Analog Modulation System (3)

LC 5-6

Lecture 9, 2008-10-10

Contents

- Linear vs. Angle Modulation
- Phase modulation and frequency modulation
- Narrowband frequency modulation
- Spectrum of angle modulation system
- Power of angle modulation system
- Angle modulator

Linear vs. Angle Modulation Methods

- Amplitude modulation (AM) methods are also called linear modulation methods, although conventional AM is not linear in the strict sense.
- In phase modulation systems, the phase of the carrier is changed according to the variations in the message signal.
- In frequency modulation system, the frequency of carrier is changed by the message signal.
- PM and FM are special cases of angle modulation methods. They are obviously quite nonlinear.

Representation of PM and FM signals

- The complex envelop is

$$g(t) = A_c e^{j\theta(t)}$$

- The angle modulated signal is

$$s(t) = A_c \cos[\omega_c t + \theta(t)]$$

- For PM, the phase is directly proportional to the message signal

$$\theta(t) = D_p m(t)$$

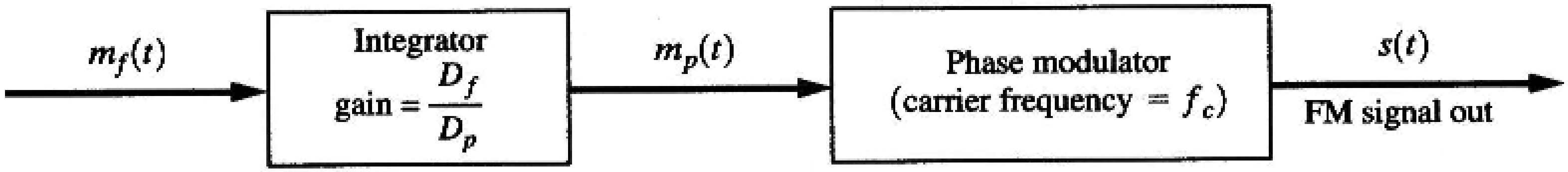
D_p is the phase sensitivity

- For FM, the phase is proportional to the integral of message signal

$$\theta(t) = D_f \int_{-\infty}^{t} m(\sigma) d\sigma \Rightarrow f_d(t) = \frac{1}{2\pi} \frac{d\theta(t)}{dt} = \frac{D_f}{2\pi} m(t)$$

