Waveguide Optics

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Chapter 2

Geometrical Optics Method

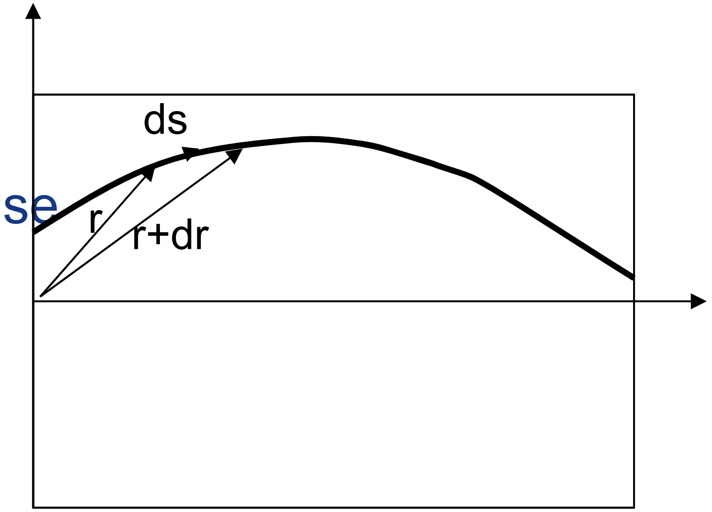
- In the geometrical optics method, the intensity and propagation direction of the light are taken into account, but ignoring the wave (phase, polarization) effects.
- The ray represents light propagation path.
- Main contents:
  - Starting from The Ray Equation, discuss one-dimensional and two-dimensional non-destructive optical waveguide, yield the basic rules of light propagation directions, as well as the classification of light (constraints).
  - In one-dimensional and two-dimensional optical waveguide, there is one and two ray invariants describing light propagation directions respectively, which correspond to "traditional" and "General" law of refraction (Snell's law).

1.The Ray Equation

- In geometrical optics, the trajectory is determined by the ray equation:

$$\frac{d}{ds}\left[n(r)\frac{dr}{ds}\right]=\nabla n(r)$$

- S is the distance along the light trails, n(r) is the spatial distribution of refractive index, r is radius vector
- The ray equation is yielded from：
  - Maxwell equation when λ → 0.
  - Fermat's principle
  - Snell's law(treat n(r) as n slices and use Snell's law at every boundary)



Fundamental theory

- When the medium is homogeneous isotropic，

$$n\frac{dr}{ds}=const$$

shows the ray (Vector r’s end track)is straight line.

- In non-homogeneous medium , the light will be bent, and point to the high refractive index region obeying refractive index rule.

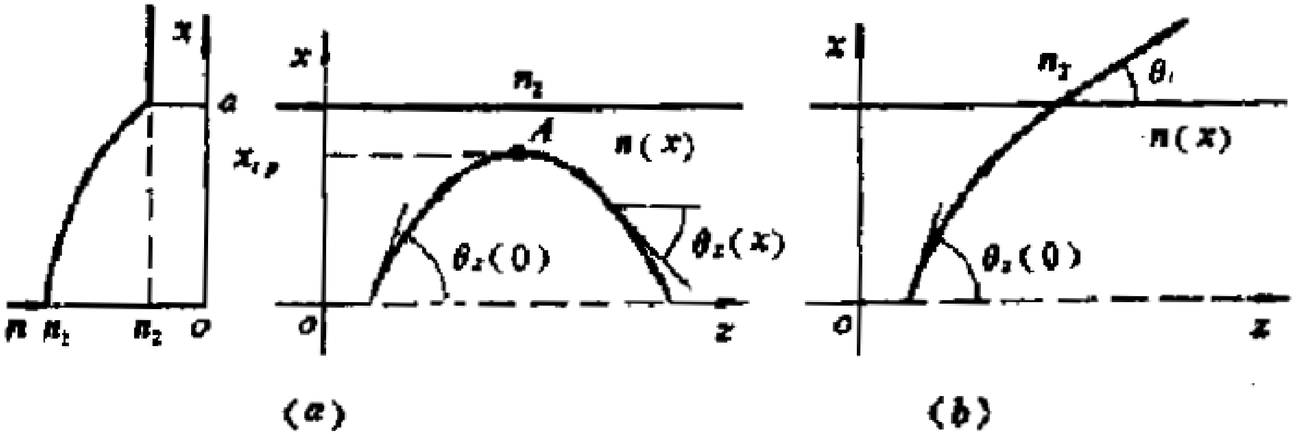
2. **渐变折射率** 设折射率只与$|x|$有关，且是$|x|$的单调降函数（图 1-2-1），其中只画出 $x>$（

