Problem Solutions for Chapter 10

10-1. In terms of wavelength, at a central wavelength of 1546 nm a 500-GHz channel spacing is

$$\Delta\lambda = \frac{\lambda^2}{c}\ \Delta f = \frac{(1546\ \text{nm})^2}{3\times10^8\ \text{m/s}}\ 500\times10^9\ \text{s}^{-1} = 4\ \text{nm}$$

The number of wavelength channels fitting into the 1536-to-1556 spectral band then is

N = (1556 – 1536 nm)/4 nm = 5

10-2. (a) We first find P_1 by using Eq. (10-6):

$$10\ \log\left(\frac{200\ \mu\text{W}}{P_1}\right) = 2.7\ \text{dB} \quad \text{yields} \quad P_1 = 10^{(\log\ 200 - 0.27)} = 107.4\ \mu\text{W}$$

Similarly, $P_2 = 10^{(\log\ 200 - 0.47)} = 67.8\ \mu\text{W}$

(b) From Eq. (10-5): Excess loss = $10\ \log\left(\frac{200}{107.4 + 67.8}\right) = 0.58\ \text{dB}$

(c) $\frac{P_1}{P_1 + P_2} = \frac{107.4}{175.2} = 61\%$ and $\frac{P_2}{P_1 + P_2} = \frac{67.8}{175.2} = 39\%$

10-3. The following coupling percents are are realized when the pull length is stopped at the designated points:

Coupling percents from input fiber to output 2

Points	**A**	**B**	**C**	**D**	**E**	**F**
1310 nm	25	50	75	90	100	0
1540 nm	50	88	100	90	50	100

10-4. From $A_{out} = s_{11}A_{in} + s_{12}B_{in}$ and $B_{out} = s_{21}A_{in} + s_{22}B_{in} = 0$, we have

$$B_{in} = -\frac{s_{21}}{s_{22}}A_{in} \qquad \text{and} \qquad A_{out} = \left[s_{11} - \frac{s_{12}s_{21}}{s_{22}}\right]A_{in}$$

Then

$$T = \left|\frac{A_{out}}{A_{in}}\right|^2 = \left|s_{11} - \frac{s_{12}s_{21}}{s_{22}}\right|^2 \qquad \text{and} \qquad R = \left|\frac{B_{in}}{A_{in}}\right|^2 = \left|\left(\frac{s_{21}}{s_{22}}\right) \div \left(s_{11} - \frac{s_{12}s_{21}}{s_{22}}\right)\right|^2$$

10-5. From Eq. (10-18)

$$\frac{P_2}{P_0} = \sin^2(0.4z)\exp(-0.06z) = 0.5$$

One can either plot both curves and find the intersection point, or solve the equation numerically to yield z = 2.15 mm.

10-6. Since $\beta_z \propto n$, then for $n_A > n_B$ we have $\kappa_A < \kappa_B$. Thus, since we need to have $\kappa_A L_A = \kappa_B L_B$, we need to have $L_A > L_B$.

10-7. From Eq. (10-6), the insertion loss L_{Ij} for output port j is

$$L_{Ij} = 10\log\left(\frac{P_{i-in}}{P_{j-out}}\right)$$

Let

$$a_j = \frac{P_{i-in}}{P_{j-out}} = 10^{L_{Ij}/10}$$, where the values of L_{Ij} are given in Table P10-7.

Exit port no.	**1**	**2**	**3**	**4**	**5**	**6**	**7**
Value of a_j	8.57	6.71	5.66	8.00	9.18	7.31	8.02

Then from Eq. (10-25) the excess loss is

$$10\ \log\left(\frac{P_{in}}{\sum P_j}\right) = 10\ \log\left(\frac{P_{in}}{P_{in}\left(\frac{1}{a_1} + \frac{1}{a_2} + \ldots + \frac{1}{a_n}\right)}\right) = 10\ \log\left(\frac{1}{0.95}\right) = 0.22\ \text{dB}$$

10-8. (a) The coupling loss is found from the area mismatch between the fiber-core endface areas and the coupling-rod cross-sectional area. If a is the fiber-core radius and R is the coupling-rod radius, then the coupling loss is