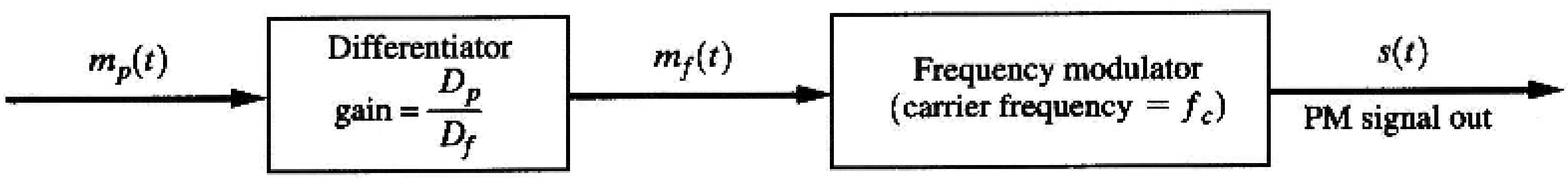
D_f is the frequency deviation constant

Comparison of PM and FM



(a) Generation of FM Using a Phase Modulator

$$m_f(t) = \frac{D_p}{D_f} \frac{dm_p(t)}{dt}$$



(b) Generation of PM Using a Frequency Modulator

$$m_p(t) = \frac{D_f}{D_p} \int_{-\infty}^{t} m_f(\sigma) d\sigma$$

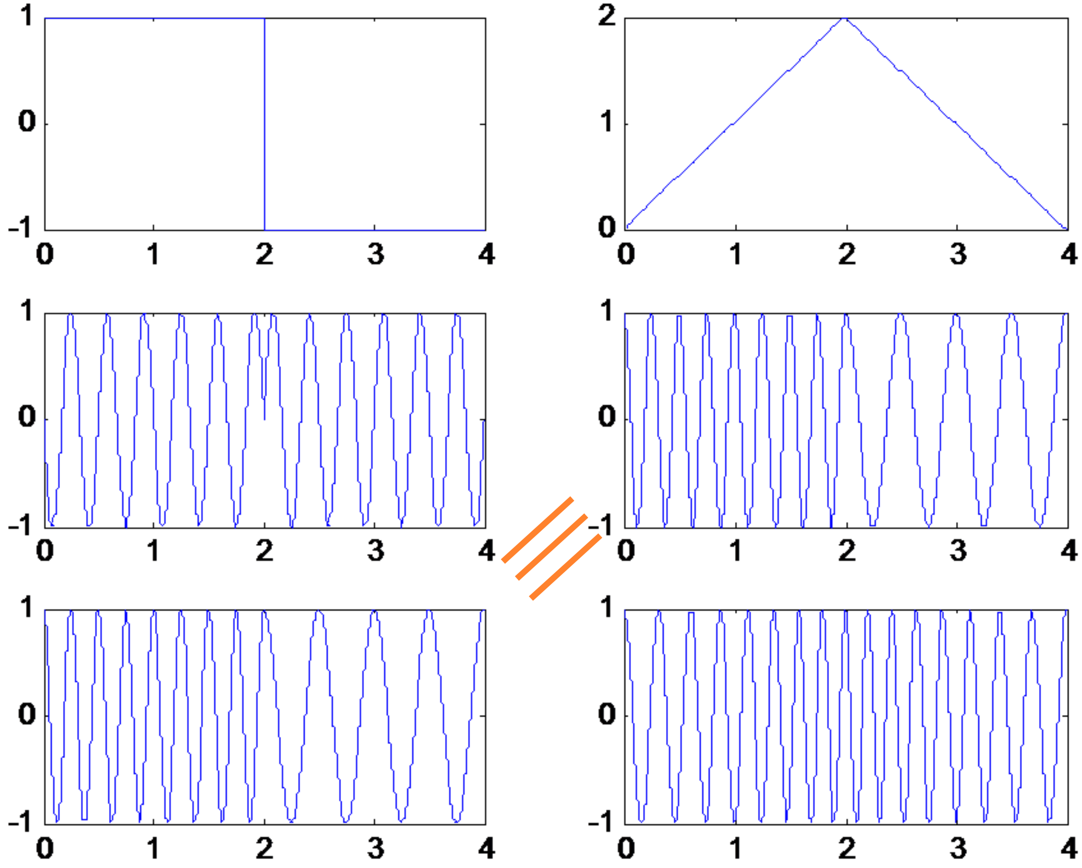
Example (1)

1 $$m_1(t) = \frac{dm_2(t)}{dt}$$ 2

m(t)

PM

FM



Example (2)

- Message signal $m(t) = V_p \cos(2\pi f_m t)$
- Carrier $A_c \cos(2\pi f_c t)$
- In PM $\theta(t) = D_p m(t) = D_p V_p \cos(2\pi f_m t) = \beta_p \cos(2\pi f_m t)$

β_p is phase modulation index

- In FM

$$\theta(t) = D_f \int_{-\infty}^{t} m(t)dt = \frac{D_f V_p}{2\pi f_m} \sin(2\pi f_m t) = \beta_f \sin(2\pi f_m t)$$

β_f is frequency modulation index

Modulation Index

- We can extend the definition for a general non-sinusoidal signal
- If a bandpass signal is represented by $s(t) = R(t)\cos[\psi(t)], \psi(t) = \omega_c t + \theta(t)$
- The instantaneous frequency of s(t) is

$$f_i(t) = \frac{1}{2\pi}\frac{d\psi(t)}{dt} = \frac{1}{2\pi}\frac{d}{dt}[\omega_c t + \theta(t)] = f_c + \frac{1}{2\pi}\frac{d\theta(t)}{dt}$$

- In FM, the peak frequency deviation from the carrier frequency is

$$\Delta F = \max[f_d(t)] = \frac{1}{2\pi}D_f \max[m(t)] = \frac{1}{2\pi}D_f V_p$$

$V_p = \max[m(t)]$

- The frequency modulation index is $\beta_f = \frac{\Delta F}{B}$

B is the bandwidth of the message signal.

