Problem Solutions for Chapter 3

3-1.

$$\alpha(\mathrm{dB/km}) = \frac{10}{z}\ \log\left[\frac{P(0)}{P(z)}\right] = \frac{10}{z}\ \log\left(e^{\alpha_p z}\right)$$

$$= 10\alpha_p \log\ e = 4.343\ \alpha_p(1/\mathrm{km})$$

3-2. Since the attenuations are given in dB/km, first find the power levels in dBm for 100 μW and 150 μW. These are, respectively,

P(100 μW) = 10 log (100 μW/1.0 mW) = 10 log (0.10) = - 10.0 dBm

P(150 μW) = 10 log (150 μW/1.0 mW) = 10 log (0.15) = - 8.24 dBm

(a) At 8 km we have the following power levels:

P_{1300}(8 km) = - 8.2 dBm – (0.6 dB/km)(8 km) = - 13.0 dBm = 50 μW

P_{1550}(8 km) = - 10.0 dBm – (0.3 dB/km)(8 km) = - 12.4 dBm = 57.5 μW

(b) At 20 km we have the following power levels:

P_{1300}(20 km) = - 8.2 dBm – (0.6 dB/km)(20 km) = - 20.2 dBm = 9.55 μW

P_{1550}(20 km) = - 10.0 dBm – (0.3 dB/km)(20 km) = - 16.0 dBm = 25.1 μW

3-3. From Eq. (3-1c) with $P_{out} = 0.45\ P_{in}$

α = (10/3.5 km) log (1/0.45) = 1.0 dB/km

3-4. (a) $P_{in} = P_{out}\ 10^{\alpha L/10} = (0.3\ \mu W)\ 10^{1.5(12)/10} = 18.9\ \mu W$

(b) $P_{in} = P_{out}\ 10^{\alpha L/10} = (0.3\ \mu W)\ 10^{2.5(12)/10} = 300\ \mu W$

3-5. With λ in Eqs. (3-2b) and (3-3) given in μm, we have the following representative points for α_{uv} and α_{IR}:

λ (μm)	α_{uv}	α_{IR}
0.5	20.3	--
0.7	1.44	--
0.9	0.33	--
1.2	0.09	2.2×10^{-6}
1.5	0.04	0.0072
2.0	0.02	23.2
3.0	0.009	7.5×10^{4}

3-6. From Eq. (3-4a) we have

$$\alpha_{scat} = \frac{8\pi^3}{3\lambda^4}(n^2-1)^2 k_B T_f \beta_T$$

$$= \frac{8\pi^3}{3(0.63\ \mu\text{m})^4}\left[(1.46)^2-1\right]^2 (1.38\times10^{-16}\ \text{dyne-cm/K})(1400\ \text{K})$$

$$\times(6.8\times10^{-12}\ \text{cm}^2\text{/dyne})$$

$$= 0.883\ \text{km}^{-1}$$

To change to dB/km, multiply by 10 log e = 4.343: α_{scat} = 3.8 dB/km

From Eq. (3-4b):

$$\alpha_{scat} = \frac{8\pi^3}{3\lambda^4} n^8 p^2 k_B T_f \beta_T = 1.16\ \text{km}^{-1} = 5.0\ \text{dB/km}$$

3-8. Plot of Eq. (3-7).

3-9. Plot of Eq. (3-9).

3-10. From Fig. 2-22, we make the estimates given in this table:

νm	P_{clad}/P	$\alpha_{\nu m} = \alpha_1 + (\alpha_2 - \alpha_1)P_{clad}/P$	$5 + 10^3 P_{clad}/P$
01	0.02	3.0 + 0.02	5 + 20 = 25
11	0.05	3.0 + 0.05	5 + 50 = 55
21	0.10	3.0 + 0.10	5 + 100 = 105
02	0.16	3.0 + 0.16	5 + 160 = 165
31	0.19	3.0 + 0.19	5 + 190 = 195
12	0.31	3.0 + 0.31	5 + 310 = 315

