Fractionally-spaced Frequency Domain Equalization for Optical Coherent Receivers with Interleaved Samplers

Yoshihiro Yasumura, Yuki Yoshida, and Ken-ichi Kitayama
Osaka University 2-1 Yamada-okas, Suita City, Osaka, Japan
yasumura@pn.comm.eng.osaka-u.ac.jp, {yuki,kitayama}@comm.eng.osaka-u.ac.jp

Abstract: We propose a fractionally-spaced frequency domain equalization technique for optical coherent receivers with interleaved samplers and experimentally demonstrate its robustness against the sampling timing mismatch and the IQ channel skew in 20Gbps coherent QPSK transmissions.

OCIS codes: (060.1660) Coherent communications; (060.2840) Heterodyne.

1. Introduction

High-speed analog-to-digital converter (ADC) with succeeding digital signal processing, such as electrical fiber dispersion compensation and carrier frequency offset mitigation, plays a key role in current digital coherent receivers. One performance limiting issue in such high-speed ADC based systems is the imperfection in the timing synchronization of the electrical front-ends, e.g., sampling timing offset, the channel skew between In- and Quadrature- phase (IQ) components, and the timing jitter [1]. So far, numbers of front-end DSP techniques has been proposed for deskewing, IQ balancing (orthogonalization), and sampling timing optimization [1,2]. Meanwhile fractionally-spaced equalizer (FSE), which exploits multiple samples resulting from the sampling at a rate higher than the symbol rate, is known to robust to the impairments in the synchronization. In FSEs, some of these impairments are taken as a part of the channel transfer function and compensated simultaneously [2].

In this paper, we propose the optimum linear fractionally-space frequency domain equalizer (FS-FDE) for coherent receivers with high-speed ADC based on interleaved sampling [1]. By adapting the delay between the time-interleaved samplers, the proposed FS-FDE can improve the tolerance for the imperfection in the synchronization, and relax the requirement for the analog and/or digital front-end of coherent receivers. Based on [1,2], we analytically derive the optimum linear FSE based on the minimum mean-square-error (MMSE) criterion and show the FSE can be efficiently implemented as a one-tap equalizer in the frequency domain. The performance of the proposed FS-FDE is experimentally demonstrated against sampling timing mismatch and IQ channel skew in 20Gbps coherent QPSK transmissions.

2. Optimum linear FS-FDE for coherent receivers with time-interleaved sampling

Fig. 1 shows the system configuration of the coherent receiver with 2 interleaved samplers and the proposed FS-FDE. For simplicity, we limit our discussion for the 2-fold oversampling case, which is commonly employed in current coherent receivers [1, 3], but the following approach can be easily extended to any oversampling rate. In Fig.1, the received optical signal is down-converted to the baseband domain and O/E converted by photo detectors (PDs). The PD output is then oversampled by 2 interleaved symbol-rate samplers. The sampling...
timing of them is assumed to be tunable and the minimum timing delay between the 2 samplers is set to \(T_s/K\) where \(T_s\) denotes the symbol duration and \(K\) is an integer. Their sampling timings might be represented as \(nT_s + \tau_1 T_s/K\) and \(nT_s + \tau_2 T_s/K\) where \(n\) in the symbol index, and the integers \(\tau_1\) and \(\tau_2\) represent the indexes for the skew as in Fig. 1. The multiple samples resulting from the oversampling are input to a linear FSE. Note that the computationally-efficient frequency domain implementation of the proposed FSE will be shown as a result of our derivation of the optimum linear FSE. To formulate the input-output relation of the system, we employ the FS-FDE model by using vectors and matrices [4]. Suppose \(s\) is \(M \times 1\) (\(M\) denotes block size) vector of the information-bearing signal block, \(r\) is a \(2M \times 1\) vector of the corresponding received signals after the oversampling and is given by

\[
r = \sum_{i=1}^{2} \Pi_{2M}^{i}U_{2M}^H \Pi_{KM}^{(i-1)} (H U_r s + n)
\]

where \(n\) is a \(KM \times 1\) noise vector which is after received filter. The channel is expressed by a \(KM \times KM\) circulant matrix \(H\) thanks to the cyclic prefix [5] whose column is given by \(KM \times 1\) channel impulse response vector \(h = [h(0), h(T_s/K), h(2T_s/K), \ldots, h(LT_s/K), h(0), \ldots, h(0)]\) where \(h(t)\) denotes the channel impulse response and \(L\) is the channel order \((M \gg L)\). The interleaved samplers can be considered to choose any 2 samples out of \(K\) samples. In Eq.1, the sample selection is expressed as the expander matrix \(U\) and shifting matrix \(\Pi\), (see [6] for their definitions). Basically, left definition of \(U^H\) equivalent to extract every \(K\)th rows of the multiplied matrix and \(\Pi^H\) represents the \(n\) times column-wise downshifting. Finally, the equalizer output \(\hat{s}\) is given by \(\hat{s} = Fr\) where the \(2M \times M\) matrix \(F\) represents a linear FSE.

