Problem Solutions for Chapter 13

13-1 (a) From the given equation, n_{air} = 1.000273. Thus,

$$\lambda_{vacuum} = \lambda_{air} n_{air} = 1.000273(1550.0 \text{ nm}) = 1550.42 \text{ nm}$$

(b) From the given equation,

$$n(T, P) = 1 + \frac{(1.000273 - 1)(0.00138823)640}{1 + 0.003671(0)} = 1.000243$$

Then $n(T, P)(1550 \text{ nm}) = 1550.38 \text{ nm}$

13-2 Since the output voltage from the photodetector is proportional to the optical power, we can write Eq. (13-1) as

$$\alpha = \frac{10}{L_1 - L_2} \log \frac{V_2}{V_1}$$

where L_1 is the length of the current fiber, L_2 is the length cut off, and V_1 and V_2 are the voltage output readings from the long and short lengths, respectively. Then the attenuation in decibels is

$$\alpha = \frac{10}{1895 - 2} \log \frac{3.78}{3.31} = 0.31 \text{ dB/km}$$

13-3 (a) From Eq. (13-1)

$$\alpha = \frac{10}{L_N - L_F} \log \frac{P_N}{P_F} = \frac{10}{L_N - L_F} \log \frac{V_N}{V_F} = \frac{10 \log e}{L_N - L_F} \ln \frac{V_N}{V_F}$$

From this we find

$$\Delta\alpha = \frac{10 \log e}{L_N - L_F}\left[\frac{\Delta V_N}{V_N} + \frac{\Delta V_F}{V_F}\right] = \frac{4.343}{L_N - L_F}(\pm 0.1\% \pm 0.1\%) = \pm \frac{8.686}{L_N - L_F} \times 10^{-3}$$

(b) If $\Delta\alpha$ = 0.05 dB/km, then

$$L = L_N - L_F \geq \frac{8.868 \times 10^{-3}}{0.05} \text{ km} = 176 \text{ m}$$

13-4 (a) From Eq. (8-11) we have

$$\frac{1}{\sqrt{2\pi}\,\sigma}\ \exp\left(-\frac{t_{1/2}^{2}}{2\sigma^2}\right) = \frac{1}{2}\frac{1}{\sqrt{2\pi}\,\sigma} \qquad \text{which yields } t_{1/2} = (2 \ln 2)^{1/2}\,\sigma$$

(b) From Eq. (8-10), the 3-dB frequency is the point at which

$$G(\omega) = \frac{1}{2}\ G(0), \qquad \text{or} \qquad \exp\left[-\frac{(2\pi f_{3dB})^2\,\sigma^2}{2}\right] = \frac{1}{2}$$

Using σ as defined in Eq. (8-13), we have

$$f_{3dB} = \frac{(2 \ln 2)^{1/2}}{2\pi\sigma} = \frac{2 \ln 2}{\pi\, t_{FWHM}} = \frac{0.44}{t_{FWHM}}$$

13-5 From Eq. (13-4), $P_{out}(f)/P_{in}(f) = |H(f)|$. To measure the frequency response, we need a constant input amplitude, that is, $P_{in}(f) = P_{in}(0)$. Thus,

$$\frac{P(f)}{P(0)} = \frac{P_{out}(f)/P_{in}(f)}{P_{out}(0)/P_{in}(0)} = \left|\frac{H(f)}{H(0)}\right| = |H(f)|$$

The following table gives some representative values of H(f) for different values of 2σ:

f (MHz)	**2σ = 2 ns**	**2σ = 1 ns**	**2σ = 0.5 ns**
100	0.821	0.952	0.988
200	0.454	0.821	0.952
300	0.169	0.641	0.895
500	0.0072	0.291	0.735
700		0.089	0.546
1000		0.0072	0.291

13-6 To estimate the value of D, consider the slope of the curve in Fig. P13-6 at λ = 1575 nm. There we have Δτ = 400 ps over the wavelength interval from 1560 nm to 1580 nm, i.e., Δλ = 20 nm. Thus

$$D = \frac{1}{L}\frac{\Delta\tau}{\Delta\lambda} = \frac{1}{10\ \text{km}}\frac{400\ \text{ps}}{20\ \text{nm}} = 2\ \text{ps}/(\text{nm}\cdot\text{km})$$

Then, using this value of D at 1575 nm and with λ_0 = 1548 nm, we have

$$S_0 = \frac{D(\lambda)}{\lambda - \lambda_0} = \frac{2\ \text{ps}/(\text{nm}\cdot\text{km})}{(1575 - 1548)\ \text{nm}} = 0.074\ \text{ps}/(\text{nm}^2\cdot\text{km})$$

13-7 With k = 1, λ_{start} = 1525 nm, and λ_{stop} = 1575 nm, we have N_e = 17 extrema.

Substituting these values into Eq. (13-14) yields 1.36 ps.

13-8 At 10 Gb/s over a 100-km link, the given equation yields:

$$P_{ISI} \approx 26\frac{(1\ \text{ps})^2\,0.5(1-0.5)}{(100\ \text{ps})^2} = 6.5\times10^{-4}\ \text{dB}$$

Similarly, at 10 Gb/s over a 1000-km link, $P_{ISI} \approx 0.065\ \text{dB}$.

This is the same result at 100 Gb/s over a 100-km link.

At 100 Gb/s over a 1000-km link, we have 6.5 dB.

