $H_{out}(0) = T_b$. Similarly, Eq. (7-33) becomes

$$B_e = \left|\frac{H_p(0)}{H_{out}(0)}\right|^2 \int_0^\infty \left|\frac{H_{out}(f)}{H_p(f)}(1 + j2\pi fRC)\right|^2 df$$

$$= \frac{1}{T_b^2} \int_0^\infty \left|\frac{H_{out}(f)}{H_p(f)}\right|^2 \left(1 + 4\pi^2 f^2 R^2 C^2\right) df$$

$$= \frac{1}{T_b^2} \int_0^\infty \left|\frac{H_{out}(f)}{H_p(f)}\right|^2 df + \frac{(2\pi RC)^2}{T_b^2} \int_0^\infty \left|\frac{H_{out}(f)}{H_p(f)}\right|^2 f^2\, df = \frac{I_2}{T_b} + \frac{(2\pi RC)^2}{T_b^3} I_3$$

(b) From Eqs. (7-29), (7-31), (7-32), and (7-34), Eq. (7-28) becomes

$$\langle v_N^2 \rangle = \langle v_s^2 \rangle + \langle v_R^2 \rangle + \langle v_I^2 \rangle + \langle v_E^2 \rangle$$

$$= 2q \langle i_0 \rangle \langle m^2 \rangle B_{bae} R^2 A^2 + \frac{4k_B T}{R_b} B_{bae} R^2 A^2 + S_I B_{bae} R^2 A^2 + S_E B_e A^2$$

$$= \left(2q \langle i_0 \rangle M^{2+x} + \frac{4k_B T}{R_b} + S_I\right) B_{bae} R^2 A^2 + S_E B_e A^2$$

$$= \frac{R^2 A^2 I_2}{T_b}\left(2q \langle i_0 \rangle M^{2+x} + \frac{4k_B T}{R_b} + S_I + \frac{S_E}{R^2}\right) + \frac{(2\pi RCA)^2}{T_b^3} S_E I_3$$

7-11. First let $x = (v - b_{off})/(\sqrt{2}\ \sigma_{off})$ with $dx = dv/(\sqrt{2}\ \sigma_{off})$ in the first part of Eq. (7-49):

$$P_e = \frac{\sqrt{2}\sigma_{off}}{\sqrt{2\pi}\sigma_{off}} \int_{\frac{v_{th} - b_{off}}{\sqrt{2}\sigma_{off}}}^\infty \exp\left(-x^2\right) dx = \frac{1}{\sqrt{\pi}} \int_{Q/\sqrt{2}}^\infty \exp\left(-x^2\right) dx$$

Similarly, let $y = (-v + b_{on})/(\sqrt{2}\ \sigma_{on})$ so that

$dy = -dv/(\sqrt{2}\sigma_{on})$ in the second part of Eq. (7-49):

$$P_e = -\frac{1}{\sqrt{\pi}} \int_{\infty}^{\frac{-v_{th}+b_{on}}{\sqrt{2}\sigma_{on}}} \exp\left(-y^2\right) dy = \frac{1}{\sqrt{\pi}} \int_{Q/\sqrt{2}}^{\infty} \exp\left(-y^2\right) dy$$

7-12. (a) Let $x = \dfrac{V}{2\sqrt{2}\,\sigma} = \dfrac{K}{2\sqrt{2}}$ For K = 10, x = 3.536. Thus

$$P_e = \frac{e^{-x^2}}{2\sqrt{\pi}\,x} = 2.97\times10^{-7} \text{ errors/bit}$$

(b) Given that $P_e = 10^{-5} = \dfrac{e^{-x^2}}{2\sqrt{\pi}\,x}$ then $e^{-x^2} = 2\sqrt{\pi}\ \ 10^{-5}\, x$.

This holds for $x \approx 3$, so that $K = 2\sqrt{2}\ \ x = 8.49$.