Next we derive the optimum linear FSE based on the MMSE criterion. The mean-square error (MSE) of the equalizer output is given by \((\text{MSE}) = E[\|s(n) - \hat{s}(n)\|^2]\). The optimum FSE might be derived by solving \(\partial(\text{MSE})/\partial \mathbf{F} = 0\). After some calculations, we have the optimum equalizer \(F_{opt}\) as

\[
F_{opt} = \Gamma H W_{2M} \left( \Gamma H + \frac{1}{\sigma} \Delta R_{nn} \Delta^H \right)^{-1} W_{2M}^H =: W_{2M} \mathbf{\Lambda}_f \mathbf{\Lambda}_f^H W_{2M}^H
\]

where \(\sigma\) is the signal to noise ratio, \(\Delta := \sum_{i=1}^{2} \Pi_{2M}^{i+1} U_{2M}^H \Pi_{KM}^{(i-1)} \Gamma := \Delta H U^H, \text{and } W_N = N\text{-points discrete Fourier transform (DFT) matrix whose } (m,n)\text{-element is given by } W_N(m,n) = 1/\sqrt{N} \exp(-j2\pi(m-1)(n-1)/N) \text{ for simplicity, we assume the noise covariance matrix is diagonal, i.e., } \Delta R_{nn} \Delta^H = \text{diag}(\xi(0); \xi(1); \ldots; \xi(2M-1)) \text{ as in } [2]. \text{ By using the useful matrix decompositions given in } [4], \text{ } F_{opt} \text{ can be further simplified and represented by using the } 2M\text{-point DFT matrix } W_{2M}, \text{ the } M\text{-point IDFT matrix } W_{2M}^H, \text{ and the two diagonal matrices of the equalizer weights } \mathbf{\Lambda}_f \text{ and } \mathbf{\Lambda}_f^H. \text{ The } n\text{-th diagonal element of } \mathbf{\Lambda}_f(i = 1,2) \text{ is given by } \mathbf{\Lambda}_f(n) = \gamma(n + (i - 1)M)/\xi(i)(\mathcal{V}(n) + K)/\sigma \text{ where } \gamma(n) = \frac{M}{\sqrt{\pi/2}} \sum_{k=1}^{K} \sum_{i=1}^{K} \exp \left( \frac{-i(n-i)(n+(i-1)K+(K-1)M)}{2^{1/2}M} \right) \lambda(n \text{ mod } M + (k - 1)M), \text{ and } \mathcal{V}(i) = \sum_{k=1}^{K} \gamma(i + (k - 1)M)/\xi(i). \text{ As in Eq. 2, the optimum linear FSE can be implemented as computationally efficient one-tap equalizer in the DFT domain, moreover the IDFT operation can be realized by the IFFT with half the size of the FFT (See Fig. 2 also).}

3. Experimental Results

Fig. 2 shows the experimental setup for the back-to-back 20Gbps coherent QPSK transmission with the proposed FS-FDE. The electrical QPSK signal is generated via an arbitrary waveform generator (AWG) at the baud rate of 10 Gsps and then E/O converted by the Mach-Zehnder IQ modulator (MZ-IQ mod.). The lasers at the transceivers have a wavelength of 1550 nm with a linewidth of 100 kHz. The polarization and the power of the optical QPSK signal are then regulated by a polarization controller (PC) and an erbium-doped fiber amplifier (EDFA). The input optical power to the coherent receiver is -8 dBm. The coherent receiver consists of the optical 90-degree hybrid, balanced photo detectors and the oscilloscope whose sampling rate is 50GSa/s. All the
digital processing for the demodulation including the FS-FDE is done in off-line manner. To evaluate the IQ channel skew tolerance and the sampling timing mismatch, we introduce timing skews artificially between I and Q channels of the AWG outputs or the sampling timing at the DPO input as in Fig. 2.

Fig. 3 represents the error vector magnitude (EVM) performances against the IQ skew (left) and the sampling timing mismatch (right). The fractional sampling timing in the FS-FDE is chosen to be \((\tau_1, \tau_2) = (3, 4)\) (FSE\(_1\), the solid blue line) and \((\tau_1, \tau_2) = (3, 2)\) (FSE\(_2\), the dashed blue line). Note that the sampling rate at the DPO is 5 times higher than the symbol rate, hence \(K = 5\) and \(T_s/K = 20\) ps here. In addition, \(\tau = 3\) denotes the optimum sampling timing for conventional symbol-spaced equalizers. The red line FSE\(_{opt}\) means the best performance achieved by the proposed FS-FDE after the sampling timing adaptation. For comparison, we also test the conventional symbol-spaced FDE (SS-FDE) with the MMSE weights [5]. From Fig. 3 (left), we can see severe performance degradation in the SS-FDE for the IQ skew larger than 15 ps, meanwhile the FS-FDE achieves the EVM of less than 10% for -27 to +27 ps skews. As for the sampling timing offset tolerance, the notable performance improvement is achieved by the FS-FDE. As in Fig. 3(right), the performance of the SS-FDE quickly deteriorates as the offset value increases. On the other hand, the FS-FDE achieves less than 13% EVM for all the offset values. By adapting the delay between the two interleaved samplers, the proposed FS-FDE jointly mitigates the IQ channel skew and/or the sampling timing offset together with the channel impairments in the computationally efficient manner. This will relax the requirement for the front-end of the high-speed ADC based coherent receivers significantly.

![Experimental setup](image)

**Fig. 2** Experimental setup

**Fig. 3** EVM performances versus the IQ skew (left) and the sampling timing offset (right).

### 4. Conclusion

We proposed the optimum linear FS-FDE for optical coherent receivers with time-interleaved samplers, and experimentally demonstrated the robustness of the FS-FDE against the IQ channel skew and the sampling timing offset in the 20 Gbps QPSK transmission.

**Acknowledgement** This work has been financially supported by the R&D program, SCOPE funded by the Ministry of Internal Affairs and Communications (MIC), Japan (FY.2011-2013).

**Reference**