半。 此时射线方程的 z 向分量仍满足 $\frac{d}{ds}\left[n(x)\frac{dz}{ds}\right]=0$。由于 $\frac{dz}{ds}=\cos\theta_z(x)$, 有

$$n(x)\cos\theta_z(x)=\text{const}=\bar{\beta}$$ (1

与（1-2-2）对应。由图 1-2-1 可以看出，可能存在一个转折点 x_{tp}，在该处$\theta_z=0$。显然 x

$$\begin{cases} n(x_{tp})=n(0)\cos\theta_z(0)=n_1\cos\theta_z(0) \\ 0<x_{tp}<a \end{cases}$$ (1



一维限制渐变光波导中的光线

确定。各 x_{tp} 的集合称为光线散焦面。当式(1-2-9)无解［对应于 $x_{tp}>a$ 或 $n_0\cos\theta_z(0)<n(a)$］时，说明在波导范围内不能形成光线的完全返回（图 1-2-1b）。由此可以定义一个临界角

$$\theta_{zc}(0)=\arccos\left[\frac{n(a)}{n(0)}\right]=\arccos\left(\frac{n_2}{n_1}\right) \tag{1-2-8}$$

以及约束光和折射光分别为：

约束光 $0\le\theta_z(0)<\theta_{zc}(0)$ 或 $n_2<\bar{\beta}\le n_1$ (1-2-9a)

折射光 $\theta_{zc}(0)<\theta_z(0)\le\pi/2$ 或 $0\le\bar{\beta}<n_2$ (1-2-9b)

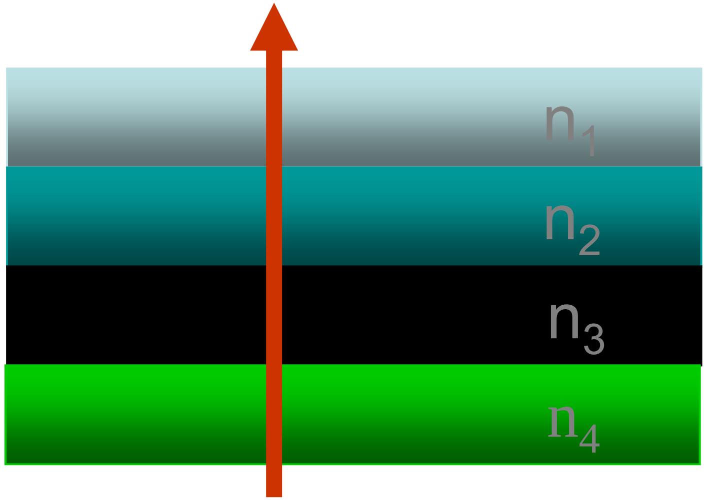
由式（1-2-6）和（1-2-7），射线不变量也可表为

$$\bar{\beta}=n(x_{tp}) \tag{1-2-10}$$

对于渐变折射率光波导，其数值孔径将是 x 的函数，在式（1-2-5）中，n_1 应改为 n(x)。

2. Light in one-dimensional waveguides

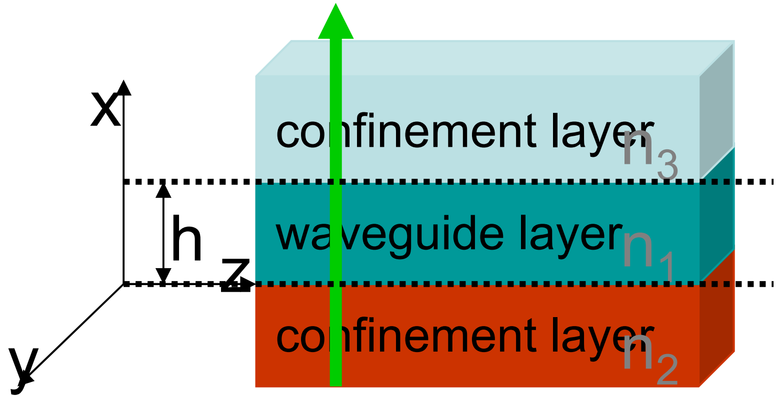
- planar optical waveguide
  - Semiconductor optoelectronic device
  - LiNbO3 waveguide device
  - Planar Lightwave Circuit (PLC)
- basic structure of one-dimensional planar optical waveguide
  - Consisting of multi-layer planar dielectric waveguide structure,
  - refractive index changes in the direction perpendicular to the dielectric boundary surface.



