Problem Solutions for Chapter 11

11-1. (a) From Eq. (11-2), the pumping rate is

$$R_p = \frac{I}{qwdL} = \frac{100\ \text{mA}}{(1.6\times10^{-19}\ \text{C})(5\ \mu\text{m})(0.5\ \mu\text{m})(200\ \mu\text{m})}$$

$$= 1.25\times10^{27}\ (\text{electrons}/\text{cm}^3)/\text{s}$$

(b) From Eq. (11-8), the maximum zero-signal gain is

$$g_0 = 0.3(1\times10^{-20}\text{m}^2)(1\ \text{ns})\left[1.25\times10^{33}\ (\text{electrons}/\ \text{m}^3)/\text{s} - \frac{1.0\times10^{24}/\text{m}^3}{1\ \text{ns}}\right]$$

$$= 750\ \text{m}^{-1} = 7.5\ \text{cm}^{-1}$$

(c) From Eq. (11-7), the saturation photon density is

$$N_{ph;sat} = \frac{1}{0.3\ (1\times10^{-20}\text{m}^2)\left(2\times10^8\ \text{m}/\text{s}\right)(1\ \text{ns})} = 1.67\times10^{15}\ \text{photons}/\text{cm}^3$$

(d) From Eq. (11-4), the photon density is

$$N_{ph} = \frac{P_{in}\ \lambda}{v_g\ hc\ (wd)} = 1.32\times10^{10}\ \text{photons}/\text{cm}^3$$

11-2. Carrying out the integrals in Eq. (11-14) yields

$$g_0L = \ln\frac{P(L)}{P(0)} + \frac{P(L)-P(0)}{P_{amp,sat}}$$

Then with $P(0) = P_{in}$, $P(L) = P_{out}$, $G = P_{out}/P_{in}$, and $G_0 = \exp(g_0L)$ from Eq. (11-10), we have

$$\ln\ G_0 = g_0L = \ln\ G + \frac{GP_{in}}{P_{amp,sat}} - \frac{P_{in}}{P_{amp,sat}} = \ln\ G + (1-G)\frac{P_{in}}{P_{amp,sat}}$$

Rearranging terms in the leftmost and rightmost parts then yields Eq. (11-15).

11-3. Plots of amplifier gains.

11-4. Let $G = G_0/2$ and $P_{in} = P_{out}/G = 2P_{out,sat}/G_0$. Then Eq. (11-15) yields

$$\frac{G_0}{2} = 1 + \frac{G_0 P_{amp.sat}}{2P_{out.sat}} \ln\ 2$$

Solving for $P_{out,sat}$ and with $G_0 >> 1$, we have

$$P_{out\ sat} = \frac{G_0 \ln\ 2}{(G_0 - 2)}\ P_{amp.sat} \approx (\ln\ 2)\ P_{amp.sat} = 0.693\ P_{amp.sat}$$

11-5. From Eq. (11-10), at half the amplifier gain we have

$$G = \frac{1}{2} G_0 = \frac{1}{2} \exp(g_0 L) = \exp(gL)$$

Taking the logarithm and substituting into the equation given in the problem,

$$g = g_0 - \frac{1}{L} \ln\ 2 = \frac{g_0}{1 + 4(\nu_{3dB} - \nu_0)^2 / (\Delta\nu)^2}$$

From this we can find that

$$\frac{2(\nu_{3dB} - \nu_0)}{\Delta\nu} = \left[\frac{g_0}{g_0 - \frac{1}{L}\ln\ 2} - 1\right]^{1/2} = \left[\frac{1}{g_0 L / \ln\ 2 - 1}\right]^{1/2} = \frac{1}{\left[\log_2\left(\frac{G_0}{2}\right)\right]^{1/2}}$$

11-6. Since

$$\ln\ G = g(\lambda)L = g_0 \exp\ \left[-(\lambda - \lambda_0)^2 / 2(\Delta\lambda)^2\right] = \ln\ G_0\ \exp\ \left[-(\lambda - \lambda_0)^2 / 2(\Delta\lambda)^2\right]$$

we have

$$\ln\left[\frac{\ln\ G_0}{\ln\ G}\right] = \frac{(\lambda - \lambda_0)^2}{2(\Delta\lambda)^2}$$

The FWHM is given by $2(\lambda - \lambda_0)$, so that from the above equation, with the 3-dB gain G = 27 dB being 3 dB below the peak gain, we have

$$\text{FWHM} = 2|\lambda - \lambda_0| = 2\left[2\ \ln\left(\frac{\ln\ G_0}{\ln\ G}\right)\right]^{1/2}\Delta\lambda$$

$$= 2\left[2\ \ln\left(\frac{\ln\ 30}{\ln\ 27}\right)\right]^{1/2}\Delta\lambda = 0.50\ \Delta\lambda$$

which is the expected result for a gaussian gain profile.