3-11. (a) We want to solve Eq. (3-12) for α_{gi}. With $\alpha = 2$ in Eq. (2-78) and letting

$$\Delta = \frac{n^2(0) - n_2^2}{2n^2(0)}$$

we have

$$\alpha(r) = \alpha_1 + (\alpha_2 - \alpha_1)\,\frac{n^2(0) - n^2(r)}{n^2(0) - n_2^2} = \alpha_1 + (\alpha_2 - \alpha_1)\,\frac{r^2}{a^2}$$

Thus

$$\alpha_{gi} = \frac{\int_0^\infty \alpha(r)\;p(r)\;r\;dr}{\int_0^\infty p(r)\;r\;dr} = \alpha_1 + \frac{(\alpha_2 - \alpha_1)}{a^2}\;\frac{\int_0^\infty \exp\left(-Kr^2\right) r^3 dr}{\int_0^\infty \exp\left(-Kr^2\right) r\;dr}$$

To evaluate the integrals, let $x = Kr^2$, so that $dx = 2Krdr$. Then

$$\frac{\int_0^\infty \exp\left(-Kr^2\right) r^3 dr}{\int_0^\infty \exp\left(-Kr^2\right) r\;dr} = \frac{\frac{1}{2K^2}\int_0^\infty e^{-x}\;x\;dx}{\frac{1}{2K}\int_0^\infty e^{-x}\;dx} = \frac{\frac{1}{K}\;1!}{0!} = \frac{1}{K}$$

Thus $\alpha_{gi} = \alpha_1 + \dfrac{(\alpha_2 - \alpha_1)}{Ka^2}$

(b) $p(a) = 0.1\,P_0 = P_0\,e^{-Ka^2}$ yields $e^{Ka^2} = 10$.

From this we have $Ka^2 = \ln 10 = 2.3$. Thus

$$\alpha_{gi} = \alpha_1 + \frac{(\alpha_2 - \alpha_1)}{2.3} = 0.57\alpha_1 + 0.43\alpha_2$$

3-12. With λ in units of micrometers, we have

$$n = \left\{1 + \frac{196.98}{(13.4)^2 - (1.24/\lambda)^2}\right\}^{1/2}$$

To compare this with Fig. 3-12, calculate three representative points, for example, λ = 0.2, 0.6, and 1.0 μm. Thus we have the following:

Wavelength λ	**Calculated n**	**n from Fig. 3-12**
0.2 μm	1.548	1.550
0.6 μm	1.457	1.458
1.0 μm	1.451	1.450

3-13. (a) From Fig. 3-13, $\frac{d\tau}{d\lambda} \approx$ 80 ps/(nm-km) at 850 nm. Therefore, for the LED we have from Eq. (3-20)

$$\frac{\sigma_{mat}}{L} = \frac{d\tau}{d\lambda}\,\sigma_\lambda = [80 \text{ ps/(nm-km)}](45 \text{ nm}) = 3.6 \text{ ns/km}$$

For a laser diode,

$$\frac{\sigma_{mat}}{L} = [80 \text{ ps/(nm-km)}](2 \text{ nm}) = 0.16 \text{ ns/km}$$

(b) From Fig. 3-13, $\frac{d\tau_{mat}}{d\lambda}$ = 22 ps/(nm-km)

Therefore, $D_{mat}(\lambda)$ = [22 ps/(nm-km)](75 nm) = 1.65 ns/km

3-14. (a) Using Eqs. (2-48), (2-49), and (2-57), Eq. (3-21) becomes

$$b = 1 - \left(\frac{ua}{V}\right)^2 = 1 - \frac{u^2a^2}{u^2a^2 + w^2a^2} = \frac{w^2}{u^2 + w^2}$$

$$= \frac{\beta^2 - k^2n_2^2}{k^2n_1^2 - \beta^2 + \beta^2 - k^2n_2^2} = \frac{\beta^2/k^2 - n_2^2}{n_1^2 - n_2^2}$$

(b) Expand b as $b = \frac{(\beta/k + n_2)(\beta/k - n_2)}{(n_1 + n_2)(n_1 - n_2)}$