Step refractive index

- 3-layer uniform one-dimensional planar optical waveguide

$$n(x)=\begin{cases} n_1, 0 \le x \le h \\ n_2, x \le 0 \qquad (n_1 > n_2 \ge n_3) \\ n_3, x \ge h \end{cases}$$



symmetrical structure： $n_2 = n_3$
asymmetric structure： $n_2 \neq n_3$

$$n\frac{dr}{ds} = const \quad \text{yields}$$

$$n\cos\theta_z = const = \bar{\beta} \Longrightarrow \text{Snell's law}$$

θ: the angle between Incident light and the interface (or z axis)
β: the invariant describing light propagation direction

- There is obviously a critical angle

$$\theta_{zc} = \arccos\left(n_2/n_1\right)$$

  - when $\theta < \theta_{zc}$，total reflection，forming constraint light；
  - when θ>θzc， partial reflection，partial refraction。

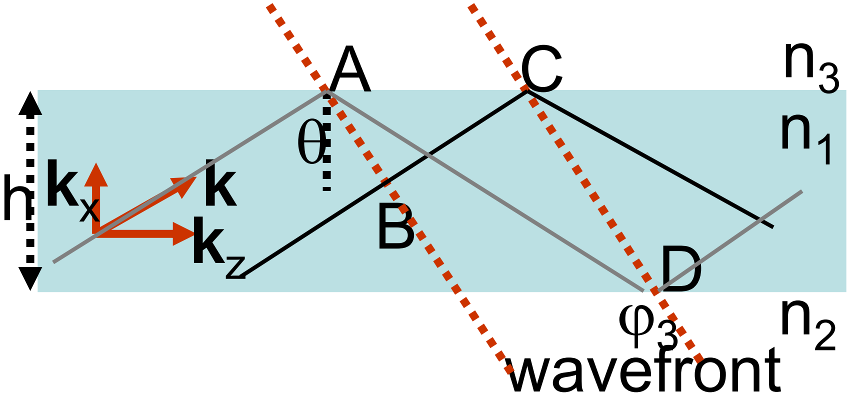
Constraint light： $0 \le \theta_z < \theta_{zc}$ 或 $n_2 < \overline{\beta} \le n_1$
refracted light： $\theta_{zc} < \theta_z \le \pi/2$ 或 $0 \le \overline{\beta} < n_2$

- Numerical Aperture(NA)：describing the ability to collect light incident. NA is defined as：the sine value of constraint light’s maximum acceptance half-width angle(measured outside the waveguide, index=n_0). For step index :

$$NA = \sin\theta_i = \frac{1}{n_0}\sqrt{n_1^2 - n_2^2}$$

- Total reflection condition

$\theta > \theta_{c12} > \theta_{c13}$ $\sin\theta > n_2/n_1$



- transmission constant

$\beta = k_z = k_0 n_1 \sin\theta$ $\mathbf{k} = \mathbf{k}_0 n_1$ $k_0 n_2 < \beta < k_0 n_1$

- coherence emphasis condition

$$\left(\overline{AD}-\overline{BC}\right)\frac{2\pi n_1}{\lambda}+\varphi_2+\varphi_3=2m\pi, m=0,1,2,\ldots$$

- characteristic equation

$$2k_0 n_1 h\cos\theta+\varphi_2+\varphi_3=2m\pi$$

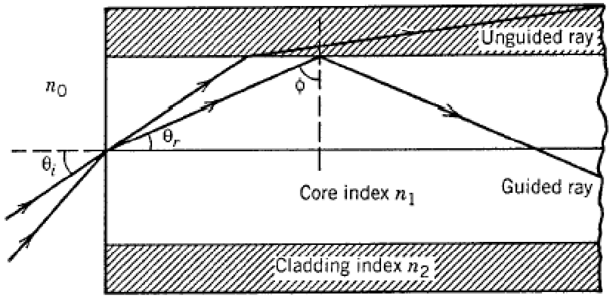
$$\overline{AD}-\overline{BC}=2h\cos\theta$$

specific incident angles make several modes

3. Light in two-dimensional waveguides

- Meridianal light, Precession light
- 2 invariants describe characteristics in meridian plane and precession characteristics respectively.
- Circular section fiber as an example
- Step Index

$$n(r)=\begin{cases} n_1 & r \le a \\ n_2 & r > a \end{cases}$$



$$\cos\theta_i = \sin\theta_z \sin\theta_\phi$$

$$r_{ic} = a\cos\theta_\phi$$

when $\theta_\phi = \pi/2$,$r_{ic}=0$, precession light turns into meridianal light.