- In PM, the peak phase deviation is $\Delta\theta = D_p \max[m(t)] = D_p V_p$
- The phase modulation index is $\beta_p = \Delta\theta$

Narrowband Frequency Modulation

If $\left| D_f \left[\int_{-\infty}^{t} m(\tau) d\tau \right] \right| << \frac{\pi}{6} (\text{or } 0.5)$, narrowband frequency modulation (NBFM)

$$s_{FM}(t) = A\cos\left[\omega_c t + D_f \int_{-\infty}^{t} m(\tau)\mathrm{d}\tau \right]$$

$$= \cos\omega_c t \cos\left[D_f \int_{-\infty}^{t} m(\tau)\mathrm{d}\tau \right] - \sin\omega_c t \sin\left[D_f \int_{-\infty}^{t} m(\tau)\mathrm{d}\tau \right]$$

$$\because \quad \cos\left[D_f \int_{-\infty}^{t} m(\tau) d\tau \right] \approx 1$$

$$\sin\left[D_f \int_{-\infty}^{t} m(\tau) d\tau \right] \approx D_f \int_{-\infty}^{t} m(\tau) d\tau$$

$$\therefore \quad s_{NBFM}(t) \approx \cos\omega_c t - \left[D_f \int_{-\infty}^{t} m(\tau) d\tau \right] \sin\omega_c t$$

NBFM vs. AM

$$\because \quad \left[\int m(t)dt\right]\sin\omega_c t \Leftrightarrow \frac{1}{2}\left[\frac{F(\omega+\omega_c)}{\omega+\omega_c}-\frac{F(\omega-\omega_c)}{\omega-\omega_c}\right]$$

$$\therefore \quad S_{NBFM}(\omega)=\pi\left[\delta(\omega+\omega_c)+\delta(\omega-\omega_c)\right]+\frac{D_f}{2}\left[\frac{F(\omega-\omega_c)}{\omega-\omega_c}-\frac{F(\omega+\omega_c)}{\omega+\omega_c}\right]$$

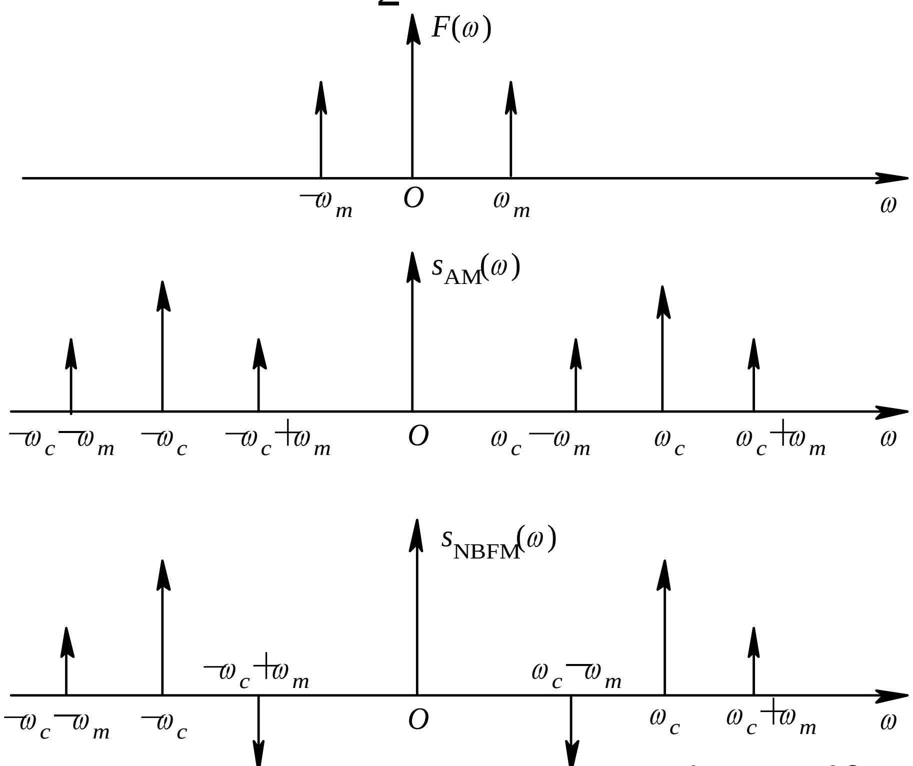
$$S_{AM}(\omega)=\pi\left[\delta(\omega+\omega_c)+\delta(\omega-\omega_c)\right]+\frac{1}{2}\left[M(\omega+\omega_c)+M(\omega-\omega_c)\right]$$

