On the LDPC-Coded Modulation for Ultra-High-Speed Optical Transport in the Presence of Phase Noise

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Abstract: A Monte Carlo integration based method to calculate symbol log-likelihood ratios for soft iterative decoding in the presence of imperfect carrier phase estimation (CPE) is proposed. The proposed method shows good robustness when CPE is imperfect, in particular for optimum modulation schemes.

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1. Introduction

As a response to never ending high bandwidth demands, the IEEE has ratified its 40/100 Gb/s Ethernet Standard IEEE 802.3ba in June 2010. The deployment of 100 Gb/s Ethernet (GbE) has already started and it is expected to accelerate in next few years. At these ultra-high data rates, the performance of fiber-optic communication systems is degraded significantly due to presence of various linear and nonlinear impairments. To deal with those channel impairments novel advanced techniques in modulation and detection, coding and signal processing have been intensively studied [1]-[3]. For carrier phase estimation (CPE), the algorithmic DSP-based approaches are highly popular, and can be categorized into two broad categories data-aided and non-data-aided. The MAP and ML approaches are particularly efficient in CPE; however, the complexity of such algorithms grows exponentially with the channel memory. Even upon compensation of chromatic dispersion and nonlinear phase distortions there will be some residual phase error as indicated in [1], [3]. It has been experimentally verified in [3], that even in beyond 100 Gb/s transmission the distribution of samples upon digital compensation of linear and nonlinear impairments (in uncompensated SMF links) is still Gaussian-like with the residual phase error that can be properly modeled as a Markov process.

In this paper, we propose a Monte Carlo integration inspired approach to calculate the symbol likelihoods in the presence of residual phase error. The proposed method is applied to conventional and optimized modulation schemes, 2-D and 4-D signaling schemes, to evaluate its efficiency. It has been demonstrated that optimized modulation schemes, when used in combination with LDPC coding, are more robust in the presence of phase error than conventional LDPC-coded QAM. Moreover, LDPC-coded 4-D signaling schemes show much better robustness compared to 2-D coded modulation schemes.

2. Description of equivalent coherent optical channel model

The equivalent channel model for coherent detection, upon compensation of linear and nonlinear impairments and CPE, can be represented as:

\[ r_k = s(a_k, \theta_k) + z, \quad r_k = [r_k^{(1)} \ldots r_k^{(N)}]^T, \quad s(a_k, \theta_k) = e^{j\theta_k} [a_k^{(1)} \ldots a_k^{(N)}]^T, \quad z_k = [z_k^{(1)} \ldots z_k^{(N)}]^T, \]

where \( r_k^{(i)} \) is the \( i \)th component of observation vector at \( k \)th symbol interval, \( a_k^{(i)} \) is the \( i \)th coordinate of transmitted symbol (at \( k \)th symbol interval), and \( z \) is the corresponding noise vector (with Gaussian-like distribution of components). The \( \theta_k \) denotes the residual phase error (at \( k \)th time instance) due to laser phase noise, nonlinear phase noise and imperfect CPE. In polarization-division multiplexing (PDM), Equation (1) corresponds to either x- or y-polarization state. In 4-D signaling, the components above represent projections along in-phase and quadrature basis functions corresponding to x- and y-polarizations. The model above is applicable to few-mode fiber (FMF) applications as well. For instance, to describe the laser phase noise and imperfect CPE, the Wiener phase noise model can be used:

\[ \theta_k = (\theta_{k-1} + \Delta \theta_k) \mod 2\pi, \]

where \( \Delta \theta_k \) is zero-mean Gaussian process of variance \( \sigma_{\theta_k}^2 = 2\pi \Delta f T_s \), with \( T_s \) denoting the symbol duration and \( \Delta f \) denoting either linewidth or frequency offset. The cyclic slips can also be modeled by Markov-like process of certain memory. The probability density function (PDF) of the phase increment in (2) is given by

\[ p_{\theta_k}(\Delta \theta_k) = \sum_{n=-\infty}^{\infty} p(0, \sigma_{\Delta \theta_k}^2, \Delta \theta_k - n2\pi), \]
where \( p(0, \sigma_{\Delta \theta}^2, \Delta \theta - n2\pi) \) denotes the Gaussian PDF of zero-mean, variance \( \sigma_{\Delta \theta}^2 \), and argument \( \Delta \theta - n2\pi \). The resulting noise process is Gaussian-like with the power spectral density of \( N_0 \) so that the corresponding conditional probability density function is given by:

\[
p_R(r | a_k, \theta_k) = \frac{1}{\pi N_0} e^{-\frac{r^2}{2N_0}}.
\]

(4)

For non-Gaussian channels, we will need to use the method of histograms to estimate the conditional probability density function \( p_R(r | a_k, \theta_k) \).