7-13. Differentiating Eq. (7-54) with respect to M and setting $db_{on}/dM = 0$, we have

$$\frac{db_{on}}{dM} = 0$$

$$= -\frac{Q(h\nu/\eta)}{M^2}\left\{\left[M^{2+x}\left(\frac{\eta}{h\nu}\right) b_{on}I_2 + W\right]^{1/2} + \left[M^{2+x}\left(\frac{\eta}{h\nu}\right) b_{on}I_2(1-\gamma\right] + W\right\}^{1/2}$$

$$+ \frac{Q(h\nu/\eta)}{M(h\nu/\eta)}\left\{\frac{\frac{1}{2}(2+x)M^{1+x}\ b_{on}I_2}{\left[M^{2+x}\left(\frac{\eta}{h\nu}\right) b_{on}I_2 + W\right]^{1/2}} + \frac{\frac{1}{2}(2+x)M^{1+x}\ b_{on}I_2(1-\gamma)}{\left[M^{2+x}\left(\frac{\eta}{h\nu}\right) b_{on}I_2(1-\gamma)+W\right]^{1/2}}\right\}^{1/2}$$

Letting $G = M^{2+x}\left(\dfrac{\eta}{h\nu}\right) b_{on}I_2$ for simplicity, yields

$$\left\{(G + W)^{1/2} + [\,G(1-\gamma) + W]^{1/2}\right\} = \frac{G}{2}(2 + x)\left\{\frac{1}{(G + W)^{1/2}} + \frac{(1-\gamma)}{[\,G(1-\gamma) + W]^{1/2}}\right\}$$

Multiply by $(G + W)^{1/2}\,[\,G(1-\gamma) + W]^{1/2}$ and rearrange terms to get

$$(G + W)^{1/2}\left[W - \frac{Gx}{2}(1-\gamma)\right] = [\,G(1-\gamma) + W]^{1/2}\left(\frac{Gx}{2} - W\right)$$

Squaring both sides and collecting terms in powers of G, we obtain the quadratic equation

$$G^2\left[\frac{x^2\gamma}{4}(1-\gamma)\right] + G\left[\frac{x^2\gamma}{4}W(2-\gamma)\right] - \gamma W^2(1+x) = 0$$

Solving this equation for G yields

$$G = \frac{-\frac{x^2}{4}W(2-\gamma) \pm \left\{\frac{x^4}{16}W^2(2-\gamma)^2 + x^2(1-\gamma)W^2(1+x)\right\}^{\frac{1}{2}}}{\frac{x^2}{2}(1-\gamma)}$$

$$= \frac{W(2-\gamma)}{2(1-\gamma)}\left\{1 - \left[1 + 16\left(\frac{1+x}{x^2}\right)\frac{1-\gamma}{(2-\gamma)^2}\right]^{\frac{1}{2}}\right\}$$

where we have chosen the "+" sign. Equation (7-55) results by letting

$$G = M_{opt}^{2+x} \quad b_{on}I_2\left(\frac{\eta}{h\nu}\right)$$

7-14. Substituting Eq. (7-55) for $M^{2+x}b_{on}$ into the square root expressions in Eq. (7-54) and solving Eq. (7-55) for M, Eq. (7-54) becomes

$$b_{on} = \frac{Q\left(\frac{h\nu}{\eta}\right)b_{on}^{1/(2+x)}}{\left[\frac{h\nu}{\eta}\,\frac{W(2-\gamma)}{2I_2(1-\gamma)}K\right]^{1/(2+x)}}\left\{\left[\frac{W(2-\gamma)}{2(1-\gamma)}K + W\right]^{\frac{1}{2}} + \left[\frac{W}{2}(2-\gamma)K + W\right]^{\frac{1}{2}}\right\}$$

Factoring out terms:

$$b_{on}^{(1+x)/(2+x)} = Q\left(\frac{h\nu}{\eta}\right)^{(1+x)/(2+x)} \quad W^{x/2(2+x)} \quad I_2^{1/(2+x)}$$

$$\times\left\{\left[\frac{(2-\gamma)}{2(1-\gamma)}K + 1\right]^{\frac{1}{2}} + \left[\frac{1}{2}(2-\gamma)K + 1\right]^{\frac{1}{2}}\right\} \quad \div \left[\frac{(2-\gamma)K}{2(1-\gamma)}\right]^{1/(2+x)}$$

or $$b_{on} = Q^{(2+x)/(1+x)}\left(\frac{h\nu}{\eta}\right) \quad W^{x/2(1+x)} \quad I_2^{1/(1+x)} \quad L$$