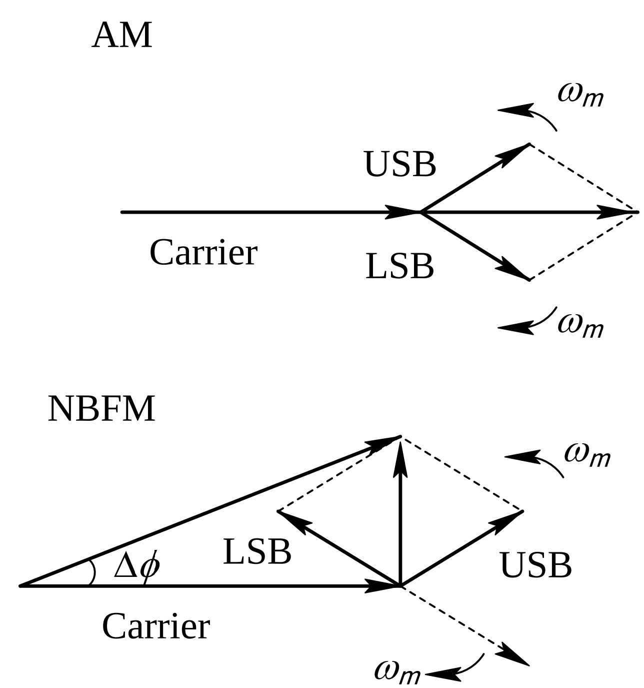
Example

$$m(t) = A_m \cos \omega_m t$$

$$s_{NBFM}(t) = \cos \omega_c t + \frac{A_m D_f}{2\omega_m}\left[\cos(\omega_c + \omega_m)t - \cos(\omega_c - \omega_m)t\right]$$

$$s_{AM}(t) = \cos \omega_c t + \frac{A_m}{2}\left[\cos(\omega_c + \omega_m)t + \cos(\omega_c - \omega_m)t\right]$$





Spectrum of Angle Modulated Signals

- Due to the nonlinearity of angle-modulation systems the precise characterization of their spectral properties, even for simple message signals, is mathematically intractable.
- We will study the spectral characteristics of an angle modulation signal in the case of a sinusoidal signal

Example 5-2

The complex envelope is

$$g(t) = A_c e^{j\theta(t)} = A_c e^{j\beta \sin \omega_m t} = \sum_{n=-\infty}^{\infty} c_n e^{jn\omega_m t}$$

