**Problem Solutions for Chapter 6**

6-1. From Eqs. (6-4) and (6-5) with $R_f = 0$, $\eta = 1 - \exp(-\alpha_s w)$

To assist in making the plots, from Fig. P6-1, we have the following representative values of the absorption coefficient:

| **λ (μm)** | **$\alpha_s$ (cm$^{-1}$)** |
|---|---|
| .60 | $4.4\times10^3$ |
| .65 | $2.9\times10^3$ |
| .70 | $2.0\times10^3$ |
| .75 | $1.4\times10^3$ |
| .80 | $0.97\times10^3$ |
| .85 | 630 |
| .90 | 370 |
| .95 | 190 |
| 1.00 | 70 |

6-2. $$I_p = qA \int_0^w G(x)\, dx = qA\, \Phi_0\, \alpha_s \int_0^w e^{-\alpha_s x}\, dx$$

$$= qA\, \Phi_0 \left[1 - e^{-\alpha_s w}\right] = qA \frac{P_0(1 - R_f)}{h\nu A}\left[1 - e^{-\alpha_s w}\right]$$

6-3. From Eq. (6-6), $$R = \frac{\eta q}{h\nu} = \frac{\eta q \lambda}{hc} = 0.8044\, \eta\lambda \quad (\lambda \text{ in } \mu m)$$

Plot R as a function of wavelength.

6-4. (a) Using the fact that $V_a \approx V_B$, rewrite the denominator as

$$1 - \left(\frac{V_a - I_M R_M}{V_B}\right)^n = 1 - \left(\frac{V_B - V_B + V_a - I_M R_M}{V_B}\right)^n$$

$$= 1 - \left(1 - \frac{V_B - V_a + I_M R_M}{V_B}\right)^n$$

Since $\dfrac{V_B - V_a + I_M R_M}{V_B} << 1$, we can expand the term in parenthesis:

$$1 - \left(1 - \frac{V_B - V_a + I_M R_M}{V_B}\right)^n \approx 1 - \left[1 - \frac{n(V_B - V_a + I_M R_M)}{V_B}\right]$$

$$= \frac{n(V_B - V_a + I_M R_M)}{V_B} \approx \frac{n I_M R_M}{V_B}$$

Therefore, $M_0 = \dfrac{I_M}{I_p} \approx \dfrac{V_B}{n(V_B - V_a + I_M R_M)} \approx \dfrac{V_B}{n I_M R_M}$

(b) $M_0 = \dfrac{I_M}{I_p} = \dfrac{V_B}{n I_M R_M}$ implies $I_M^2 = \dfrac{I_p V_B}{n R_M}$ , so that $M_0 = \left(\dfrac{V_B}{n I_p R_M}\right)^{1/2}$

6-5. $$\langle i_s^2(t) \rangle = \frac{1}{T}\int_0^T i_s^2(t)dt = \frac{\omega}{2\pi}\int_0^{2\pi/\omega} R_0^2 P^2(t)\, dt \qquad \text{(where } T = 2\pi/\omega\text{)},$$

$$= \frac{\omega}{2\pi} R_0^2 P_0^2 \int_0^{2\pi/\omega} (1 + 2m \cos \omega t + m^2 \cos^2 \omega t)\, dt$$

Using

$$\int_0^{2\pi/\omega} \cos \omega t\, dt = \frac{1}{\omega} \sin \omega t \Big|_{t=0}^{t=2\pi/\omega} = 0$$

and $$\int_0^{2\pi/\omega} \cos^2 \omega t\, dt = \frac{1}{\omega}\int_0^{2\pi}\left(\frac{1}{2} + \frac{1}{2}\cos 2x\right)dx = \frac{\pi}{\omega}$$

we have $$\langle i_s^2(t) \rangle = R_0^2 P_0^2 \left(1 + \frac{m^2}{2}\right)$$

6-6. Same problem as Example 6-6: compare Eqs. (6-13), (6-14), and (6-17).