7-15. In Eq. (7-59) we want to evaluate

$$\lim_{\gamma\to 1}\left[\frac{(2-\gamma)K}{2(1-\gamma)L}\right]^{\frac{1+x}{2+x}} = \lim_{\gamma\to 1}\left[\frac{(2-\gamma)K}{2(1-\gamma)}\right]^{\frac{1+x}{2+x}} \lim_{\gamma\to 1}\left(\frac{1}{L}\right)^{\frac{1+x}{2+x}}$$

Consider first

$$\lim_{\gamma\to 1}\left[\frac{(2-\gamma)K}{2(1-\gamma)}\right] = \lim_{\gamma\to 1}\frac{(2-\gamma)}{2(1-\gamma)}\left\{-1+\left[1+B\frac{(1-\gamma)}{(2-\gamma)^2}\right]^{\frac{1}{2}}\right\}$$

where B = 16(1+x)/x^2 . Since $\gamma\to 1$, we can expand the square root term in a binomial series, so that

$$\lim_{\gamma\to 1}\left[\frac{(2-\gamma)K}{2(1-\gamma)}\right] = \lim_{\gamma\to 1}\frac{(2-\gamma)}{2(1-\gamma)}\left\{-1+\left[1+\frac{1}{2}B\frac{(1-\gamma)}{(2-\gamma)^2}-\text{Order}(1-\gamma)^2\right]\right\}$$

$$= \lim_{\gamma\to 1}\frac{B}{4(2-\gamma)} = \frac{B}{4} = 4\,\frac{1+x}{x^2}$$

Thus $$\lim_{\gamma\to 1}\left[\frac{(2-\gamma)K}{2(1-\gamma)}\right]^{\frac{1+x}{2+x}} = \left(4\,\frac{1+x}{x^2}\right)^{\frac{1+x}{2+x}}$$

Next consider, using Eq. (7-58)

$$\lim_{\gamma\to 1}\left(\frac{1}{L}\right)^{\frac{1+x}{2+x}}$$

$$= \lim_{\gamma\to 1}\left[\frac{(2-\gamma)K}{2(1-\gamma)}\right]^{1/(2+x)} \div \left\{\left[\frac{(2-\gamma)}{2(1-\gamma)}K+1\right]^{\frac{1}{2}}+\left[\frac{1}{2}(2-\gamma)K+1\right]^{\frac{1}{2}}\right\}$$

From the above result, the first square root term is

$$\left[\frac{(2-\gamma)}{2(1-\gamma)}K+1\right]^{\frac{1}{2}} = \left[4\,\frac{1+x}{x^2}+1\right]^{\frac{1}{2}} = \left(\frac{x^2+4x+4}{x^2}\right)^{\frac{1}{2}} = \frac{x+2}{x}$$

From the expression for K in Eq. (7-55), we have that $\lim_{\gamma\to 1} K = 0$, so that

$$\lim_{\gamma \to 1}\left[\frac{1}{2}(2-\gamma)K+1\right]^{\frac{1}{2}} = 1 \qquad \text{Thus}$$

$$\lim_{\gamma \to 1}\left\{\left[\frac{(2-\gamma)}{2(1-\gamma)}K+1\right]^{\frac{1}{2}} + \left[\frac{1}{2}(2-\gamma)K+1\right]^{\frac{1}{2}}\right\} = \frac{x+2}{x} + 1 = \frac{2(1+x)}{x}$$

Combining the above results yields

$$\lim_{\gamma \to 1}\left[\frac{(2-\gamma)K}{2(1-\gamma)L}\right]^{\frac{1+x}{2+x}} = \left(4\,\frac{1+x}{x^2}\right)^{\frac{1+x}{2+x}}\left(4\,\frac{1+x}{x^2}\right)^{\frac{1}{2+x}}\frac{2(1+x)}{x} = \frac{2}{x}$$

so that $\displaystyle \lim_{\gamma \to 1} M_{opt}^{1+x} = \frac{W^{1/2}}{QI_2}\,\frac{2}{x}$