where

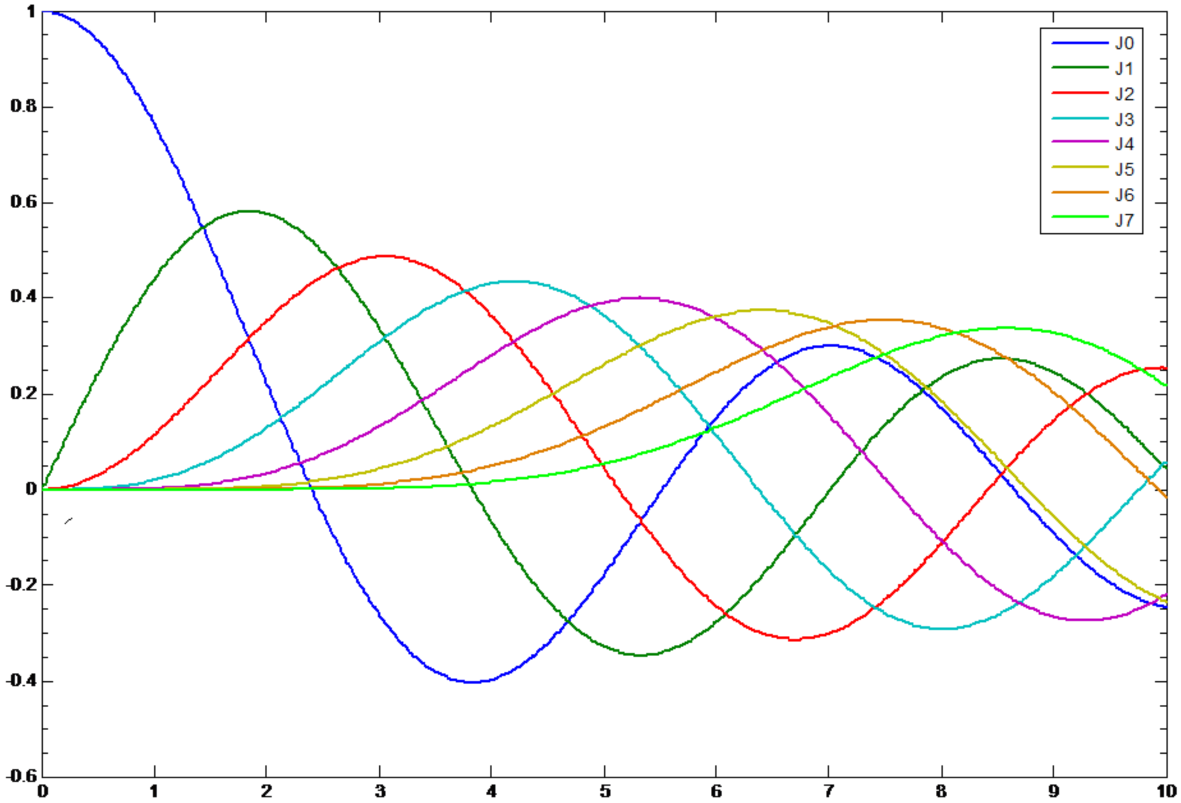
$$c_n = \frac{1}{T_m}\int_{-T_m/2}^{T_m/2} g(t) e^{-jn\omega_m t} dt = \frac{1}{T_m}\int_{-T_m/2}^{T_m/2} A_c e^{j\beta \sin \omega_m t} e^{-jn\omega_m t} dt$$

$$= \frac{1}{\omega_m T_m}\int_{-\omega_m T_m/2}^{\omega_m T_m/2} A_c e^{j\beta \sin \omega_m t} e^{-jn\omega_m t} d\omega_m t$$

$$= \frac{1}{2\pi}\int_{-\pi}^{\pi} A_c e^{j(\beta \sin\theta - n\theta)} d\theta = A_c\left[\frac{1}{2\pi}\int_{-\pi}^{\pi} e^{j(\beta \sin\theta - n\theta)} d\theta\right] = A_c J_n(\beta)$$

$J_n(\beta)$: the Bessel function of the first kind of the nth order.

Angle Modulation by a Sinusoidal Signal



Bessel Function

1. $J_n(\beta) = (-1)^n J_{-n}(\beta)$

2. When is β very small

$J_0(\beta) \approx 1$

$J_1(\beta) \approx \beta / 2$

$J_n(\beta) \approx 0, \ n > 2$

3. $\sum_{n=-\infty}^{\infty} J_n^2(\beta) = 1$

The spectrum of the complex envelope is

$$G(f) = \sum_{n=-\infty}^{\infty} c_n \delta(f - nf_m) = A_c \sum_{n=-\infty}^{\infty} J_n(\beta)\delta(f - nf_m)$$