11-7. From Eq. (11-17), the maximum PCE is given by

$$\text{PCE} \le \frac{\lambda_p}{\lambda_s} = \frac{980}{1545} = 63.4\%$$ for 980-nm pumping, and by

$$\text{PCE} \le \frac{\lambda_p}{\lambda_s} = \frac{1475}{1545} = 95.5\%$$ for 1475-nm pumping

11-8. (a) 27 dBm = 501 mW and 2 dBm = 1.6 mW.

Thus the gain is

$$G = 10\ \log\left(\frac{501}{1.6}\right) = 10\ \log\ 313 = 25\ \text{dB}$$

(b) From Eq. (11-19),

$313 \le 1 + \frac{980}{1542}\ \frac{P_{p,in}}{P_{s,\,in}}$. With a 1.6-mW input signal, the pump power needed is

$$P_{p,in} \ge \frac{312(1542)}{980}\ (1.6\ \text{mW}) = 785\ \text{mW}$$

11-9. (a) Noise terms:

From Eq. (6-17), the thermal noise term is

$$\sigma_T^2 = \frac{4k_BT}{R_L}B = \frac{4\ (1.38\times10^{-23}\ \text{J/K})(293\ \text{K})}{1000\ \Omega}\ 1\ \text{GHz} = 1.62\times10^{-14}\ \text{A}^2$$

From Eq. (11-26), we have

$$\sigma^2_{shot-s} = 2q\mathcal{R}\, GP_{s,in}B$$

$$= 2\left(1.6\times10^{-19}\ \text{C}\right)(0.73\ \text{A/W})(100)\left(1\ \mu\text{W}\right)1\ \text{GHz}$$

$$= 2.34\times10^{-14}\ \text{A}^2$$

From Eqs. (11-26) and (11-24), we have

$$\sigma^2_{shot-ASE} = 2q\mathcal{R}\, S_{ASE}\Delta\nu_{opt}B = 2q\mathcal{R}\,\frac{hc}{\lambda}\, n_{sp}G\Delta\nu_{opt}B$$

$$= 2\left(1.6\times10^{-19}\ \text{C}\right)(.73\ \text{A/W})\left(6.626\times10^{-34}\ \text{J/K}\right)$$

$$\times\left(3\times10^{8}\,\text{m/s}\right)2(100)(3.77\ \text{THz})(1\ \text{GHz})/1550\text{nm}$$

$$= 2.26\times10^{-14}\ \text{A}^2$$

From Eq. (11-27) and (11-24), we have

$$\sigma^2_{s-ASE} = 4\left[(0.73\ \text{A/W})(100)(1\ \mu\text{W})\right]$$

$$\times\left[(.73\ \text{A/W})\frac{\left(6.626\times10^{-34}\ \text{J/K}\right)\left(3\times10^{8}\,\text{m/s}\right)}{1550\,\text{nm}}2(100)(1\ \text{GHz})\right]$$

$$= 5.47\times10^{-12}\ \text{A}^2$$

From Eq. (11-28), we have

$$\sigma^2_{ASE-ASE} = (.73\text{A/W})^2\left[\frac{\left(6.626\times10^{-34}\ \text{J/K}\right)\left(3\times10^{8}\,\text{m/s}\right)}{1550\text{nm}}2(100)\right]^2$$

$$\times\left[2(1\ \text{THz}) - 1\ \text{GHz}\right](1\ \text{GHz})$$

$$= 7.01\times10^{-13}\ \text{A}^2$$

11-10. Plot of penalty factor from Eq. (11-36).

11-11. (a) Using the transparency condition Gexp(-αL) = 1 for a fiber/amplifier segment, we have

$$\langle P \rangle_{path} = \frac{1}{L} \int_0^L P(z)\ dz = \frac{P_{in}}{L} \int_0^L e^{-\alpha z}\ dz$$

$$= \frac{P_{in}}{\alpha L} \left[1 - e^{-\alpha L}\right] = \frac{P_{in}}{\alpha L} \left[1 - \frac{1}{G}\right] = \frac{P_{in}}{G} \left(\frac{G-1}{\ln\ G}\right)$$

since ln G = αL from the transparency condition.

(b) From Eq. (11-35) and using Eq. (11-24),

$$\langle P_{ASE} \rangle_{path} = \frac{NP_{ASE}}{L} \int_0^L e^{-\alpha z}\ dz = \frac{NP_{ASE}}{\alpha L} \left(1 - e^{-\alpha L}\right)$$

$$= \frac{\alpha\ (NL)}{(\alpha L)^2}\ P_{ASE} \left(1 - \frac{1}{G}\right) = \frac{\alpha L_{tot}}{(\ln\ G)^2}\ h\nu n_{sp}(G-1)\Delta\nu_{opt} \left(1 - \frac{1}{G}\right)$$

$$= \alpha L_{tot}\ h\nu n_{sp}\Delta\nu_{opt}\ \frac{1}{G}\left(\frac{G-1}{\ln\ G}\right)^2$$

11-12. Since the slope of the gain-versus -input power curve is –0.5, then for a 6-dB drop in the input signal, the gain increases by +3 dB.