7-16. Using $H_p^{'}(f) = 1$ from Eq. (7-69) for the impulse input and Eq. (7-66) for the raised cosine output, Eq. (7-41) yields

$$I_2 = \int_0^{\infty}\left|H_{out}^{'}(\phi)\right|^2 d\phi = \frac{1}{2}\int_{-\infty}^{\infty}\left|H_{out}^{'}(\phi)\right|^2 d\phi$$

$$= \frac{1}{2}\int_{-\frac{1-\beta}{2}}^{\frac{1-\beta}{2}} d\phi + \int_{\frac{1-\beta}{2}}^{\frac{1+\beta}{2}}\frac{1}{8}\left[1-\sin\left(\frac{\pi\phi}{\beta}-\frac{\pi}{2\beta}\right)\right]^2 d\phi + \int_{-\frac{1+\beta}{2}}^{-\frac{1-\beta}{2}}\frac{1}{8}\left[1-\sin\left(\frac{\pi\phi}{\beta}-\frac{\pi}{2\beta}\right)\right]^2 d\phi$$

Letting $y = \dfrac{\pi\phi}{\beta} - \dfrac{\pi}{2\beta}$ we have

$$I_2 = \frac{1}{2}(1-\beta) + \frac{\beta}{4\pi}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}[1 - 2\sin y + \sin^2 y]\,dy$$

$$= \frac{1}{2}(1-\beta) + \frac{\beta}{4\pi}\left[\pi - 0 + \frac{\pi}{2}\right] = \frac{1}{2}\left(1 - \frac{\beta}{4}\right)$$

Use Eq. (7-42) to find I_3:

$$I_3 = \int_0^{\infty} \left|H'_{out}(\phi)\right|^2 \phi^2 \, d\phi = \frac{1}{2} \int_{-\infty}^{\infty} \left|H'_{out}(\phi)\right|^2 \phi^2 \, d\phi$$

$$= \frac{1}{2} \int_{-\frac{1-\beta}{2}}^{\frac{1-\beta}{2}} \phi^2 \, d\phi \quad + \int_{\frac{1-\beta}{2}}^{\frac{1+\beta}{2}} \frac{1}{8}\left[1\text{-sin}\left(\frac{\pi\phi}{\beta} - \frac{\pi}{2\beta}\right)\right]^2 \phi^2 d\phi \quad + \int_{-\frac{1+\beta}{2}}^{-\frac{1-\beta}{2}} \frac{1}{8}\left[1\text{-sin}\left(\frac{\pi\phi}{\beta} - \frac{\pi}{2\beta}\right)\right]^2 \phi^2 d\phi$$

Letting $y = \frac{\pi\phi}{\beta} - \frac{\pi}{2\beta}$

$$I_3 = \frac{1}{3}\left(\frac{1-\beta}{2}\right)^3 \quad + \frac{\beta}{4\pi}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} [\,1 - 2\sin y + \sin^2 y]\left[\frac{\beta^2 y^2}{\pi^2} + \frac{\beta y}{\pi} + \frac{1}{4}\right] \, dy$$

$$= \frac{1}{3}\left(\frac{1-\beta}{2}\right)^3 \quad + \frac{\beta}{4\pi}\left[\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left(\frac{\beta^2 y^2}{\pi^2} + \frac{1}{4}\right)(1 + \sin^2 y)\, dy - \frac{2\beta}{\pi}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} y \sin y \, dy\right]$$

where only even terms in "y" are nonzero. Using the relationships

$$\int_0^{\frac{\pi}{2}} \sin^2 y \quad dy = \frac{\pi}{4} \quad ; \qquad \int y \sin y dy \quad = -y \cos y + \sin y$$

and $\int x^2 \sin x^2 \, dx \quad = \frac{x^3}{6} \quad - \left(\frac{x^2}{4} - \frac{1}{8}\right) \quad \sin 2x - \frac{x \cos 2x}{4}$