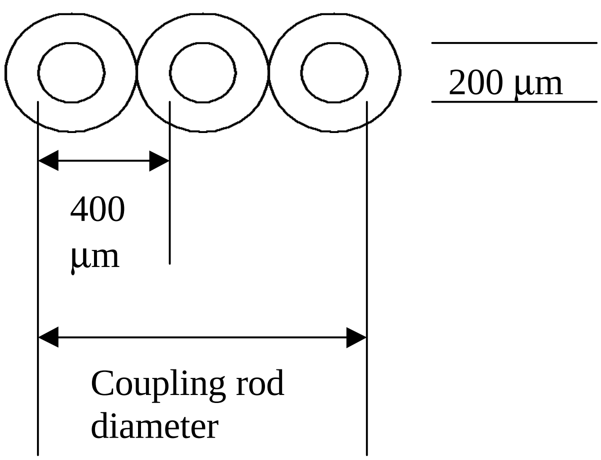
$$L_{coupling} = 10 \log \frac{P_{out}}{P_{in}} = 10 \log \frac{7\pi a^2}{\pi R^2} = 10 \log \frac{7(25)^2}{(150)^2} = -7.11 \text{ dB}$$

(b) Similarly, for the linear-plate coupler

$$L_{coupling} = 10 \log \frac{7\pi a^2}{l \infty w} = 10 \log \frac{7\pi(25)^2}{800(50)} = -4.64 \text{ dB}$$

10-9. (a) The diameter of the circular coupling rod must be 1000 µm, as shown in the figure below. The coupling loss is

$$L_{coupling} = 10 \log \frac{7\pi a^2}{\pi R^2} = 10 \log \frac{7(100)^2}{(500)^2} = -5.53 \text{ dB}$$



(b) The size of the plate coupler must be 200 µm by 2600 µm.

The coupling loss is $10 \log \frac{7\pi(100)^2}{200(2600)} = -3.74$ dB

10-10. The excess loss for a 2-by-2 coupler is given by Eq. (10-5), where $P_1 = P_2$ for a 3-dB coupler. Thus,

$$\text{Excess loss} = 10 \log\left(\frac{P_0}{P_1 + P_2}\right) = 10 \log\left(\frac{P_0}{2P_1}\right) = 0.1\text{ dB}$$

This yields

$$P_1 = \left(\frac{P_0}{2}\right) \div 10^{0.01} = 0.977 \left(\frac{P_0}{2}\right)$$

Thus the fractional power traversing the 3-dB coupler is F_T = 0.977.

Then, from Eq. (10-27),

$$\text{Total loss} = -10 \log\left(\frac{\log F_T}{\log 2} - 1\right)\log N = -10 \log\left(\frac{\log 0.977}{\log 2} - 1\right)\log 2^n \le 30$$

Solving for n yields

$$n \le \frac{-3}{\log 2\left(\frac{\log 0.977}{\log 2} - 1\right)} = 9.64$$

Thus, n = 9 and $N = 2^n = 2^9 = 512$

10-11. For details, see Verbeek et al., Ref. 34, p. 1012

For the general case, from Eq. (10-29) we find

$$M_{11} = \cos(2\kappa d)\cdot\cos(k\Delta L/2) + j\sin(k\Delta L/2)$$

$$M_{12} = M_{21} = j\sin(2\kappa d)\cdot\cos(k\Delta L/2)$$

$$M_{22} = \cos(2\kappa d)\cdot\cos(k\Delta L/2) - j\sin(k\Delta L/2)$$

The output powers are then given by

$$P_{out,1} = \left[\cos^2(2\kappa d)\cdot\cos^2(k\Delta L/2) + \sin^2(k\Delta L/2)\right]P_{in,1}$$

$$+\left[\sin^2(2\kappa d)\cdot\cos^2(k\Delta L/2)\right]P_{in,2}$$

$$P_{out,2} = \left[\sin^2(2\kappa d)\cdot\cos^2(k\Delta L/2)\right]P_{in,1}$$

$$+\left[\cos^2(2\kappa d)\cdot\cos^2(k\Delta L/2)+\sin^2(k\Delta L/2)\right]P_{in,2}$$

10-12. (a) The condition $\Delta\nu$ = 125 GHz is equivalent to having $\Delta\lambda$ = 1 nm. Thus the other three wavelengths are 1549, 1550, and 1551 nm.