3. Proposed LDPC coded modulation scheme suitable for ultra-high-speed optical transport in the presence of residual phase noise

For convenience, we can use the likelihood function defined as

\[
L(a_k, \theta_k) = \frac{p_R(r | a_k, \theta_k)}{p_R(r | a_k = 0)}.
\]

(5)

If the sequence of \( L=T/T_s \) statistically independent symbols, \( a=[a_1 \ldots a_L]^T \), is transmitted, the corresponding likelihood function will be:

\[
L(a, \theta) = \prod_{k=1}^{L} L(a_k, \theta_k).
\]

(6)

To avoid numerical overflow problems, the log-likelihood function should be used instead, so we have that

\[
I(a, \theta) = \log L(a, \theta).
\]

(7)

Clearly, the ML approach leads to exponential increase in complexity as sequence length \( L \) increases. Some other potential approaches to be used include factor graph based approaches [4], expectation-maximization (EM) algorithm [5], and blind turbo equalization [6], to mention few.

Here we propose a different strategy, which is inspired by Monte Carlo integration method. Namely, to calculate the log-likelihood function we will need to perform the following \( L \)-dimensional numerical integration:

\[
I(a) = \log \left[ \int \cdots \int \exp[I(a, \theta)] p_\theta(\theta) d\theta \right].
\]

(8)

Instead of numerical integration, we propose to use Monte Carlo integration. Namely, by using the Monte Carlo integration, the log-likelihood function \( I(a) \) can be estimated as:

\[
I(a) = \log E_\theta \left[ \exp[I(a, \theta)] \right],
\]

(9)

where the expectation averaging \( E_\theta \) is performed for different phase noise realizations. This method is particularly simple for memoryless phase noise processes, Wiener phase noise process and cyclic slip phase noise process described as a Markov process of reasonable memory. It can be shown that complexity of this method is \( O((m^2+L)N_F) \), where \( m \) is the channel memory, \( L \) is the sequence length and \( N_F \) is the number of phase noise realizations. Compared to the ML method, whose complexity is \( O(M^F) \), where \( M \) is the signal constellation size, complexity is significantly lower for long sequences to be detected. This method requires the knowledge of Markov phase noise process, which is quite easy to characterize by training. In particular, for the Wiener phase noise process and memoryless phase noise process, the Gaussian noise generator is only needed.

In Fig. 1, we depict one possible coded modulation scheme employing the proposed strategy. This scheme is suitable for 4-D signaling; however, it can be straightforwardly generalized for FMF applications and PDM. The \( b \) independent data streams are \((n,k)\) LDPC encoded, and corresponding encoder codewords are written row-wise into \( bxn \) block-interleaver. The \( b \) bits are taken from block-interleaver column-wise and used to select the 4-D constellation point in 4-D mapper. After digital-to-analog conversion (DAC) and pulse shaping, the corresponding pulse, representing a 4-D constellation point, is converted into optical domain by 4-D modulator; composed of polarization beam splitter (PBS), two I/Q modulators and polarization beam combiner; and transmitted to remote destination over an optical transmission system of interest. On receiver side, after conventional polarization-diversity receiver, four signals, representing in-phase and quadrature signals in two orthogonal polarizations, are upon analog-to-digital conversion (ADC) used as input to linear/nonlinear impairments compensator block, followed by CPE. After that the log-likelihood function is calculated based on Equation (9), and passed to the a posteriori probability demapper (calculating symbol and bit log-likelihood ratios (LLRs)). Once bit LLRs are calculated, the
LDPC decoders perform decoding operating in parallel. To improve the overall BER performance, the extrinsic information is iterated back and forward between APP demapper and LDPC decoders.

4. Performance Analysis

To demonstrate the efficiency of this method, it is applied to conventional and optimized modulation schemes, 2-D and 4-D signaling schemes, and the results of simulations (for optical channel model (1)) are summarized in Fig. 2 for symbol rate of $R_s=31.25$ GS/s by using a quasi-cyclic LDPC code of rate 0.8. The PDM 2-D constellation obtained by OSCD algorithm [7] and 4-D OSCD constellation, both of size 16, are evaluated against PDM 16-QAM in the presence of phase noise for two phase noise variances ($2\pi\Delta f_T$). It is evident that optimized modulation schemes, when used in combination with LDPC coding, are more robust in the presence of phase error than the conventional LDPC-coded QAM. Moreover, LDPC-coded 4-D signaling schemes show much better robustness compared to 2-D coded modulation schemes. For instance, PDM 16-ary OSCD coded modulation schemes is facing 0.33 dB degradation in the presence of phase noise at BER of $10^{-7}$ (for $\Delta f_T=10^{-3}$). On the other hand, the LDPC coded PDM 16-QAM is facing ~0.5 dB degradation.

5. Conclusion

The Monte Carlo integration is used to calculate symbol LLRs needed in soft iterative decoding in the presence of phase noise. Both conventional and optimized modulation schemes, 2-D and 4-D signaling schemes, have been evaluated by using the proposed method. The optimized modulation schemes, when used in combination with LDPC coding, have shown better robustness to the residual phase noise compared to the conventional QAM-based schemes.

References