(a) First from Eq. (6-6), $I_p = \dfrac{\eta q \lambda}{hc} P_0 = 0.593\ \mu A$

Then $\sigma_Q^2 = 2qI_pB = 2(1.6 \times 10^{-19}\ C)(0.593\ \mu A)(150 \times 10^6\ Hz) = 2.84 \times 10^{-17}\ A^2$

(b) $\sigma_{DB}^2 = 2qI_D B = 2\left(1.6\times10^{-19}\ \text{C}\right)(1.0\ \text{nA})(150\times10^{6}\ \text{Hz}) = 4.81\times10^{-20}\ \text{A}^2$

(c) $\sigma_T^2 = \frac{4k_B T}{R_L}B = \frac{4\left(1.38\times10^{-23}\ \text{J/K}\right)(293\ \text{K})}{500\ \Omega}\left(150\times10^{6}\ \text{Hz}\right) = 4.85\times10^{-15}\ \text{A}^2$

6-7. Using $R_0 = \frac{\eta q\lambda}{hc}$ = 0.58 A/W, we have from Eqs. (6-4), (6-11b), (6-15), and (6-17)

$$\left(\frac{S}{N}\right)_Q = \frac{\frac{1}{2}\left(R_0P_0m\right)^2M^2}{2qI_pBM^{1/2}M^2} = \frac{R_0P_0m^2}{4qBM^{1/2}} = 6.565\times10^{12}\ P_0$$

$$\left(\frac{S}{N}\right)_{DB} = \frac{(R_0P_0m)^2}{4qI_DBM^{1/2}} = 3.798\times10^{22}\ P_0^2$$

$$\left(\frac{S}{N}\right)_{DS} = \frac{(R_0P_0m)^2M^2}{4qI_LB} = 3.798\times10^{26}\ P_0^2$$

$$\left(\frac{S}{N}\right)_T = \frac{\frac{1}{2}(R_0P_0m)^2M^2}{4k_BTB/R_L} = 7.333\times10^{22}\ P_0^2$$

where $P_0$ is given in watts. To convert $P_0 = 10^{-n}$ W to dBm, use $10\log\left(\frac{P_0}{10^{-3}}\right) = 10(3-n)$ dBm

6-8. Using Eq. (6-18) we have

$$\frac{S}{N} = \frac{\frac{1}{2}(R_0P_0m)^2M^2}{2qB(R_0P_0 + I_D)M^{5/2} + 2qI_LB + 4k_BTB/R_L}$$

$$= \frac{1.215\times10^{-16}M^2}{2.176\times10^{-23}M^{5/2} + 1.656\times10^{-19}}$$

The value of M for maximum S/N is found from Eq. (6-19), with x = 0.5: $M_{optimum} = 62.1$.

6-9. $0 = \frac{d}{dM}\left(\frac{S}{N}\right) = \frac{d}{dM}\left[\frac{\frac{1}{2} I_p^2 M^2}{2q(I_p + I_D)M^{2+x} + 2qI_L + 4k_B T/R_L}\right]$

$$0 = I_p^2 \; M - \frac{(2+x)M^{1+x} \, 2q(I_p + I_D)\frac{1}{2} I_p^2 M^2}{2q(I_p + I_D)M^{2+x} + 2qI_L + 4k_B T/R_L}$$

Solving for M: $M_{opt}^{2+x} = \frac{2qI_L + 4k_B T/R_L}{xq(I_p + I_D)}$

6-10. (a) Differentiating $p_n$, we have

$$\frac{\partial p_n}{\partial x} = \frac{1}{L_p}\left(p_{n0} + Be^{-\alpha_s w}\right) e^{(w-x)/L_p} - \alpha_s Be^{-\alpha_s x}$$

$$\frac{\partial^2 p_n}{\partial x^2} = -\frac{1}{L_p^2}\left(p_{n0} + Be^{-\alpha_s w}\right) e^{(w-x)/L_p} + \alpha_s^2 \; Be^{-\alpha_s x}$$

Substituting $p_n$ and $\frac{\partial^2 p_n}{\partial x^2}$ into the left side of Eq. (6-23):

$$-\frac{D_p}{L_p^2}\left(p_{n0} + Be^{-\alpha_s w}\right) e^{(w-x)/L_p} + D_p \alpha_s^2 \; Be^{-\alpha_s x}$$

$$+ \frac{1}{\tau_p}\left(p_{n0} + Be^{-\alpha_s w}\right) e^{(w-x)/L_p} - \frac{B}{\tau_p} e^{-\alpha_s x} + \Phi_0 \, \alpha_s \, e^{-\alpha_s x}$$

$$= \left[B\left(D_p \, \alpha_s^2 - \frac{1}{\tau_p}\right) + \Phi_0 \, \alpha_s\right] e^{-\alpha_s x}$$

where the first and third terms cancelled because $L_p^2 = D_p \tau_p$ .