The spectrum of the angle-modulated signal is

$$S(f) = \tfrac{1}{2}[G(f - f_c) + G^*(-f - f_c)]$$

For $f > 0$

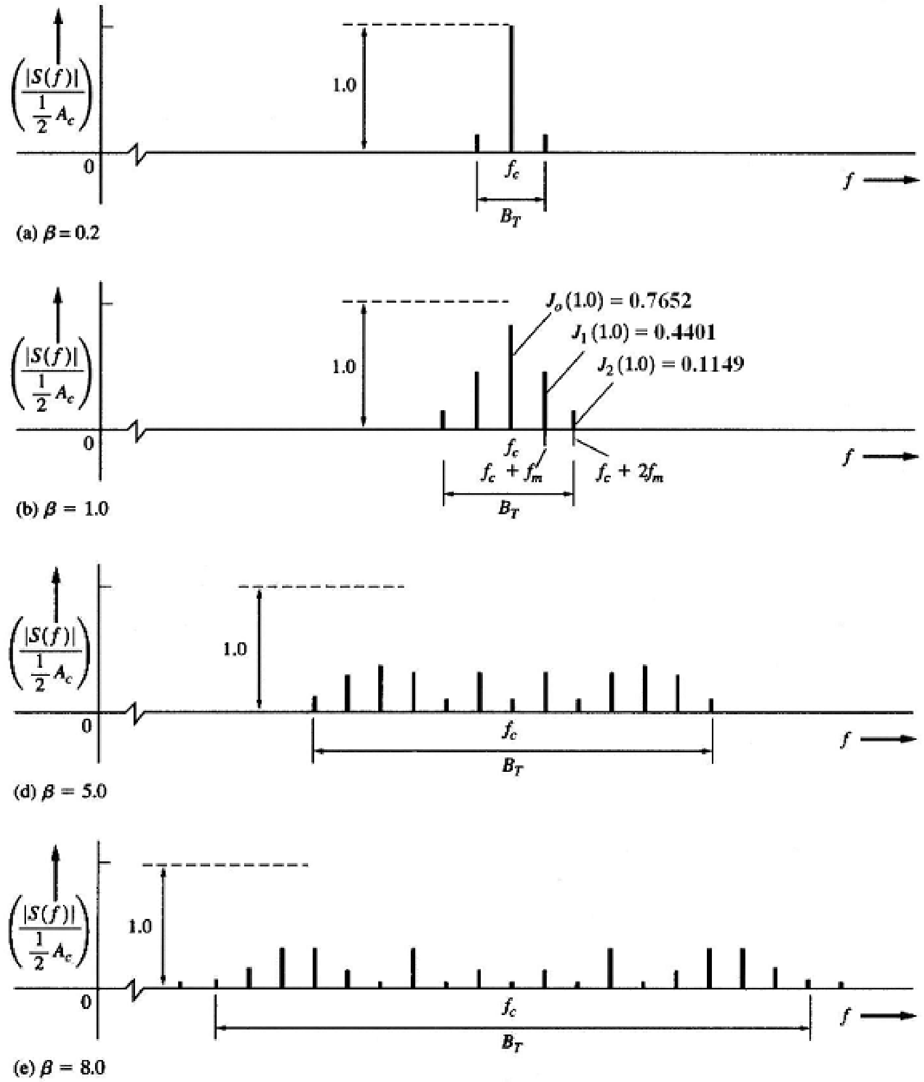
$$|S(f)| = \tfrac{1}{2} A_c \sum_{n=-\infty}^{\infty} |J_n(\beta)| \delta(f - f_c - nf_m)$$

Carson's Rule

- Carson's rule

$$B_T = 2(\beta + 1)B$$

The effective bandwidth of an angle-modulated signal, which contains at least 98% of the signal power.