1. Thus at the first amplifier, a –10.1-dBm signal now arrives and experiences a +10.1-dB gain. This gives a 0-dBm output (versus a normal +3-dBm output).
2. At the second amplifier, the input is now –7.1 dBm (down 3 dB from the usual –4.1 dBm level). Hence the gain is now 8.6 dB (up 1.5 dB), yielding an output of

   –7.1 dBm + (7.1 + 1.5) dB = 1.5 dBm

3. At the third amplifier, the input is now –5.6 dBm (down 1.5 dB from the usual –4.1 dBm level). Hence the gain is up 0.75 dB, yielding an output of
–5.6 dBm + (7.1 + 0.75) dB = 2.25 dBm
4. At the fourth amplifier, the input is now –4.85 dBm (down 0.75 dB from the usual –4.1 dBm level). Hence the gain is up 0.375 dB, yielding an output of
–4.85 dBm + (7.1 + 0.375) dB = 2.63 dBm
which is within 0.37 dB of the normal +3 dBm level.

11-13. First let $2\pi\nu_i t + \phi_i = \theta_i$ for simplicity. Then write the cosine term as

$\cos\ \theta_i = \dfrac{e^{j\theta_i} + e^{-j\theta_i}}{2}$, so that

$$P = E_i(t)E_i^*(t) = \left[\sum_{i=1}^{N}\sqrt{2P_i}\,\frac{e^{j\theta_i} + e^{-j\theta_i}}{2}\right]\times\left[\sum_{k=1}^{N}\sqrt{2P_k}\,\frac{e^{j\theta_k} + e^{-j\theta_k}}{2}\right]$$

$$= \frac{1}{4}\sum_{i=1}^{N}\sum_{k=1}^{N}\sqrt{2P_i}\sqrt{2P_k}\ \left[e^{j\theta_i}e^{-j\theta_k} + e^{j\theta_k}e^{-j\theta_i} + e^{j\theta_i}e^{j\theta_k} + e^{-j\theta_i}e^{-j\theta_k}\right]$$

$$= \frac{1}{4}\sum_{i=1}^{N}\sum_{k=1}^{N}\sqrt{2P_i}\sqrt{2P_k}\ \left[e^{j(\theta_i-\theta_k)} + e^{-j(\theta_i-\theta_k)} + e^{j(\theta_i+\theta_k)} + e^{-j(\theta_i+\theta_k)}\right]$$

$$= \sum_{i=1}^{N}P_i + \frac{1}{2}\sum_{i=1}^{N}\sum_{k\neq i}^{N}\sqrt{2P_i}\sqrt{2P_k}\ \left[e^{j(\theta_i-\theta_k)} + e^{-j(\theta_i-\theta_k)}\right]$$

where the last two terms in the second-last line drop out because they are beyond the response frequency of the detector. Thus,

$$P = \sum_{i=1}^{N}P_i + \sum_{i=1}^{N}\sum_{k\neq i}^{N}2\sqrt{P_iP_k}\ \left[\cos(\theta_i - \theta_k)\right]$$

11-14. (a) For N input signals, the output signal level is given by

$$P_{s,out} = G\sum_{i=1}^{N}P_{s,in}(i) \le 1\ \text{mW}.$$

The inputs are 1 μW (-30 dBm) each and the gain is 26 dB (a factor of 400).

Thus for one input signal, the output is (400)(1 μW) = 400 μW or –4 dBm.

For two input signals, the total output is 800 μW or –1 dBm. Thus the level of each individual output signal is 400 μW or –4 dBm.

For four input signals, the total input level is 4 μW or –24 dBm. The output then reaches its limit of 0 dBm, since the maximum gain is 26 dB. Thus the level of each individual output signal is 250 μW or –6 dBm.

Similarly, for eight input channels the maximum output level is o dBm, so the level of each individual output signal is 1/8(1 mW) = 125 μW or –9 dBm.
(b) When the pump power is doubled, the outputs for one and two inputs remains at the same level. However, for four inputs, the individual output level is 500 μW or –3 dBm, and for 8 inputs, the individual output level is 250 μW or –6 dBm.

11-15. Substituting the various expressions for the variances from Eqs. (11-26) through (11-30) into the expression given for Q in the problem statement, we find

$$Q = \frac{AP}{\left(HP + D^2\right)^{1/2} + D}$$

where we have defined the following terms for simplicity

$$A = 2\mathcal{R}\,G$$

$$H = 4q\mathcal{R}\,GB + 8\mathcal{R}^{\,2}GS_{ASE}B \quad \text{and} \qquad D^2 = \sigma_{off}^2$$

Rearrange terms in the equation for Q to get

$$Q^2\left(HP + D^2\right)^{1/2} = AP - QD$$

Squaring both sides and solving for P yields $P = \frac{2QD}{A} + \frac{Q^2H}{A^2}$

Substituting the expressions for A, H, and D into this equation, and recalling the expression for the responsivity from Eq. (6-6), then produces the result stated in the problem, where

$$F = \frac{1 + 2\eta n_{sp}(G - 1)}{\eta G}$$