Substituting in for B:

$$\text{Left side} = \left[\frac{\Phi_0}{D_p}\frac{\alpha_s L_p^2}{1 - \alpha_s^2 \, L_p^2}\left(D_p \, \alpha_s^2 - \frac{1}{\tau_p}\right) + \Phi_0 \, \alpha_s\right] e^{-\alpha_s x}$$

$$= \frac{\Phi_0}{D_p}\left[\frac{\alpha_s L_p^2}{1 - \alpha_s^2 L_p^2}\left(\frac{\alpha_s^2 L_p - 1}{\tau_p}\right) + D_p\alpha_s\right] e^{-\alpha_s x}$$

$$= \frac{\Phi_0}{D_p}\left(-D_p\alpha_s + D_p\alpha_s\right) e^{-\alpha_s x} = 0$$ Thus Eq. (6-23) is satisfied.

(b) $J_{diff} = qD_p \left(\frac{\partial p_n}{\partial x}\right)_{x=w}$

$$= qD_p\left[\frac{1}{L_p}\left(p_{n0} + Be^{-\alpha_s w}\right) - \alpha_s Be^{-\alpha_s w}\right]$$

$$= qD_p B\left(\frac{1}{L_p} - \alpha_s\right) e^{-\alpha_s w} + qp_{n0}\frac{D_p}{L_p}$$

$$= q\Phi_0\left(\frac{\alpha_s L_p^2}{1 - \alpha_s^2 L_p^2}\right)\left(\frac{1 - \alpha_s L_p}{L_p}\right) e^{-\alpha_s w} + qp_{n0}\frac{D_p}{L_p}$$

$$= q\Phi_0 \frac{\alpha_s L_p}{1 + \alpha_s L_p} e^{-\alpha_s w} + qp_{n0}\frac{D_p}{L_p}$$

c) Adding Eqs. (6-21) and (6-25), we have

$$J_{total} = J_{drift} + J_{diffusion} = q\Phi_0\left[\left(1 - e^{-\alpha_s w}\right) + \left(\frac{\alpha_s L_p}{1 + \alpha_s L_p}\right)e^{-\alpha_s w}\right] + qp_{n0}\frac{D_p}{L_p}$$

$$= q\Phi_0\left(1 - \frac{e^{-\alpha_s w}}{1 + \alpha_s L_p}\right) e^{-\alpha_s w} + qp_{n0}\frac{D_p}{L_p}$$

6-11. (a) To find the amplitude, consider

$$\left(J_{tot} J_{tot}^*\right)_{sc}^{1/2} = q\Phi_0 (S S^*)^{1/2} \quad \text{where } S = \frac{1 - e^{-j\omega t_d}}{j\omega t_d}$$

We want to find the value of $\omega t_d$ at which $(S S^*)^{1/2} = \frac{1}{\sqrt{2}}$ .

Evaluating $(S S^*)^{1/2}$ , we have

$$(S\,S^*)^{1/2} = \left[\left(\frac{1 - e^{-j\omega t_d}}{j\omega t_d}\right)\left(\frac{1 - e^{+j\omega t_d}}{-j\omega t_d}\right)\right]^{1/2}$$

$$= \frac{\left[1 - \left(e^{+j\omega t_d} + e^{-j\omega t_d}\right) + 1\right]^{1/2}}{\omega t_d} = \frac{(2 - 2\cos \omega t_d)^{1/2}}{\omega t_d}$$

$$= \frac{[(1 - \cos \omega t_d)/2]^{1/2}}{\omega t_d/2} = \frac{\sin\left(\frac{\omega t_d}{2}\right)}{\frac{\omega t_d}{2}} = \text{sinc}\left(\frac{\omega t_d}{2}\right)$$

We want to find values of $\omega t_d$ where $(S\,S^*)^{1/2} = \frac{1}{\sqrt{2}}$ .

| **x** | **sinc x** | | **x** | **sinc x** |
|---|---|---|---|---|
| 0.0 | 1.000 | | 0.5 | 0.637 |
| 0.1 | 0.984 | | 0.6 | 0.505 |
| 0.2 | 0.935 | | 0.7 | 0.368 |
| 0.3 | 0.858 | | 0.8 | 0.234 |
| 0.4 | 0.757 | | 0.9 | 0.109 |

By extrapolation, we find sinc x = 0.707 at x = 0.442.