we have

$$I_3 = \frac{1}{3}\left(\frac{1-\beta}{2}\right)^3 \quad + \frac{\beta}{2\pi}\left\{\frac{\beta^2}{\pi^2}\left[\frac{1}{3}\left(\frac{\pi}{2}\right)^3 + \frac{1}{6}\left(\frac{\pi}{2}\right)^3 + \frac{\pi}{8}\right] + \frac{1}{4}\left(\frac{\pi}{2} + \frac{\pi}{4}\right) - \frac{2\beta}{\pi}(1)\right\}$$

$$= \frac{\beta^3}{16}\left(\frac{1}{\pi^2} - \frac{1}{6}\right) - \beta^2\left(\frac{1}{\pi^2} - \frac{1}{8}\right) - \frac{\beta}{32} + \frac{1}{24}$$

7-17. Substituting Eq. (7-64) and (7-66) into Eq. (7-41), with $s^2 = 4\pi^2\alpha^2$ and $\beta = 1$, we have

$$I_2 = \int_0 \left| \frac{H'_{out}(\phi)}{H'_p(\phi)} \right|^2 d\phi = \frac{1}{4}\int_0^1 e^{s^2\phi^2}\left[1\text{-}\sin\left(\pi\phi - \frac{\pi}{2}\right)\right]^2 d\phi$$

$$= \int_0^1 e^{s^2\phi^2}\left(\frac{1 + \cos\pi\phi}{2}\right)^2 d\phi = \int_0^1 e^{s^2\phi^2}\cos^4\left(\frac{\pi\phi}{2}\right) d\phi$$

Letting $x = \frac{\pi\phi}{2}$ yields $\qquad I_2 = \frac{2}{\pi}\int_0^{\frac{\pi}{2}} e^{16\alpha^2x^2}\cos^4 x \;\; dx$

Similarly, using Eqs. (7-64) and (7-66), Eq. (7-42) becomes

$$I_3 = \int_0^1 e^{s^2\phi^2}\cos^4\left(\frac{\pi\phi}{2}\right)\phi^2 d\phi = \left(\frac{2}{\pi}\right)^3 \int_0^{\frac{\pi}{2}} x^2 e^{16\alpha^2 x^2} \cos^4 x \; dx$$

7-18. Plot of I_2 versus α for a gaussian input pulse:

7-19. Plot of I_3 versus α for a gaussian input pulse:

7-20. Consider first $\lim\limits_{\gamma \to 1} K$:

$$\lim_{\gamma\to 1} K = \lim_{\gamma\to 1}\left\{-1+\left[1+16\left(\frac{1+x}{x^2}\right)\frac{1-\gamma}{(2-\gamma)^2}\right]^{\frac{1}{2}}\right\} = -1 + 1 = 0$$

Also $\lim_{\gamma\to 1}(1-\gamma) = 0$. Therefore from Eq. (7-58)

$$\lim_{\gamma\to 1} L = \lim_{\gamma\to 1}\left[\frac{2(1-\gamma)}{(2-\gamma)K}\right]^{1/(1+x)}\left\{\left[\frac{(2-\gamma)}{2(1-\gamma)}K + 1\right]^{\frac{1}{2}} + 1\right\}^{\frac{2+x}{1+x}}$$

Expanding the square root term in K yields

$$\lim_{\gamma\to 1}\frac{2-\gamma}{1-\gamma}K = \lim_{\gamma\to 1}\left(\frac{2-\gamma}{1-\gamma}\right)\left\{-1+\left[1+\frac{16}{2}\left(\frac{1+x}{x^2}\right)\frac{1-\gamma}{(2-\gamma)^2} + \text{order}(1-\gamma)^2\right]\right\}$$

$$= \lim_{\gamma\to 1}\left[\frac{8(1+x)}{x^2}\frac{1}{2-\gamma}\right] = \frac{8(1+x)}{x^2}$$

Therefore

$$\lim_{\gamma\to 1} L = \left[\frac{2x^2}{8(1+x)}\right]^{1/(1+x)}\left\{\left[\frac{4(1+x)}{x^2} + 1\right]^{\frac{1}{2}} + 1\right\}^{\frac{2+x}{1+x}}$$

$$= \left[\frac{2x^2}{8(1+x)}\right]^{1/(1+x)}\left\{\frac{x+2}{x} + 1\right\}^{\frac{2+x}{1+x}} \quad = (1+x)\left(\frac{2}{x}\right)^{\frac{x}{1+x}}$$