Power in an Angle Modulation Signal

- The normalized power of angle modulation signal:

$$\left\langle s^2(t) \right\rangle = A_c^2 \left\langle \cos^2\left[\omega_c t + \theta(t)\right] \right\rangle$$

$$= \frac{1}{2} A_c^2 + \frac{1}{2} A_c^2 \left\langle \cos 2\left[\omega_c t + \theta(t)\right] \right\rangle$$

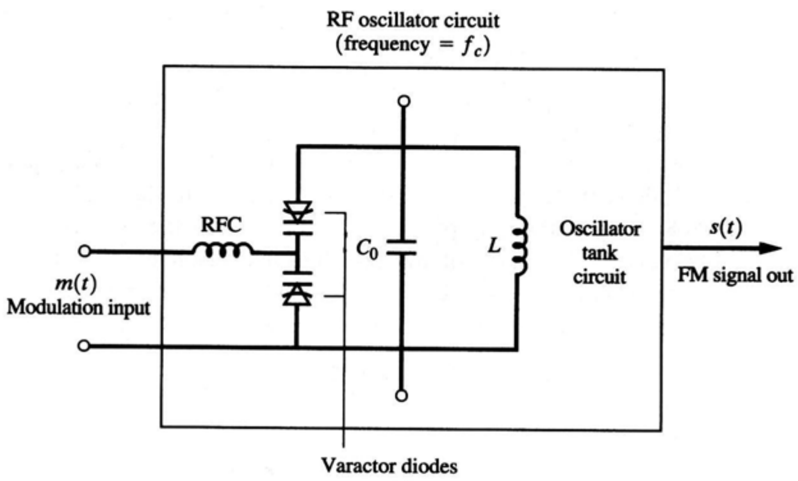
- If the carrier frequency is large so that s(t) has negligible frequency content in the region of dc, the second term is negligible

$$\left\langle s^2(t) \right\rangle = \frac{1}{2} A_c^2$$

- Thus the power contained in the output of an angle modulator is independent of the message signal.

Angle Modulators

- Direct FM – Varactor Diodes



(b) A Frequency Modulator Circuit

Figure 5–8 Angle modulator circuits. RFC = radio-frequency choke.

Assume that $\Delta C = K_c m(t)$ and $\frac{\Delta C}{C_0} << 1$

$$f_c = \frac{1}{2\pi\sqrt{LC_0}},\ f_0 = \frac{1}{2\pi\sqrt{L(C_0+\Delta C)}}$$

$$\Delta f = f_0 - f_c = \frac{1}{2\pi\sqrt{L(C_0+\Delta C)}} - \frac{1}{2\pi\sqrt{LC_0}}$$