Thus $\frac{\omega t_d}{2} = 0.442$ which implies $\omega t_d = 0.884$

(b) From Eq. (6-27) we have $t_d = \frac{w}{v_d} = \frac{1}{\alpha_s v_d}$ . Then

$$\omega t_d = 2\pi f_{3\text{-dB}}\, t_d = 2\pi f_{3\text{-dB}} \frac{1}{\alpha_s v_d} = 0.884 \qquad \text{or}$$

$$f_{3\text{-dB}} = 0.884\, \alpha_s v_d / 2\pi$$

6-12. (a) The RC time constant is

$$RC = \frac{R\varepsilon_0 K_s A}{w} = \frac{(10^4\,\Omega)(8.85\times10^{-12}\,F/m)(11.7)(5\times10^{-8}\,m^2)}{2\times10^{-5}m} = 2.59 \text{ ns}$$

(b) From Eq. (6-27), the carrier drift time is

$$t_d = \frac{w}{v_d} = \frac{20\times10^{-6}\,m}{4.4\times10^{4}\,m/s} = 0.45\text{ ns}$$

c) $\frac{1}{\alpha_s} = 10^{-3}\text{ cm} = 10\ \mu m = \frac{1}{2}\ w$

Thus since most carriers are absorbed in the depletion region, the carrier diffusion time is not important here. The detector response time is dominated by the RC time constant.

6-13. (a) With $k_1 \approx k_2$ and $k_{eff}$ defined in Eq. (6-10), we have

(1) $$1 - \frac{k_1(1 - k_1)}{1 - k_2} = 1 - \frac{k_1 - k_1^2}{1 - k_2} \approx 1 - \frac{k_2 - k_1^2}{1 - k_2} = 1 - k_{eff}$$

(2) $$\frac{(1 - k_1)^2}{1 - k_2} = \frac{1 - 2k_1 + k_1^2}{1 - k_2} \approx \frac{1 - 2k_2 + k_1^2}{1 - k_2}$$

$$= \frac{1 - k_2}{1 - k_2} - \frac{k_2 - k_1^2}{1 - k_2} = 1 - k_{eff}$$

Therefore Eq. (6-34) becomes Eq. (6-38):

$$F_e = k_{eff}M_e + 2(1 - k_{eff}) - \frac{1}{M_e}(1 - k_{eff}) = k_{eff}M_e + \left(2 - \frac{1}{M_e}\right)(1 - k_{eff})$$

(b) With $k_1 \approx k_2$ and $k'_{eff}$ defined in Eq. (6-40), we have

(1) $$\frac{k_2(1 - k_1)}{k_1^2(1 - k_2)} \approx \frac{k_2 - k_1^2}{k_1^2(1 - k_2)} = k'_{eff}$$

(2) $$\frac{(1 - k_1)^2 k_2}{k_1^2(1 - k_2)} = \frac{k_2 - 2k_1k_2 + k_2k_1^2}{k_1^2(1 - k_2)}$$

$$\approx \frac{\left(k_2 - k_1^2\right) - \left(k_1^2 - k_2k_1^2\right)}{k_1^2(1 - k_2)} = k'_{eff} - 1$$

Therefore Eq. (6-35) becomes Eq. (6-39): $F_h = k'_{eff} \; M_h - \left(2 - \frac{1}{M_h}\right)(k'_{eff} - 1)$

6-14. (a) If only electrons cause ionization, then $\beta = 0$, so that from Eqs. (6-36) and (6-37), $k_1 = k_2 = 0$ and $k_{eff} = 0$. Then from Eq. (6-38)

$$F_e = 2 - \frac{1}{M_e} \approx 2 \text{ for large } M_e$$

(b) If $\alpha = \beta$, then from Eqs. (6-36) and (6-37), $k_1 = k_2 = 1$ so that

$k_{eff} = 1$. Then, from Eq. (6-38), we have $F_e = M_e$.