Since $n_2 < \beta/k < n_1$, let $\beta/k = n_1(1 - \delta)$ where $0 < \delta < \Delta << 1$. Thus,

$$\frac{\beta/k+n_2}{n_1+n_2} = \frac{n_1(1-\delta)+n_2}{n_1+n_2} = 1 - \frac{n_1}{n_1+n_2}\delta$$

Letting $n_2 = n_1(1 - \Delta)$ then yields

$$\frac{\beta/k+n_2}{n_1+n_2} = 1 - \frac{\delta}{2-\Delta} \approx 1 \text{ since } \frac{\delta}{2-\Delta} << 1$$

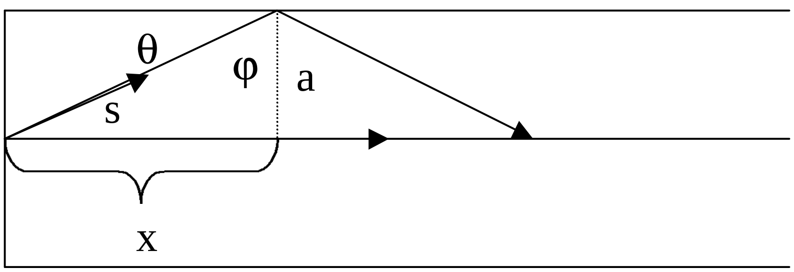
Therefore, $b \approx \frac{\beta/k-n_2}{n_1-n_2}$ or $\beta = k[bn_1\Delta + n_2]$

From $n_2 = n_1(1 - \Delta)$ we have

$$n_1 = n_2(1-\Delta)^{-1} = n_2(1+\Delta+\Delta^2+...) \approx n_2(1+\Delta)$$

Therefore, $\beta = k[b\, n_2(1+\Delta)\Delta + n_2] \approx k\, n_2(b\Delta + 1)$

3-16. The time delay between the highest and lowest order modes can be found from the travel time difference between the two rays shown here.



The travel time of each ray is given by

$$\sin\phi = \frac{x}{s} = \frac{n_2}{n_1} = \frac{n_1(1-\Delta)}{n_1} = (1-\Delta)$$

The travel time of the highest order ray is thus

$$T_{max} = \frac{n_1}{c}\left[s\left(\frac{L}{x}\right)\right] = \frac{n_1 L}{c}\frac{1}{1-\Delta}$$

For the axial ray the travel time is $T_{min} = \frac{Ln_1}{c}$

Therefore

$$T_{min} - T_{max} = \frac{Ln_1}{c}\left(\frac{1}{1-\Delta} - 1\right) = \frac{Ln_1}{c}\frac{\Delta}{1-\Delta} \approx \frac{Ln_1\Delta}{c}$$

3-17. Since $n_2 = n_1(1-\Delta)$, we can rewrite the equation as

$$\frac{\sigma_{mod}}{L} = \frac{n_1\Delta}{c}\left(1 - \frac{\pi}{V}\right)$$

where the first tern is Equation (3-30). The difference is then given by the factor

$$1 - \frac{\pi}{V} = 1 - \frac{\pi\lambda}{2a}\frac{1}{\left(n_1^2 - n_2^2\right)^{1/2}} \approx 1 - \frac{\pi\lambda}{2a}\frac{1}{n_1\sqrt{2\Delta}}$$

At 1300 nm this factor is $1 - \frac{\pi(1.3)}{2(62.5)}\frac{1}{1.48\sqrt{2(0.015)}} = 1 - 0.127 = 0.873$

3-18. For $\varepsilon = 0$ and in the limit of $\alpha \to \infty$ we have

$$C_1 = 1, \quad C_2 = \frac{3}{2}, \quad \frac{\alpha}{\alpha+1} = 1, \quad \frac{\alpha+2}{3\alpha+2} = \frac{1}{3}, \quad \frac{\alpha+1}{2\alpha+1} = \frac{1}{2},$$

and $\frac{(\alpha+1)^2}{(5\alpha+2)(3\alpha+2)} = \frac{1}{15}$

Thus Eq. (3-41) becomes

$$\sigma_{intermodal} = \frac{Ln_1\Delta}{2\sqrt{3}c}\left(1 + 3\Delta + \frac{12}{5}\Delta^2\right)^{1/2} \approx \frac{Ln_1\Delta}{2\sqrt{3}c}$$