13-9 For a uniform attenuation coefficient, β is independent of y. Thus, Eq. (13-16) becomes

$$P(x) = P(0)\exp\left[-\beta\int_0^x dy\right] = P(0)e^{-\beta x}$$

Writing this as $\exp(-\beta x) = P(0)/P(x)$ and taking the logarithm on both sides yields

$\beta x\ \log\ e = \log\ \frac{P(0)}{P(x)}$. Since $\alpha = \beta(10\ \log\ e)$, this becomes

$$\alpha x = 10\ \log\ \frac{P(0)}{P(x)}$$

For a fiber of length x = L with P(0) = P_N being the near-end input power, this equation reduces to Eq. (13-1).

13-10 Consider an isotropically radiating point source in the fiber. The power from this point source is radiated into a sphere that has a surface area $4\pi r^2$. The portion of this power captured by the fiber in the backward direction at a distance r from the point source is the ratio of the area $A = \pi a^2$ to the sphere area $4\pi r^2$. If θ is the acceptance angle of the fiber core, then $A = \pi a^2 = \pi(r\theta)^2$. Therefore S, as defined in Eq. (13-18), is given by

$$S = \frac{A}{4\pi r^2} = \frac{\pi r^2 \theta^2}{4\pi r^2} = \frac{\theta^2}{4}$$

From Eq. (2-23), the acceptance angle is

$$\sin\ \theta \approx \theta = \frac{NA}{n}\text{, so that} \qquad S = \frac{\theta^2}{4} = \frac{(NA)^2}{4n^2}$$

13-11 The attenuation is found from the slope of the curve, by using Eq. (13-22):

$$\text{Fiber a:}\quad \alpha = \frac{10\ \log\ \dfrac{P_D(x_1)}{P_D(x_2)}}{2(x_2 - x_1)} = \frac{10\ \log\ \dfrac{70}{28}}{2(0.5\ \text{km})} = 4.0\ \text{dB/km}$$

$$\text{Fiber b:}\quad \alpha = \frac{10\ \log\ \dfrac{25}{11}}{2(0.5\ \text{km})} = 3.6\ \text{dB/km}$$

$$\text{Fiber c:}\quad \alpha = \frac{10\ \log\ \dfrac{7}{1.8}}{2(0.5\ \text{km})} = 5.9\ \text{dB/km}$$

To find the final splice loss, let P_1 and P_2 be the input and output power levels, respectively, at the splice point. Then for

$$\text{For splice 1:}\quad L_{splice} = 10\ \log\ \frac{P_2}{P_1} = 10\ \log\ \frac{25}{28} = -0.5\ \text{dB}$$

$$\text{For splice 2:}\quad L_{splice} = 10\ \log\ \frac{7}{11} = -2.0\ \text{dB}$$

13-12 See Ref. 42, pp. 450-452 for a detailed and illustrated derivation.

Consider the light scattered from an infinitesimal interval dz that is located at L = Tv_{gr}. Light scattered from this point will return to the OTDR at time t = 2T. Upon inspection of the pulse of width W being scattered form the point L, it can be deduced that the back-scattered power seen by the OTDR at time 2T is the integrated sum of the light scattered from the locations z = L – W/2 to z = L.

Thus, summing up the power from infinitesimal short intervals dz from the whole pulse and taking the fiber attenuation into account yields

$$P_s(L) = \int_0^W S\alpha_s P_0 \exp\left[-2\alpha\left(L + \frac{z}{2}\right)\right] dz$$

$$= S\,\frac{\alpha_s}{\alpha}\,P_0\,e^{-2\alpha L}\left(1 - e^{-\alpha W}\right)$$

which holds for L ≥ W/2. For distances less than W/2, the lower integral limit gets replaced by W – 2L.

13-13 For very short pulse widths, we have that $\alpha W \ll 1$. Thus the expression in parenthesis becomes

$$\frac{1}{\alpha}\left(1 - e^{-\alpha W}\right) \approx \frac{1}{\alpha}\left[1 - (1 - \alpha W)\right] = W$$

Thus

$$P_s(L) \approx S\,\alpha_s\,W\,P_0\,e^{-2\alpha L}$$

13-14 (a) From the given equation, for an 0.5-dB accuracy, the SNR is 4.5 dB.

The total loss of the fiber is (0.33 dB/km)(50 km) = 16.5 dB.

The OTDR dynamic range D is

$$D = SNR + \alpha L + \text{splice loss}$$

$$= 4.5\ \text{dB} + 16.5\ \text{dB} + 0.5\ \text{dB} = 21.5\ \text{dB}$$

Here the splice loss is added to the dynamic range because the noise that limits the achievable accuracy shows up after the event.

(b) For a 0.05-dB accuracy, the OTDR dynamic range must be 26.5 dB.

13-15 To find the fault-location accuracy dL with an OTDR, we differentiate Eq. (13-23):

$$dL = \frac{c}{2n} dt$$

where is the accuracy to which the time difference between the original and reflected pulses must be measured. For dL ≤ 1 m, we need

$$dt = \frac{2n}{c} dL \le \frac{2(1.5)}{3\times10^{8}\ \mathrm{m/s}}(0.5\ \mathrm{m}) = 5\ \mathrm{ns}$$

To measure dt to this accuracy, the pulse width must be ≤ 0.5dt (because we are measuring the time difference between the original and reflected pulse widths). Thus we need a pulse width of 2.5 ns or less to locate a fiber fault within 0.5 m of its true position.