An effective signal detection algorithm with low complexity is presented for multiple-input multiple-output orthogonal frequency division multiplexing systems. The proposed technique, QR-MLD, combines the conventional maximum likelihood detection (MLD) algorithm and the QR algorithm, resulting in much lower complexity compared to MLD. The proposed technique is compared with a similar algorithm, showing that the complexity of the proposed technique with \( T=1 \) is a 95% improvement over that of MLD, at the expense of about a 2-dB signal-to-noise-ratio (SNR) degradation for a bit error rate (BER) of \( 10^{-3} \). Additionally, with \( T=2 \), the proposed technique reduces the complexity by 73% for multiplications and 80% for additions and enhances the SNR performance about 1 dB for a BER of \( 10^{-3} \).

Keywords: MIMO-OFDM, QR decomposition, QR-MLD algorithm, maximum likelihood detection (MLD).
from minimum to maximum. Then, the first \( T \) signals are detected by the QR algorithm. Finally, the conventional MLD algorithm is used to detect the last \( N_r - T \) signals, where \( N_r \) represents the number of transmitter antennas. If the value of \( T \) is small, the system performs well and has high computational complexity, which is similar with the MLD algorithm. On the other hand, when the value of \( T \) is large, the performance quality diminishes and the complexity of the detection scheme is reduced. Hence, the performance and the computational complexity of the QR-MLD algorithm can be controlled by adjusting the value of this new parameter \( T \). The proposed technique is compared with MLD and QRDM-QR. Even though QRDM-QR \([4]\) is a suitable signal detection algorithm in both performance and complexity, it is much worse than QR-MLD at a given value of \( T \). Compared with MLD, QR-MLD significantly reduces the computational complexity at the expense of minimal degradation of the bit error rate (BER) performance. Therefore, the proposed QR-MLD detection algorithm demonstrates desirable performance and low complexity.

From simulation and computational complexity comparison results, the complexity of the proposed technique with \( T=1 \) is reduced by 95% compared with that of MLD at the expense of about a 2-dB signal-to-noise-ratio (SNR) degradation for a BER of \(10^{-3} \). Compared with MLD, QR-MLD significantly reduces the computational complexity at the expense of minimal degradation of the bit error rate (BER) performance. Therefore, the proposed QR-MLD detection algorithm demonstrates desirable performance and low complexity.

The rest of this paper is organized as follows. The model for MIMO-OFDM systems is introduced in section II. In section III, two conventional MIMO detection algorithms are briefly described. In section IV, the proposed technique combining MLD and the QR detection technique is presented. A computational complexity comparison is described in section V. The simulation results and discussion are presented in section VI, and the conclusion is presented in section VII.

II. System Model

1. System Model

Consider a wireless MIMO-OFDM system with \( N_t \) transmitting antennas and \( N_r \) receiving antennas, supposing \( N_t \geq N_r \). This system can be denoted by \((N_t, N_r)\). The transmitter and receiver structures of the MIMO-OFDM system are shown in Figs. 1 and 2. For the transmitter side, the input information source is multiplexed to \( N_r \) symbol streams by serial-to-parallel (S/P) converting. Then, the symbols are passed through the constellation mapping, the inverse fast Fourier transform (IFFT) modulators, and the cyclic prefix (CP) insertion. Finally, the \( N_r \) OFDM signals are transmitted by \( N_t \) antennas.

For the receiver side, after the cyclic prefix is removed, the \( N_r \) symbols are launched into the fast Fourier transform (FFT) for the demodulation and detection. Finally, the \( N_r \) symbols are output by parallel-to-serial (P/S) converting. After the demodulation, the receiver signals can be given by

\[
R(k) = HS(k) + W(k),
\]

where \( R(k) \) is the \( N_r \times 1 \) vector of received signals, \( S(k) \) is the \( N_r \times 1 \) vector of transmitted signals, \( W(k) \) is the \( N_r \times 1 \) vector additive white Gaussian noise (AWGN), and \( H \) is the \( N_r \times N_t \) MIMO channel matrix. We can denote \( R(k)=[R_0(k), \ldots, R_{N_r}(k)]^T \), \( S(k)=[S_0(k), \ldots, S_{N_r}(k)]^T \), and \( W(k)=[W_0(k), \ldots, W_{N_r}(k)]^T \), where \( S_0(k) \) and \( W_0(k) \) are the \( k \)-th subcarrier of the complex baseband signal transmitted by the \( i \)-th antenna and \( K \) is the length of the OFDM data sequence. \( H \) can be represented as