3-19. For $\varepsilon = 0$ we have that $\alpha = 2(1 - \frac{6}{5}\Delta)$. Thus C_1 and C_2 in Eq. (3-42) become (ignoring small terms such as Δ^3, Δ^4, ...)

$$C_1 = \frac{\alpha-2}{\alpha+2} = \frac{2\left(1-\frac{6}{5}\Delta\right)-2}{2\left(1-\frac{6}{5}\Delta\right)+2} = \frac{-\frac{3}{5}\Delta}{1-\frac{3}{5}\Delta} \approx -\frac{3}{5}\Delta\left(1+\frac{3}{5}\Delta\right)$$

$$C_2 = \frac{3\alpha-2}{2(\alpha+2)} = \frac{3\left[2\left(1-\frac{6}{5}\Delta\right)\right]-2}{2\left[2\left(1-\frac{6}{5}\Delta\right)+2\right]} = \frac{1-\frac{9}{5}\Delta}{2\left(1-\frac{3}{5}\Delta\right)}$$

Evaluating the factors in Eq. (3-41) yields:

(a) $C_1^2 \approx \frac{9}{25}\ \Delta^2$

(b) $$\frac{4C_1C_2(\alpha+1)\Delta}{2\alpha+1} = \frac{4\Delta\left(-\frac{3}{5}\Delta\right)\left(1-\frac{9}{5}\Delta\right)\left[2\left(1-\frac{6}{5}\Delta\right)+1\right]}{\left(1-\frac{3}{5}\Delta\right)2\left(1-\frac{3}{5}\Delta\right)\left[4\left(1-\frac{6}{5}\Delta\right)+1\right]}$$

$$= \frac{-\frac{18\Delta}{\quad}\left(1-\frac{11}{\quad}\Delta+\frac{18}{25}\quad\right)}{5\left(1\quad\frac{}{5}\Delta\right)^2\left(\quad\frac{24}{25}\Delta\right)} \qquad \frac{18}{25}\ \Delta^2$$

(c) $$\frac{16\Delta^2C_2^2(\alpha+1)^2}{(5\alpha+2)(3\alpha+2)}$$

$$= \frac{16\Delta^2\left(1-\frac{9}{5}\Delta\right)^2\left[2(1-\frac{6}{5}\Delta)+1\right]^2}{4\left(1-\frac{3}{5}\Delta\right)^2\left[10(1-\frac{6}{5}\Delta)+2\right]\left[6(1-\frac{6}{5}\Delta)+2\right]}$$

$$= \frac{16\Delta^2\left(1-\frac{9}{5}\Delta\right)^2 9\left(1-\frac{4}{5}\Delta\right)^2}{96(1-\Delta)(1-\frac{9}{10}\Delta)4\left(1-\frac{3}{5}\Delta\right)^2} \approx \frac{9}{24}\ \Delta^2$$

Therefore,

$$\sigma_{\text{int er mod al}} = \frac{Ln_1\Delta}{2c}\left(\frac{\alpha}{\alpha+1}\right)\left(\frac{\alpha+2}{3\alpha+2}\right)^{1/2}\left(\frac{9}{25}\Delta^2-\frac{18}{25}\Delta^2+\frac{9}{24}\Delta^2\right)^{1/2}$$

$$= \frac{Ln_1\Delta^2}{2c}\frac{2(1-\frac{6}{5}\Delta)}{\left[2(1-\frac{6}{5}\Delta)+1\right]}\left[\frac{2(1-\frac{6}{5}\Delta)+2}{6(1-\frac{6}{5}\Delta)+2}\right]^{1/2}\frac{3}{10\sqrt{6}} \approx \frac{n_1\Delta^2L}{20\sqrt{3}c}$$