(b) From Eqs. (10-42) and (10-43), we have

$$\Delta L_1 = \frac{c}{2n_{eff}(2\Delta\nu)} = 0.4\ \text{mm} \quad \text{and} \qquad \Delta L_3 = \frac{c}{2n_{eff}\Delta\nu} = 0.8\ \text{mm}$$

10-13. An 8-to-1 multiplexer consists of three stages of 2×2 MZI multiplexers. The first stage has four 2×2 MZIs, the second stage has two, and the final stage has one 2×2 MZI. Analogous to Fig. 10-14, the inputs to the first stage are (from top to bottom) ν, $\nu + 4\Delta\nu$, $\nu + 2\Delta\nu$, $\nu + 6\Delta\nu$, $\nu + \Delta\nu$, $\nu + 5\Delta\nu$, $\nu + 3\Delta\nu$, $\nu + 7\Delta\nu$.
In the first stage

$$\Delta L_1 = \frac{c}{2n_{eff}(4\Delta\nu)} = 0.75\ \text{mm}$$

In the second stage

$$\Delta L_2 = \frac{c}{2n_{eff}(2\Delta\nu)} = 1.5\ \text{mm}$$

In the third stage

$$\Delta L_3 = \frac{c}{2n_{eff}(\Delta\nu)} = 3.0\ \text{mm}$$

10-14. (a) For a fixed input angle ϕ, we differentiate both sides of the grating equation to get

$$\cos\theta\, d\theta = \frac{k}{n'\Lambda}\, d\lambda \qquad \text{or} \qquad \frac{d\theta}{d\lambda} = \frac{k}{n'\Lambda\cos\theta}$$

If $\phi \approx \theta$, then the grating equation becomes $2\sin\theta = \dfrac{k\lambda}{n'\Lambda}$.

Solving this for $\frac{k}{n'\Lambda}$ and substituting into the $\frac{d\theta}{d\lambda}$ equation yields

$$\frac{d\theta}{d\lambda} = \frac{2\sin\theta}{\lambda\cos\theta} = \frac{2\tan\theta}{\lambda}$$

(b) For S = 0.01,

$$\tan\theta = \left[\frac{S\lambda}{2\Delta\lambda\,(1+m)}\right]^{1/2} = \left[\frac{0.01(1350)}{2(26)(1+3)}\right]^{1/2} = 0.2548$$

or $\theta = 14.3°$

10-15. For 93% reflectivity

$R = \tanh^2(\kappa L) = 0.93$ yields $\kappa L = 2.0$, so that L = 2.7 mm for $\kappa = 0.75\ \text{mm}^{-1}$.

10-16. See Bennion et al., Ref. 42, Fig. 2a.

10-17. Derivation of Eq. (10-49).

10-18. (a) From Eq. (10-45), the grating period is

$$\Lambda = \frac{\lambda_{uv}}{2\ \sin\frac{\theta}{2}} = \frac{244\ \text{nm}}{2\ \sin(13.5°)} = \frac{244}{2(0.2334)}\ \text{nm} = 523\ \text{nm}$$

(b) From Eq. (10-47), $\lambda_{Bragg} = 2\Lambda n_{eff} = 2(523\ \text{nm})\ 1.48 = 1547\ \text{nm}$

(c) Using $\eta = 1 - 1/\sqrt{2} = 0.827$, we have from Eq. (10-51),

$$\kappa = \frac{\pi\ \delta n\ \eta}{\lambda_{Bragg}} = \frac{\pi\left(2.5\times10^{-4}\right)(0.827)}{1.547\times10^{-4}\ \text{cm}} = 4.2\ \text{cm}^{-1}$$

(d) From Eq. (10-49), $\Delta\lambda = \frac{(1.547\ \mu\text{m})^2}{\pi\ (1.48)\ 500\ \mu\text{m}}\left[(2.1)^2 + \pi^2\right]^{1/2} = 3.9\ \text{nm}$

(e) From Eq. (10-48), $R_{max} = \tanh^2(\kappa L) = \tanh^2(2.1) = (0.97)^2 = 94\%$

10-19. Derivation of Eq. (10-55).