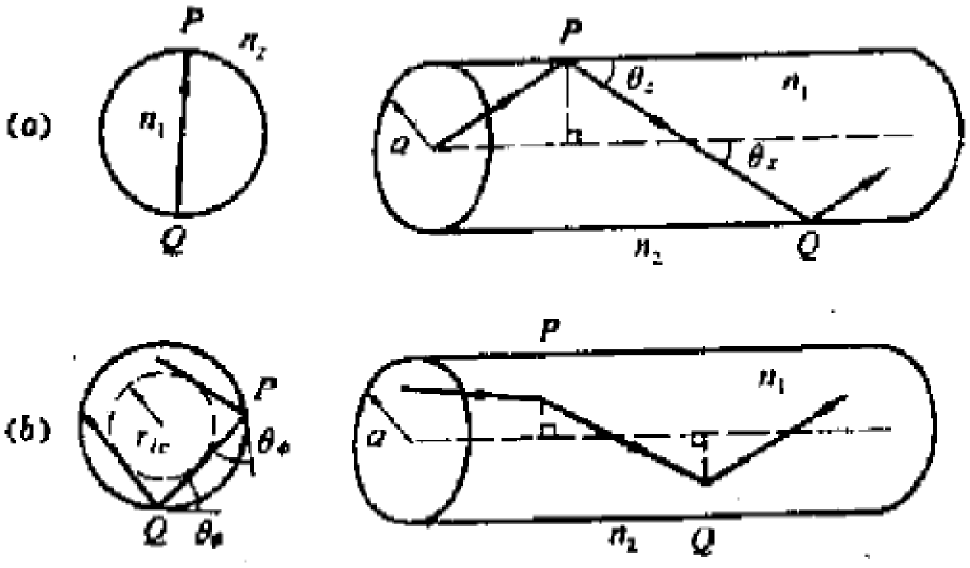


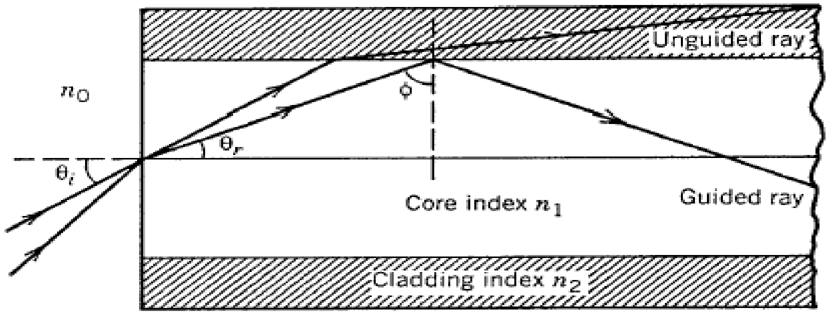
图 1-2-2　　阶跃折射率分布光纤中的(a)子午光和(b)旋进光

NA

relative refractive index difference

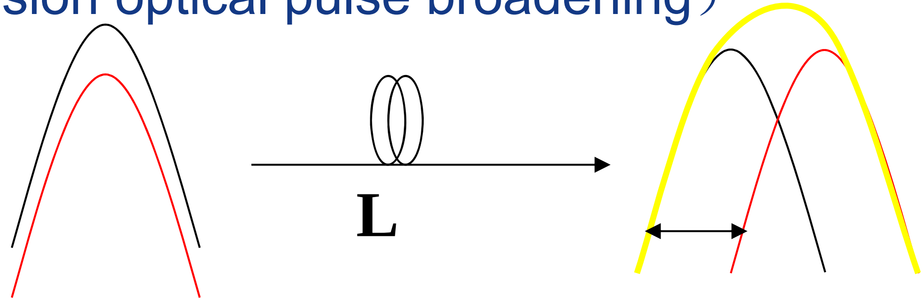
$$\left.\begin{aligned} NA &= \frac{1}{n_0}\sqrt{n_1^2 - n_2^2} \\ \Delta &= \frac{n_1^2 - n_2^2}{2n_1^2} \approx \frac{n_1 - n_2}{n_1} \end{aligned}\right\} \Rightarrow NA = n_1(2\Delta)^{1/2}$$