3-20. We want to plot Eq. (3-30) as a function of σ_λ, where $\sigma_{intermodal}$ and $\sigma_{intramodal}$ are given by Eqs. (3-41) and (3-45). For $\varepsilon = 0$ and $\alpha = 2$, we have $C_1 = 0$ and $C_2 = 1/2$. Since $\sigma_{intermodal}$ does not vary with σ_λ, we have

$$\frac{\sigma_{intermodal}}{L} = \frac{N_1\Delta}{2c}\left(\frac{\alpha}{\alpha+2}\right)\left(\frac{\alpha+2}{3\alpha+2}\right)^{1/2}\frac{4\Delta C_2(\alpha+1)}{\sqrt{(5\alpha+2)(3\alpha+2)}} = 0.070 \text{ ns/km}$$

With $C_1 = 0$ we have from Eq. (3-45)

$$\sigma_{intramodal} = \frac{1}{c}\frac{\sigma_\lambda}{\lambda}\left(-\lambda^2\frac{d^2n_1}{d\lambda^2}\right) = \begin{cases} 0.098\sigma_\lambda \text{ ns/km at } 850 \text{ nm} \\ 1.026\times10^{-2}\sigma_\lambda \text{ ns/km at } 1300 \text{ nm}\end{cases}$$

3-21. Using the same parameter values as in Prob. 3-18, except with $\Delta = 0.001$, we have from Eq. (3-41) $\sigma_{intermodal}$ /L = 7 ps/km, and from Eq. (3.45)

$$\frac{\sigma_{intramodal}}{L} = \begin{cases} 0.098\sigma_\lambda \text{ ns/km at } 850 \text{ nm} \\ 0.0103\sigma_\lambda \text{ ns/km at } 1300 \text{ nm}\end{cases}$$

The plot of $\frac{\sigma}{L} = \frac{1}{L}\left(\sigma_{inter}^2 + \sigma_{intra}^2\right)^{1/2}$ vs σ_λ :

3-22. Substituting Eq. (3-34) into Eq. (3-33)

$$\tau_g = \frac{L}{c}\frac{d\beta}{dk} = \frac{L}{c}\frac{1}{2\beta}\left\{2kn_1^2 + 2k^2n_1\frac{dn_1}{dk}\right.$$

$$- 2\left(\frac{\alpha+2}{\alpha}\frac{m}{a^2}\right)^{\frac{\alpha}{\alpha+2}}\frac{2}{\alpha+2}\left(n_1^2k^2\Delta\right)^{\frac{2}{\alpha+2}-1}$$

$$\left.\times\ \Delta\left[2k^2n_1\frac{dn_1}{dk} + 2kn_1^2 + \frac{n_1^2k^2}{\Delta}\frac{d\Delta}{dk}\right]\right\}$$

$$= \frac{L}{c}\frac{kn_1}{\beta}\left[N_1 - \frac{4\Delta}{\alpha+2}\left(\frac{\alpha+2}{\alpha}\frac{m}{a^2}\frac{1}{n_1^2k^2\Delta}\right)^{\frac{\alpha}{\alpha+2}}\left(N_1 + \frac{n_1k}{2\Delta}\frac{d\Delta}{dk}\right)\right]$$

$$= \frac{LN_1}{c}\frac{kn_1}{\beta}\left[1-\frac{4\Delta}{\alpha+2}\left(\frac{m}{M}\right)^{\frac{\alpha}{\alpha+2}}\left(1+\frac{\varepsilon}{4}\right)\right]$$

with $N_1 = n_1 + k\,\frac{dn_1}{dk}$ and where M is given by Eq. (2-97) and ε is defined in Eq. (3-36b).