7-21. (a) First we need to find L and L'. With x = 0.5 and $\gamma = 0.9$, Eq. (7-56) yields K = 0.7824, so that from Eq. (7-58) we have L = 2.89. With $\varepsilon = 0.1$, we have $\gamma' = \gamma(1 - \varepsilon) = 0.9\gamma = 0.81$. Thus L' = 3.166 from Eq. (7-80). Substituting these values into Eq. (7-83) yields

$$y(\varepsilon) = (1+\varepsilon)\left(\frac{1}{1-\varepsilon}\right)^{\frac{2+x}{1+x}}\frac{L'}{L} = 1.1\left(\frac{1}{.9}\right)^{5/3}\frac{3.166}{2.89} = 1.437$$

Then $10 \log y(\varepsilon) = 10 \log 1.437 = 1.57$ dB

(b) Similarly, for $x = 1.0$, $\gamma = 0.9$, and $\varepsilon = 0.1$, we have $L = 3.15$ and $L' = 3.35$, so that

$$y(\varepsilon) = 1.1\left(\frac{1}{.9}\right)^{3/2} \frac{3.35}{3.15} = 1.37$$

Then $10 \log y(\varepsilon) = 10 \log 1.37 = 1.37$ dB

7-22. (a) First we need to find L and L'. With $x = 0.5$ and $\gamma = 0.9$, Eq. (7-56) yields $K = 0.7824$, so that from Eq. (7-58) we have $L = 2.89$. With $\varepsilon = 0.1$, we have $\gamma' = \gamma(1 - \varepsilon) = 0.9\gamma = 0.81$. Thus $L' = 3.166$ from Eq. (7-80). Substituting these values into Eq. (7-83) yields

$$y(\varepsilon) = (1 + \varepsilon)\left(\frac{1}{1 - \varepsilon}\right)^{\frac{2+x}{1+x}} \frac{L'}{L} = 1.1\left(\frac{1}{.9}\right)^{5/3} \frac{3.166}{2.89} = 1.437$$

Then $10 \log y(\varepsilon) = 10 \log 1.437 = 1.57$ dB

(b) Similarly, for $x = 1.0$, $\gamma = 0.9$, and $\varepsilon = 0.1$, we have $L = 3.15$ and $L' = 3.35$, so that

$$y(\varepsilon) = 1.1\left(\frac{1}{.9}\right)^{3/2} \frac{3.35}{3.15} = 1.37$$

Then $10 \log y(\varepsilon) = 10 \log 1.37 = 1.37$ dB

7-23. Consider using a Si JFET with $I_{gate} = 0.01$ nA. From Fig. 7-14 we have that $\alpha = 0.3$ for $\gamma = 0.9$. At $\alpha = 0.3$, Fig. 7-13 gives $I_2 = 0.543$ and $I_3 = 0.073$. Thus from Eq. (7-86)

$$W_{JFET} = \frac{1}{B}\left[\frac{2(.01\text{nA})}{1.6\times10^{-19}\text{C}} + \frac{4(1.38\times10^{-23}\text{J/K})(300\text{K})}{(1.6\times10^{-19}\text{C})^2 10^5\,\Omega}\right] 0.543$$

$$+ \frac{1}{B}\left[\frac{4(1.38\times10^{-23}\text{ J/K})(300\text{ K})(.7)}{(1.6\times10^{-19}\text{ C})^2(.005\text{ S})(10^5\ \Omega)^2}\right] 0.543$$

$$+ \left[\frac{2\pi(10\text{ pF})}{1.6\times10^{-19}\text{ C}}\right]^2 \frac{4(1.38\times10^{-23}\text{ J/K})(300\text{ K})(.7)}{(.005\text{ S})} 0.073\text{ B}$$

or

$$W_{JFET} \approx \frac{3.51\times10^{12}}{B} + 0.026B$$

and from Eq. (7-92) $\qquad W_{BP} = \dfrac{3.39\times10^{13}}{B} + 0.0049B$

7-24. We need to find b_{on} from Eq. (7-57). From Fig. 7-9 we have Q = 6 for a 10^{-9} BER. To evaluate Eq. (7-57) we also need the values of W and L. With $\gamma = 0.9$, Fig. 7-14 gives $\alpha = 0.3$, so that Fig. 7-13 gives $I_2 = 0.543$ and $I_3 = 0.073$. Thus from Eq. (7-86)

$$W = \frac{3.51\times10^{12}}{B} + 0.026B = 3.51\times10^5 + 2.6\times10^5 = 6.1\times10^5$$