\[
H = \begin{bmatrix}
h_{11} & h_{12} & \cdots & h_{1N_t} \\
h_{21} & h_{22} & \cdots & h_{2N_t} \\
\vdots & \vdots & \ddots & \vdots \\
h_{N_r,1} & h_{N_r,2} & \cdots & h_{N_r,N_t}
\end{bmatrix} = (h_{ij})_{N_r \times N_t},
\]

where \( h_{ij} \) is the channel impulse response from transmitter \( j \) to receiver antenna \( i \). Here are some assumptions:

(A) \( W(k) \) is zero mean, circularly symmetric complex Gaussian with \( \mathcal{E}(W(k)W(k)^H) = \sigma^2 I_{N_r} \);

(B) \( S(k) \) is zero mean and with \( \mathcal{E}(S(k)S(k)^H) = I_{N_r} \).
(C) $W(k)$ and $S(k)$ are independent of each other.

2. Channel Model

Let $H$ be the perfect channel state information and well known by the detector. Each element of $H$ is an asymmetric complex Gaussian distribution with zero mean and unit variance, which are independent of each other. We consider the MIMO channel to be a quasistatic flat Rayleigh fading channel. From [12], we know that the envelope response at any time has a Rayleigh probability distribution and that the phase is uniformly distributed in the interval $(0, 2\pi)$. That is, $E(r)=0$ (zero mean).

From [4], we know that the Rayleigh fading can be defined as:

$$h(t) = \frac{1}{\rho} \sum_{j=1}^{\rho} \rho_j e^{-j\pi n t} \cdot S(t - \tau_j), \quad (4)$$

where $S(t)$ is the input signal, $\theta$ is the phase shift from the scattering of the $l$-th path, $\rho$ is the attenuation of the $l$-th path, $\tau$ is the relative delay of the $l$-th path, and $\mu$ is the total number of multipaths.

III. Conventional MLD and QR Detection Algorithms

In this section, we give an overview of the conventional optimal MLD algorithm and QR algorithm.

1. MLD Algorithm

   The MLD can be written as
   
   $$S(k)_{\text{MLD}} = \arg \min_{S(k) \in \Omega} \| R(k) - HS(k) \|^2, \quad (5)$$

   where $\Omega$ denotes the modulation constellation. To obtain the MLD, an exhaustive search is conducted $|\Omega|^N$ times, where $|\Omega|$ represents the total number of elements of the constellation set. When both $N_t$ and $\Omega$ are large, the complexity of the MLD algorithm is extremely high. Therefore, it is difficult to use in the practical detection process.

2. QR Algorithm

   We apply QR decomposition to (1) with $H=QR$, where the $N_t \times N$ matrix $Q$ is an orthonormal matrix satisfying $Q^H Q = I_N$ and matrix $R$ is an $N \times N_t$ upper triangular matrix with $R_{jj} (j \geq i)$ denoting its non-zero elements and $R_{jj} (j=1, 2, \ldots, N_t)$ being a positive real number. We have
   
   $$R(k) = QRS(k) + W(k). \quad (6)$$

   Therefore,
   
   $$Y(k) = R(k) + n(k), \quad (7)$$

   where $Y(k)=Q^H R(k)$, $n(k)=Q^H W(k)$, and $n(k)$ are still AWGN.

   From [4], we know that the Rayleigh fading can be defined as:
   
   $$Y(k) = Q_{N_t} S_N (k) + n_N (k). \quad (8)$$

   Finally, the decision statistic (9) can be used to estimate $S_N (k)$ as
   
   $$\hat{S}_N (k) = \text{quant}(Y_N (k) / R_{N_t}), \quad (10)$$

   The process of QR detecting can be described by using the following recursive algorithm.

   $$\hat{S}_i (k) = \text{quant}((Y_i (k) - \sum_{j=1}^{N_t} R_{ij} \hat{S}_j (k)) / R_{ii}), \quad (11)$$

   Although the QR detection scheme has low computational complexity, it suffers from error propagation. If the $i$-th layer is detected successively and incorrectly, those faulty decisions will affect the subsequent decisions.