10-20. (a) From Eq. (10-54),

$$\Delta L = m \frac{\lambda_0}{n_c} = 118 \frac{1.554\ \mu m}{1.451} = 126.4\ \mu m$$

(b) From Eq. (10-57),

$$\Delta \nu = \frac{x}{L_f} \frac{n_s c d}{m\lambda^2} \frac{n_c}{n_g}$$

$$= \frac{25\ \mu m}{9.36 \times 10^3\ \mu m} \frac{1.453\ (3 \times 10^8\ m/s)(25 \times 10^{-6}\ m)}{118\ (1.554 \times 10^{-6}\ m)^2} \frac{1.451}{1.475} = 100.5\ GHz$$

$$\Delta\lambda = \frac{\lambda^2}{c}\ \Delta\nu = \frac{(1.554 \times 10^{-6}\ m)^2}{3 \times 10^8\ m/s}\ 100.5\ GHz = 0.81\ nm$$

(c) From Eq. (10-60),

$$\Delta\nu_{FSR} = \frac{c}{n_g \Delta L} = \frac{3 \times 10^8\ m/s}{1.475(126.4\ \mu m)} = 1609\ GHz$$

Then

$$|\Delta\lambda| = \frac{\lambda^2}{c}\ \Delta\nu_{FSR} = \frac{(1.554 \times 10^{-6}\ m)^2}{3 \times 10^8\ m/s}\ 1609\ GHz = 12.95\ nm$$

(d) Using the conditions

$$\sin\theta_i \approx \theta_i = \frac{2(25\ \mu m)}{9380\ \mu m} = 5.33 \times 10^{-3}\ \text{radians}$$

and

$$\sin\theta_o \approx \theta_o = 21.3 \times 10^{-3}\ \text{radians}$$

then from Eq. (10-59),

$$\Delta\nu_{FSR} \approx \frac{c}{n_g[\Delta L + d(\theta_i + \theta_o)]}$$

$$= \frac{3\times 10^8 \text{ m/s}}{1.475\left[(126.4\times 10^{-6}\text{ m}) + (25\times 10^{-6}\text{ m})(5.33+21.3)\times 10^{-3}\right]} = 1601\text{ GHz}$$

10-21. The source spectral width is

$$\Delta\lambda_{signal} = \frac{\lambda^2\nu}{c} = \frac{(1550\text{ nm})^2(1.25\times 10^9\text{ s}^{-1})}{(3\times 10^8\text{ m/s})(10^9\text{ nm/m})} = 1\times 10^{-2}\text{ nm}$$

Then from Eq. (10-61)

$$\Delta\lambda_{tune} = \lambda\frac{\Delta n_{eff}}{n_{eff}} = (1550\text{ nm})(0.5\%) = 7.75\text{ nm}$$

Thus, from Eq. (10-63)

$$N = \frac{\Delta\lambda_{tune}}{10\ \lambda_{signal}} = \frac{7.75\text{ nm}}{10(0.01\text{ nm})} = 77$$

10-22. (a) From Eq. (10-64), the grating period is

$$\Lambda = \frac{\lambda_{Bragg}}{2n_{eff}} = \frac{1550\text{ nm}}{2(3.2)} = 242.2\text{ nm}$$

(b) Again, from the grating equation,

$$\Delta\Lambda = \frac{\Delta\lambda}{2n_{eff}} = \frac{2.0\text{ nm}}{2(3.2)} = 0.3\text{ nm}$$

10-23. (a) From Eq. (10-43)

$$\Delta L = \frac{c}{2n_{eff}\Delta\nu} = \frac{\lambda^2}{\Delta\lambda}\frac{1}{2n_{eff}} = 4.0\text{ mm}$$

(b) $\Delta L_{eff} = \Delta n_{eff} L$ implies that $\Delta n_{eff} = \frac{4\text{ mm}}{100\text{ mm}} = 0.04 = 4\%$

10-24. For example, see C. R. Pollock, *Fundamentals of Optoelectronics*, Irwin, 1995, Fig. 15.11, p. 439.

10-25. (a) The driving frequencies are found from

$$f_a = \nu_o \frac{v_a \Delta n}{c} = \frac{v_a \Delta n}{\lambda}$$

Thus we have

Wavelength (nm)	1300	1546	1550	1554
Acoustic frequency (MHz)	56.69	47.67	47.55	47.43

(b) The sensitivity is (4 nm)/(0.12 MHz) = 0.033 nm/kHz