3-23. From Eq. (3-39), ignoring terms of order Δ^2,

$$\lambda\frac{d\tau}{d\lambda} = \frac{L}{c}\frac{dN_1}{d\lambda}\left[1+\frac{\alpha-\varepsilon-2}{\alpha+2}\left(\frac{m}{M}\right)^{\frac{\alpha}{\alpha+2}}\right]$$

$$+\ \frac{LN_1}{c}\frac{\alpha-\varepsilon-2}{\alpha+2}\frac{d}{d\lambda}\left[\Delta\left(\frac{m}{M}\right)^{\frac{\alpha}{\alpha+2}}\right]$$

where

$N_1 = n_1 - \lambda\,\frac{dn_1}{d\lambda}$ and $M = \frac{\alpha}{\alpha+2}\,a^2k^2n_1^2\Delta$

(a) $$\frac{dN_1}{d\lambda} = \frac{d}{d\lambda}\left(n_1-\lambda\frac{dn_1}{d\lambda}\right) = -\ \lambda\,\frac{d^2n_1}{d\lambda^2}$$

Thus ignoring the term involving $\Delta\,\frac{d^2n_1}{d\lambda^2}$, the first term in square brackets becomes $-\frac{L}{c}\lambda^2\,\frac{d^2n_1}{d\lambda^2}$

(b) $$\frac{d}{d\lambda}\left[\Delta\left(\frac{m}{M}\right)^{\frac{\alpha}{\alpha+2}}\right] = m^{\frac{\alpha}{\alpha+2}}\left[\frac{d\Delta}{d\lambda}\left(\frac{1}{M}\right)^{\frac{\alpha}{\alpha+2}} + \Delta\frac{-\alpha}{\alpha+2}\frac{dM}{d\lambda}\left(\frac{1}{M}\right)^{\frac{\alpha}{\alpha+2}+1}\right]$$

(c) $$\frac{dM}{d\lambda} = \frac{\alpha}{\alpha+2}\,a^2\,\frac{d}{d\lambda}\left(k^2n_1^2\Delta\right)$$

$$= \frac{\alpha}{\alpha+2}\,a^2\left(\frac{d\Delta}{d\lambda}k^2n_1^2 + 2k^2\Delta n_1\frac{dn_1}{d\lambda} + 2kn_1^2\Delta\frac{dk}{d\lambda}\right)$$

Ignoring $\frac{d\Delta}{d\lambda}$ and $\frac{dn_1}{d\lambda}$ terms yields

$$\frac{dM}{d\lambda} = \frac{2\alpha}{\alpha+2}\, a^2k^2n_1^2\Delta\left(-\frac{1}{\lambda}\right) = -\frac{2M}{\lambda} \qquad \text{so that}$$

$$\frac{d}{d\lambda}\left[\Delta\left(\frac{m}{M}\right)^{\frac{\alpha}{\alpha+2}}\right] = \frac{\Delta}{\lambda}\frac{2\alpha}{\alpha+2}\left(\frac{m}{M}\right)^{\frac{\alpha}{\alpha+2}}. \qquad \text{Therefore}$$

$$\lambda\frac{d\tau}{d\lambda} = -\frac{L}{c}\lambda^2\frac{d^2n_1}{d\lambda^2} + \frac{LN_1}{c}\frac{\alpha-\varepsilon-2}{\alpha+2}\frac{2\alpha\Delta}{\alpha+2}\left(\frac{m}{M}\right)^{\frac{\alpha}{\alpha+2}}$$

3-24. Let $a = \lambda^2\frac{d^2n_1}{d\lambda^2}$; $b = N_1C_1\Delta\frac{2\alpha}{\alpha+2}$; $\gamma = \frac{\alpha}{\alpha+2}$

Then from Eqs. (3-32), (3-43), and (3-44) we have

$$\sigma^2_{\text{intramodal}} = L^2\left(\frac{\sigma_\lambda}{\lambda}\right)^2\frac{1}{M}\sum_{m=0}^{M}\left(\lambda\frac{d\tau_g}{d\lambda}\right)^2$$

$$= \left(\frac{L}{c}\right)^2\left(\frac{\sigma_\lambda}{\lambda}\right)^2\frac{1}{M}\sum_{m=0}^{M}\left[-a+b\left(\frac{m}{M}\right)^{\gamma}\right]^2$$