Using Eq. (7-58) to find L yields L = 2.871 at $\gamma = 0.9$ and x = 0.5. Substituting these values into eq. (7-57) we have

$$b_{on} = (6)^{5/3}\,(1.6\times10^{-19}/0.7)\,(6.1\times10^5)^{.5/3}\,(0.543)^{1/1.5}\,2.871 = 7.97\times10^{-17}\text{ J}$$

Thus $\qquad P_r = b_{on}B = (7.97\times10^{-17}\text{ J})(10^7\text{ b/s}) = 7.97\times10^{-10}\text{ W}$

or

$P_r(\text{dBm}) = 10 \log 7.97\times10^{-10} = -61.0\text{ dBm}$

7-25. From Eq. (7-96) the difference in the two amplifier designs is given by

$$\Delta W = \frac{1}{Bq^2}\frac{2k_BT}{R_f}\; I_2 = 3.52\times10^6 \text{ for } I_2 = 0.543 \text{ and } \gamma = 0.9.$$

From Eq. (7-57), the change in sensitivity is found from

$$10 \log\left[\frac{W_{HZ} + \Delta W}{W_{HZ}}\right]^{\frac{x}{2(1+x)}} = 10 \log\left[\frac{1.0 + 3.52}{1.0}\right]^{\frac{.5}{3}} = 10 \log 1.29 = 1.09\text{ dB}$$

7-26. (a) For simplicity, let

$$D = M^{2+x}\left(\frac{\eta}{h\nu}\right) I_2 \quad\text{and}\quad F = \frac{Q}{M}\left(\frac{\eta}{h\nu}\right)$$

so that Eq. (7-54) becomes, for $\gamma = 1$,

$b = F[(Db + W)^{1/2} + W^{1/2}]$

Squaring both sides and rearranging terms gives

$$\frac{b^2}{F^2} - Db - 2W = 2W^{1/2}(Db + W)^{1/2}$$

Squaring again and factoring out a "b^2" term yields

$$b^2 - (2DF^2)b + (F^4D^2 - 4WF^2) = 0$$

Solving this quadratic equation in b yields

$$b = \frac{1}{2}\left[2DF^2 \pm \sqrt{4F^4D^2 - 4F^4D^2 + 16WF^2}\right] = DF^2 + 2F\sqrt{W}$$

(where we chose the "+" sign) $\qquad = \dfrac{h\nu}{\eta}\left(M^xQ^2I_2 + \dfrac{2Q}{M}W^{1/2}\right)$

(b) With the given parameter values, we have

$$b_{on} = 2.286\times10^{-19}\left(39.1M^{0.5} + \frac{1.7\times10^4}{M}\right)$$

The receiver sensitivity in dBm is found from

$$P_r = 10\log\left[b_{on}(50\times10^6\,b/s)\right]$$

Representative values of P_r for several values of M are listed in the table below:

M	**P_r(dBm)**		**M**	**P_r(dBm)**
30	- 50.49		80	-51.92
40	-51.14		90	-51.94
50	-51.52		100	-51.93
60	-51.74		110	-51.90
70	-51.86		120	-51.86

7-27. Using Eq. (E-10) and the relationship

$$\int_0^\infty \frac{1}{1+\left(\frac{x}{a}\right)^2}\,dx = \frac{\pi a}{2}$$

from App. B, we have from Eq. (7-97)

$$B_{HZ} = \frac{1}{(AR)^2}\int_0^\infty \frac{(AR)^2}{1+(2\pi RC)^2 f^2}\,df = \frac{\pi}{2}\,\frac{1}{2\pi RC} = \frac{1}{4RC}$$

where H(0) = AR. Similarly, from Eq. (7-98)

$$B_{TZ} = \frac{1}{1}\int_0^\infty \frac{1}{1+\left(\frac{2\pi RC}{A}\right)^2 f^2}\,df = \frac{\pi}{2}\,\frac{A}{2\pi RC} = \frac{A}{4RC}$$