IV. Combined MLD and QR Detection Algorithm

1. Algorithm Development

   To lower the complexity of the MLD algorithm and improve the performance of the QR algorithm, a combined MLD and QR detection technique is proposed for the MIMO-OFDM systems. We denote this proposed detection algorithm as QR-MLD. During the detection process, a new parameter $T$ is adopted to decrease the detection complexity. The value of parameter $T$ determines the number of signals detected by the QR algorithm, and the number $N_t - T$ determines the number of signals detected by the MLD algorithm. Obviously, the value of $T$ is a set. That is, $T=\{0, 1, \ldots, N_t\}$. The whole process of the QR-MLD detection algorithm is as follows.

   **Step 1:** We sort the columns of the channel matrix $H$ by the column norm from minimum to maximum. That is,
\[ H_{\text{new}} = \text{sort}((\|H_1\|^2, \|H_2\|^2, ..., \|H_N\|^2)), \]  

where \( \text{sort}(\cdot) \) represents a sorting from minimum to maximum.  
\( \|H_i\|^2 \) is the norm of the \( i \)-th column of the channel matrix \( H \).  
The sorting order can be recorded in a set, denoted by \( \Omega \).

**Step 2**: The detection process with QR is executed for the signals.  
Note that the channel matrix \( H \) is represented by \( H_{\text{new}} \) and that parameters \( R(k), S(k), \) and \( W(k) \) are ordered by set \( B \).  
The first \( T \) signals can be obtained by (13)  
\[ S_i(k) = \text{quant}(\frac{1}{R_{ij}} Y_i(k) - \sum_{j=1}^{N_t} R_{ij} S_j(k)), \quad i = 1, 2, ..., N_t - T + 1, \]  
where \( S_i(k) = 0 \) if the value of \( T \) is zero.  
Reserve the \( T \) detected signals as  
\[ S_{\text{QR}}(k) = [S_{N_t-T+1}(k), ..., S_N(k)]^T. \]  

**Step 3**: The last \( N_t - T \) signals are detected by the MLD algorithm.  
\[ S_{\text{MLD}}(k) = \arg \min_{S(k) \in \Omega^{N_t+T}} \| R(k) - H[ S_{\text{MLD}}(k)^T + S_{\text{QR}}(k)^T ] \|^2, \]  
where \( \Omega \) denotes the modulation constellation set and an exhaustive search will be conducted \( |\Omega|^{N_t+T} \) times.

**Step 4**: Finally, the estimate of the transmitted signal is  
\[ S(k) = [S_{\text{MLD}}(k)^T, S_{\text{QR}}(k)^T]^T. \]  
According to the first sorting order recorded in set \( B \) in Step 1, sort \( S(k) \) by \( S_{\text{last}} = \text{sort}(S) \) and then the output \( S_{\text{last}} \) is the result of the QR-MLD detection algorithm.

2. Algorithm Analysis

At the beginning of the QR-MLD algorithm, the channel matrix \( H \) is sorted as the column norm from minimum to maximum.  
In the first detection stage, when the QR detection technique is executed, its reliability has greatly enhanced since the maximum column norm represents the maximum SNR.  
The bigger the SNR is, the more reliable the detection signal is.  
In the second stage, the conventional optimal MLD algorithm is executed.  
Its performance is the best among other detection algorithms.  
Due to the first stage of the QR detection scheme, the exhaustive search will be conducted \( |\Omega|^{N_t+T} \) times, rather than \( |\Omega|^N \).  
When \( T \neq 0 \), its complexity will be reduced significantly.  
Obviously, when \( T = 0 \), the proposed algorithm works as if it is the MLD algorithm, and its performance is much better than that of the QR algorithm, considering that channel \( H \) is sorted at the beginning of the QR-MLD algorithm.  
The comparison of the computational complexity among different algorithms will be given in section V.