$$\approx \left(\frac{L}{c}\right)^2\left(\frac{\sigma_\lambda}{\lambda}\right)^2\frac{1}{M}\int_0^M\left[-a+b\left(\frac{m}{M}\right)^{\gamma}\right]^2 dm$$

$$= \left(\frac{L}{c}\right)^2\left(\frac{\sigma_\lambda}{\lambda}\right)^2\frac{1}{M}\int_0^M\left[a^2-2ab\left(\frac{m}{M}\right)^{\gamma}+b^2\left(\frac{m}{M}\right)^{2\gamma}\right] dm$$

$$= \left(\frac{L}{c}\right)^2\left(\frac{\sigma_\lambda}{\lambda}\right)^2\left[a^2-\frac{2ab}{\gamma+1}+\frac{b^2}{2\gamma+1}\right]$$

$$= \left(\frac{L}{c}\right)^2\left(\frac{\sigma_\lambda}{\lambda}\right)^2\left[\left(-\lambda^2\frac{d^2n_1}{d\lambda^2}\right)^2\right.$$

$$- \quad 2\left(\lambda^2 \frac{d^2 n_1}{d\lambda^2}\right) N_1 C_1 \Delta \quad \frac{\alpha}{\alpha+1} + (N_1 C_1 \Delta)^2 \frac{4\alpha^2}{(\alpha+2)(3\alpha+2)} \Bigg]$$

3-25. Plot of Eq. (3-57).

3-26. (a) $D = (\lambda - \lambda_0) S_0 = -50(0.07) = -3.5 \ \text{ps}/(\text{nm}-\text{km})$

(b) $D = \frac{1500(0.09)}{4}\left[1 - \left(\frac{1310}{1500}\right)^4\right] = 14.1 \ \text{ps}/(\text{nm}-\text{km})$

3-27. (a) From Eq. (3-48)

$$\frac{\sigma_{step}}{L} = \frac{n_1 \Delta}{2\sqrt{3}\ c} = \frac{1.49(0.01)}{2\sqrt{3}\ (3\times 10^8)} = 14.4 \ \text{ns}/\text{km}$$

(b) From Eq. (3-47)

$$\frac{\sigma_{opt}}{L} = \frac{n_1 \Delta^2}{20\sqrt{3}\ c} = \frac{1.49(0.01)^2}{20\sqrt{3}\ (3\times 10^8)} = 14.3 \ \text{ps}/\text{km}$$

(c) 3.5 ps/km

3-28. (a) From Eq. (3-29)

$$\sigma_{mod} = T_{max} - T_{min} = \frac{n_1 \Delta L}{c} = \frac{(1.49)(0.01)(5\times 10^3\,\text{m})}{3\times 10^8\,\text{m}/\text{s}} = 248 \ \text{ns}$$

(b) From Eq. (3-48)

$$\sigma_{step} = \frac{n_1 \Delta L}{2\sqrt{3}\ c} = \frac{248}{2\sqrt{3}} = 71.7 \ \text{ns}$$

(c) $B_T = \frac{0.2}{\sigma_{step}} = 2.8 \ \text{Mb}/\text{s}$

(d) $B_T \cdot L = (2.8 \ \text{MHz})(5 \ \text{km}) = 13.9 \ \text{MHz}\cdot\text{km}$

3-29. For $\alpha = 0.95\alpha_{opt}$, we have

$$\frac{\sigma_{int\,er}(\alpha \neq \alpha_{opt})}{\sigma_{int\,er}(\alpha = \alpha_{opt})} = \frac{(\alpha - \alpha_{opt})}{\Delta(\alpha + 2)} = -\frac{0.05}{(0.015)(1.95)} = -170\%$$

For $\alpha = 1.05\alpha_{opt}$, we have

$$\frac{\sigma_{int\,er}(\alpha \neq \alpha_{opt})}{\sigma_{int\,er}(\alpha = \alpha_{opt})} = \frac{(\alpha - \alpha_{opt})}{\Delta(\alpha + 2)} = +\frac{0.05}{(0.015)(2.05)} = +163\%$$