7-28. To find the optimum value of M for a maximum S/N, differentiate Eq. (7-105) with respect to M and set the result equal to zero:

$$\frac{d(S/N)}{dM} = \frac{(I_p m)^2 M}{2q(I_p+I_D)M^{2+x}B + \frac{4k_B TB}{R_{eq}}F_T}$$

$$- \frac{q(I_p+I_D)\,(2+x)\,M^{1+x}\,B\,(I_p m)^2 M^2}{\left[2q(I_p+I_D)M^{2+x}B + \frac{4k_B TB}{R_{eq}}F_T\right]^2} = 0$$

Solving for M,

$$M_{opt}^{2+x} = \frac{4k_B TBF_T/R_{eq}}{q(I_p+I_D)x}$$

7-29. (a) For computational simplicity, let $K = 4k_BTBF_T/R_{eq}$; substituting M_{opt} from Problem 7-28 into Eq. (7-105) gives

$$\frac{S}{N} = \frac{\frac{1}{2}(I_p m)^2 M_{opt}^2}{2q(I_p+I_D)M_{opt}^{2+x} B + KB} = \frac{\frac{1}{2}(I_p m)^2 \left[\frac{K}{q(I_p+I_D)x}\right]^{\frac{2}{2+x}}}{\frac{2q(I_p+I_D)K}{q(I_p+I_D)x} B + KB}$$

$$= \frac{xm^2 I_p^2}{2B(2+x)\,[q(I_p+I_D)x]^{2/(2+x)}} \left(\frac{R_{eq}}{4k_B T F_T}\right)^{x/(2+x)}$$

(b) If $I_p >> I_D$, then

$$\frac{S}{N} = \frac{xm^2 I_p^2}{2B(2+x)\,(qx)^{2/(2+x)}\, I_p^{2/(2+x)}} \left(\frac{R_{eq}}{4k_B T F_T}\right)^{x/(2+x)}$$

$$= \frac{m^2}{2Bx(2+x)} \left[\frac{(xI_p)^{2(1+x)}}{q^2(4k_B T F_T/R_{eq})^x}\right]^{1/(2+x)}$$

7-30. Substituting $I_p = R_0 P_r$ into the S/N expression in Prob. 7-29a,

$$\frac{S}{N} = \frac{xm^2}{2B(2+x)} \frac{(R_0 P_r)^2}{[q(R_0 P_r + I_D)x]^{2/(2+x)}} \left(\frac{R_{eq}}{4k_B T F_T}\right)^{x/(2+x)}$$

$$= \frac{(0.8)^2 (0.5\ \mathrm{A/W})^2 P_r^2}{2(5\times 10^6/\mathrm{s})\ 3\ [1.6\times 10^{-19}\mathrm{C}(0.5P_r + 10^{-8})\ \mathrm{A}]^{2/3}} \left(\frac{10^4\ \Omega/\mathrm{J}}{1.656\times 10^{-20}}\right)^{1/3}$$

$$= \frac{1.530\times 10^{12} P_r^2}{(0.5P_r + 10^{-8})^{2/3}}$$ where P_r is in watts.

We want to plot 10 log (S/N) versus 10 log $\frac{P_r}{1 \text{ mW}}$. Representative values are shown in the following table:

P_r (W)	P_r (dBm)	S/N	10 log (S/N) (dB)
2×10^{-9}	- 57	1.237	0.92
4×10^{-9}	- 54	4.669	6.69
1×10^{-8}	- 50	25.15	14.01
4×10^{-8}	- 44	253.5	24.04
1×10^{-7}	- 40	998.0	29.99
1×10^{-6}	- 30	2.4×10^{4}	43.80
1×10^{-5}	- 20	5.2×10^{5}	57.18
1×10^{-4}	- 10	1.13×10^{7}	70.52