V. Computational Complexity Comparison

In this section, we calculate the computational complexity for the proposed QR-MLD detection scheme, conventional MLD algorithm, and QRDM-QR algorithm proposed in [4].  
Real multiplications and additions are considered for the purpose of computational complexity comparison.  
From [4], we know that we count the total number of multiplications as four if there is one complex multiplication operation and we count the total number of additions as two if there is one complex addition operation.  
For simplicity, we suppose \( N_{c}^r = N_{c} \) and \( L\)-QAM modulation is used.  
From [13], [14], we know that when \( M = 2L \), the performance of the QRDM algorithm is similar to that of the MLD algorithm, where \( M \) represents the number of branches kept in every layer.  
Therefore, to compare the complexity between QR-MLD and QRDM-QR, we suppose \( M = 2L \).

The number of multiplications, \( N_{\text{mul}}^{\text{MLD}} \), for the MLD detection algorithm can be written as  
\[ N_{\text{mul}}^{\text{MLD}} = 4L^N(N + 1). \]  
The number of additions, \( N_{\text{add}}^{\text{MLD}} \), for the MLD detection algorithm can be written as  
\[ N_{\text{add}}^{\text{MLD}} = 4L^N(N + 1) + 2(N + 1)N + 2N + (N - 1). \]  
The number of multiplications, \( N_{\text{mul}}^{\text{QR}} \), for the QR detection algorithm can be written as  
\[ N_{\text{mul}}^{\text{QR}} = 4N^3 + 4N^2 + 8L + 4L^2 + \sum_{n=2}^{N} 4Ln. \]  
The number of additions, \( N_{\text{add}}^{\text{QR}} \), for the QR detection algorithm can be written as  
\[ N_{\text{add}}^{\text{QR}} = 4N^3 + 4N^2 + 10L + \sum_{n=2}^{N} (4Ln + 2Ln) + 4L^2 + 2N. \]  
Therefore, the number of the multiplications, \( N_{\text{mul}}^{\text{QR-MLD}} \), for the QR-MLD detection algorithm can be written as  
\[ N_{\text{mul}}^{\text{QR-MLD}} = 4N^3 + 4N^2 + 8L + 4L^2 + \sum_{n=2}^{N} 4Lt \]  
\[ + 4L^{N-T} N(N - T + 1) \]  
\[ + 4N^2 \]  
\[ + 4N^2 \]  
The number of additions, \( N_{\text{add}}^{\text{QR-MLD}} \), for the QR-MLD detection algorithm can be written as  
\[ N_{\text{add}}^{\text{QR-MLD}} = 4N^3 + 4N^2 + 10L + \sum_{n=2}^{N} (4Lt + 2Lt) + 4L^2 + 2N \]
$$+ 4L^{N-T}(N-T+1) + 2(N-T-1) + 2(N-T) + (N-T-1)$$

\[c_5\]

$$+ 4N^2 + 2N(N-1).$$

(21)

From [4], we know that the computational complexity of the QRDM-QR algorithm is as follows:

$$N_{\text{mul}}^{\text{QRDM-QR}} = 4N^3 + 4N^2 + 8L + \sum_{j=2}^{N-1} 4D_jL + \sum_{j=2}^{N-1} 4D_jL$$

\[c_7\]

\[
+ 4D_jN^2 + \sum_{i=N-j+1}^{N} D_i4t,
\]

(22)

$$N_{\text{add}}^{\text{QRDM-QR}} = 4N^3 + 4N^2 + 10L + \sum_{j=2}^{N-1} (4D_jL + 4D_jL)$$

\[c_9\]

\[
+ \sum_{i=2}^{N} (4D_jL + 4L) + \sum_{i=N-j+1}^{N} D_i(4t + 2t) + 4D_jN^2 + 2N,
\]

(23)

where \(D_i = ML = L^2\) and \(D_k = L\).

In (20) through (23), \(C_1\) and \(C_4\) are the number of multiplications and additions of the QR process in the QR-MLD algorithm. Similarly, \(C_2\) and \(C_5\) represent the number of multiplications and additions of the MLD process in the proposed algorithm.

Finally, \(C_3\) and \(C_6\) are the number of multiplications and additions in the sorting process in the QR-MLD detection scheme. Furthermore, \(C_7\) and \(C_9\) are the number of operations to calculate multiplications and additions of the QRDM process in the QRDM-QR algorithm, and \(C_8\) and \(C_{10}\) are the number of operations to calculate multiplications and additions of the QR process in the QRDM-QR technique.

The computational complexity of the QR-MLD detection algorithm compared with MLD and the QRDM-QR is illustrated in Table 1 and Table 2 with 16-QAM modulation and \(N_t=N_r=4\).