- intermodal dispersion
- maximum time delay: transfer time difference between Shortest and longest path
- Shortest path:L $\theta_i = 0$
- Longest path: $L/\sin\phi_c$, $n_0 \sin\theta_i = n_1 \cos\theta_c$



- Maximum time delay（Transmission optical pulse broadening）

$$\frac{\Delta T}{L} = \frac{n_1}{c}\left(\frac{1}{\sin\theta_c} - 1\right) = \frac{1}{c}\frac{n_1^2}{n_2}\Delta$$



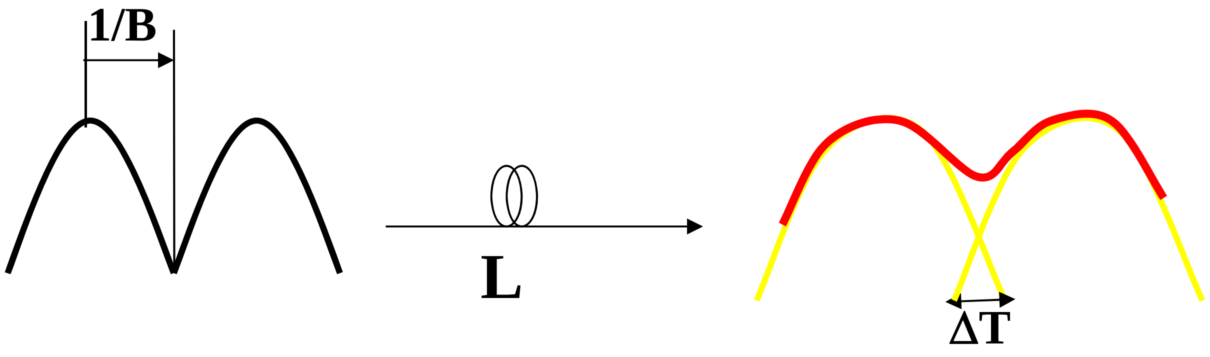
L——fiber length

c——light speed in a vacuum

Transmission capacity limitation

Transmission optical pulse broadening caused by dispersion.

$$\Delta T < \frac{1}{B} \Rightarrow BL < \frac{n_2}{n_1^2}\frac{c}{\Delta}$$



i.e. $n_1 = 1.5$ $\Delta = 2\times10^{-3}$ $BL < 100(Mb/s)$ Signal bit rate

Graded index fiber

- motivation:
  - Intermodal dispersion is the major constraint on multimode fiber transmission
  - Mode number in multimode optical fiber depends on the optical fiber core diameter, numerical aperture (relative refractive index), as well as optical wavelength

$$Modes = 0.5\left(\frac{corediameter \times NA \times \pi}{wvelength}\right)$$

  - the shorter (longer) the wavelength, the larger (smaller) the mode number
  - Reduce modes to single mode is a method to increase fiber optic transmission capacity
  - Difficult to make such a small diameter core fiber at an early time.
  - Grade-index fiber is proposed to decrease the intermodal dispersion.

- refractive-index profile

$$n(r) = \begin{cases} n_1\left[1 - 2\Delta\left(\dfrac{r}{a}\right)^{\alpha}\right]^{1/2} & r \le a \\ n_1(1 - 2\Delta)^{1/2} = n_2 & r > a \end{cases}$$

a-- core radius， α=1~∞
when α»10， approaching step index
when α=1， triangular form（ Dispersion-shift ）
when α=2， Square law distribution(Parabolic form)
Δ--relative refractive index difference

Two types of fiber core diameter: 50um and 62.5um

Intermodal dispersion

- Intermodal dispersion has a direct relationship with α.
- Intermodal dispersion becomes zero when the refractive-index profile is hyperbolic secant form.
- For graded-index fiber, intermodal dispersion becomes the smallest when α=2(1-Δ)：

$$\Delta T/L = n_1\Delta^2/8c$$

Transmission capacity limitation is:

$$\Delta T < 1/B \Rightarrow BL < 8c/n_1\Delta^2$$

B—signal bit rate，
L--transmission distance，
c-- light speed in a vacuum

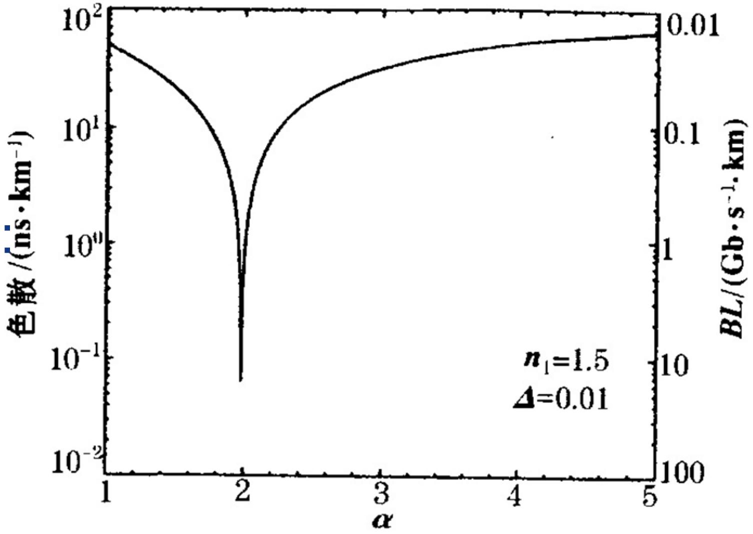


图 2.4 渐变光纤模间色散和 BL 积随α的变化

Light propagation

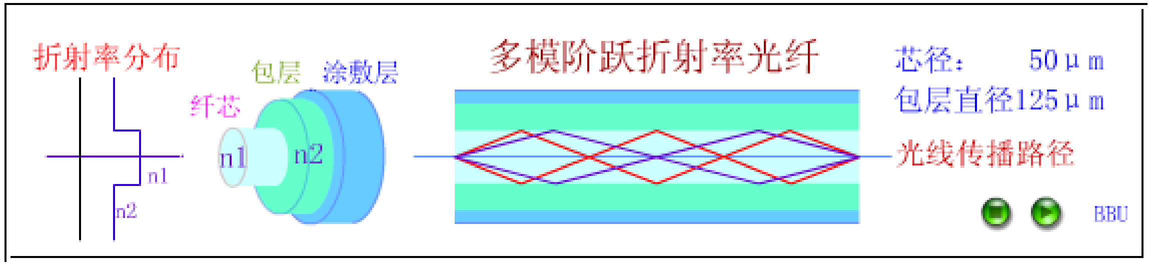


图 2-2 a

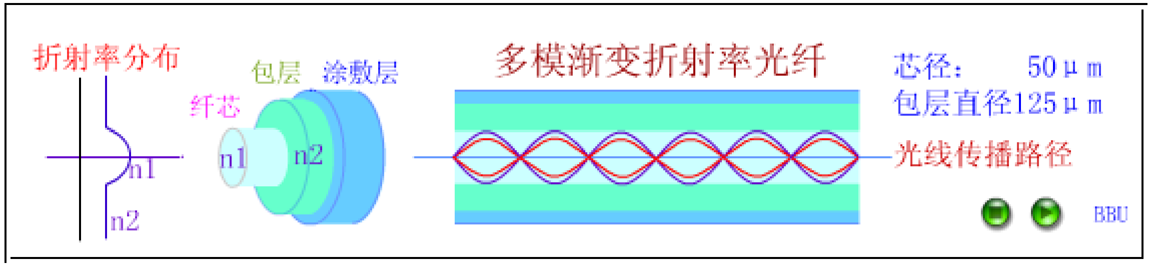
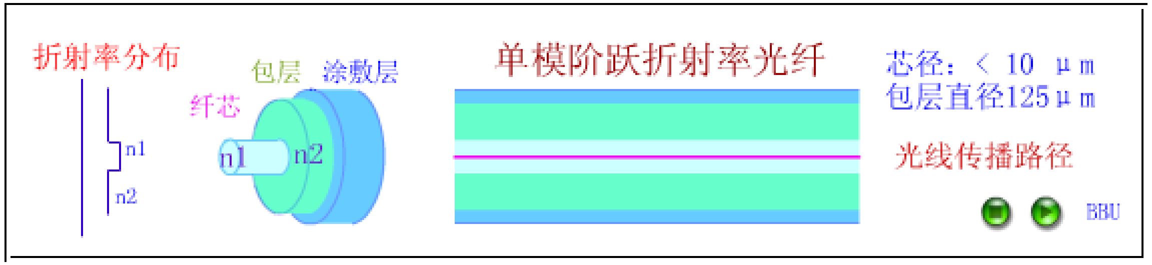


图 2-2 b



Further Reading

- 光波导理论与技术，李玉权，人民邮电出版社，第三章

Thank You！