From Tables 1 and 2, it is interesting to note that the complexity of the proposed detection algorithm is significantly reduced for both multiplications and additions compared with the conventional MLD algorithm whenever \(T\) is equal to 1, 2, 3, or 4. Furthermore, compared with the QRDM-QR algorithm proposed in [4], the complexity of the QR-MLD algorithm is reduced by 73% for multiplications and 80% for additions when the value of \(T\) is 2. When \(T\) is equal to 3 or 4, the complexity of the QR-MLD algorithm is similar with that of QRDM-QR. The principle to choose the proper value of \(T\) for different \(N_t\) and \(N_r\) and the performance simulation of the proposed technique are presented in section VI.

VI. Simulation Results and Discussion

1. Simulation Results

In this subsection, computer simulations are presented to
evaluate the performance of the proposed QR-MLD algorithm for MIMO-OFDM systems. Assuming perfect channel estimation, the receiver detection is based on a five-path Rayleigh fading environment without channel coding. During the simulation, the length of the OFDM data sequence \( K \) is equal to 512, \( N_t = N_r = 4 \), and 16-QAM modulation is used. Figure 3 shows the BER performance of the MLD algorithm and the proposed QR-MLD algorithm with \( T \) being equal to 1, 2, 3, and 4, respectively. Figure 4 plots the BER performance of the proposed QR-MLD algorithm and the QRDM-QR algorithm.

2. Discussion

First, the BER performance of the QR-MLD algorithm is compared with that of the MLD algorithm in Fig. 3. The lower the value of \( T \) is, the better the BER performance of the QR-MLD algorithm is. From Table 1, Table 2, and Fig. 3, it can be concluded that in the case of \( N_t = N_r = 4 \), with \( T = 1 \), the performance of the proposed QR-MLD algorithm reflects only a 2-dB degradation for a BER of \( 10^{-3} \) and a 95% decrease in complexity compared with the MLD algorithm. It is reasonable to decrease the complexity of the algorithm by 95% at the expense of a 2-dB performance degradation because the conventional MLD algorithm is difficult to use in practical systems.

As shown in Fig. 4, although the BER performance of the QRDM-QR detection algorithm is similar to that of the proposed QR-MLD detection algorithm, there are notable differences. For example, when the value of \( T \) is 2, compared with that of the QRDM-QR algorithm, the performance of the QR-MLD algorithm is enhanced about 1 dB for a BER of \( 10^{-3} \). Furthermore, considering the complexity of the algorithm, it is amazing to find that the complexity of the proposed technique is decreased by 73% for multiplications and 80% for additions compared with the QRDM-QR detection algorithm. When the value of \( T \) is 3 or 4, although the computational complexity of QR-MLD is about 5% higher than that of QRDM-QR, the performance of QR-MLD is about 1 dB improved for a BER of \( 10^{-3} \) compared with the QRDM-QR algorithm.

Finally, we can conclude that even though QRDM-QR is a suitable detection algorithm regarding performance and complexity, the level of performance and complexity of QR-MLD when an appropriate value of \( T \) is selected is better. Compared with the conventional MLD algorithm, it is reasonable to use the QR-MLD detection technique with much lower complexity. Therefore, the proposed selective technique can be efficiently used for practical MIMO-OFDM systems requiring very low complexity by assigning the proper value of \( T \).

VII. Conclusion

In this paper, a highly efficient and minimally complex algorithm for MIMO-OFDM systems was proposed. The proposed QR-MLD algorithm enhances the detection performance by applying the MLD technique and decreases the computational complexity by using the QR algorithm. Simulation results show that the complexity of the proposed technique with \( T = 1 \) is reduced by 95% compared with that of MLD, at the expense of about a 2-dB SNR degradation for a BER of \( 10^{-3} \). Its performance and complexity are much better than those of the effective QRDM-QR algorithm by appropriately selecting \( T \). Considering that the conventional MLD scheme is difficult to use in practical MIMO-OFDM systems because of its high complexity, the proposed QR-MLD detection scheme can be effectively used. In conclusion, by assigning the proper value of \( T \), the proposed combined MLD and QR detection technique can be used for MIMO-OFDM systems requiring high-level performance and very low complexity.

References


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