$$= \frac{1}{2\pi\sqrt{LC_0}}[(1+\frac{\Delta C}{C_0})^{-\frac{1}{2}} - 1]$$

$$= f_c[(1+\frac{\Delta C}{C_0})^{-\frac{1}{2}} - 1]$$

$$\approx f_c[(1-\frac{1}{2}\frac{\Delta C}{C_0}) - 1]$$

$$= -\frac{1}{2}\frac{f_c K_c}{C_0} m(t)$$

Simple LC circuit
Large frequency offset
But, unstable

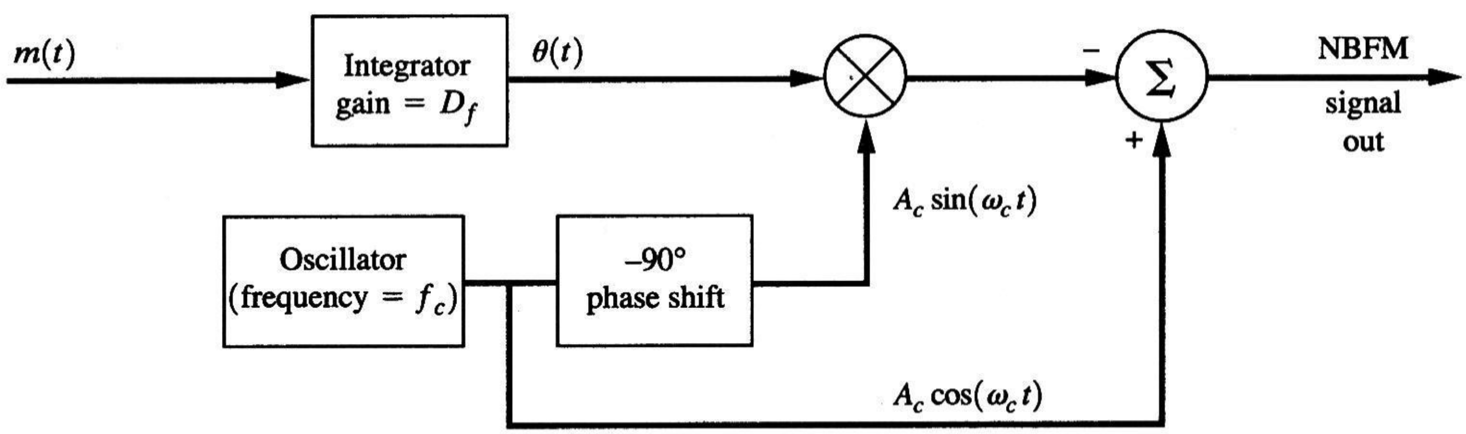
Angle Modulators

- Indirect Method - narrowband frequency modulation (NBFM)

When $\theta(t)$ is restricted to a small value

$$g(t) = A_c e^{j\theta(t)} \approx A_c[1 + j\theta(t)]$$

$$s(t) = x(t)\cos\omega_c t - y(t)\sin\omega_c t = \underbrace{A_c \cos\omega_c t}_{\text{carrier term}} - \underbrace{A_c\theta(t)\sin\omega_c t}_{\text{sideband term}}$$



(a) Generation of NBFM Using a Balanced Modulator

Angle Modulators

- Indirect Method – Narrowband-to-Wideband Conversion

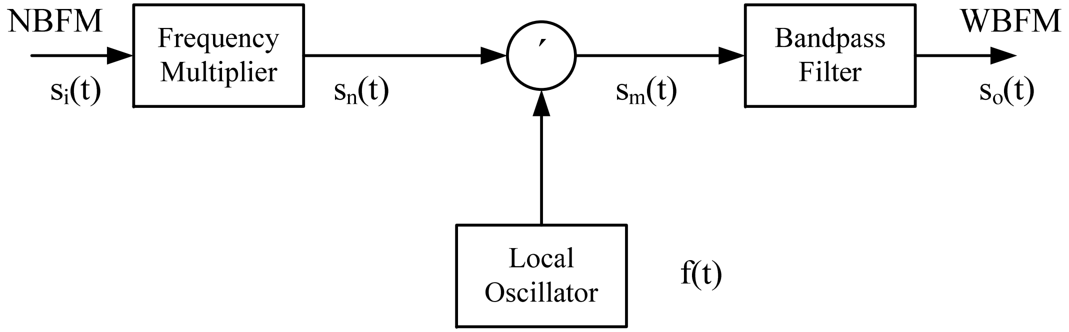
$$s_i(t) = A_c \cos[\omega_c t + \theta(t)]$$

$$s_n(t) = A_c \cos[n\omega_c t + n\theta(t)]$$

$$f(t) = 2\cos(\omega_{LO} t)$$

$$s_m(t) = A_c \cos[(n\omega_c + \omega_{LO})t + n\theta(t)] + A_c \cos[(n\omega_c - \omega_{LO})t + n\theta(t)]$$

$$s_o(t) = A_c \cos[(n\omega_c - \omega_{LO})t + n\theta(t)]$$



The central idea is that frequency multiplier changes both the carrier frequency and the deviation ratio by a factor of n, whereas the mixer changes the effective frequency but does not affect the deviation ratio.

FM Demodulation

- Differentiator and Envelope Detector

$$s'(t) = A_c[2\pi f_c + 2\pi k_f m(t)]\sin[2\pi f_c t + 2\pi k_f \int_0^t m(\tau)d\tau]$$

- Zero Crossing Detector
  - Uses rate of zero crossings to estimate f_i
- Phase Lock Loop (PLL)
  - Uses VCO and feedback to extract m(t)

Theorem

For WBFM signaling, where

$$s(t) = A_c \cos\left[\omega_c t + D_f \int_{-\infty}^{t} m(\sigma)d\sigma\right]$$

$$\beta_f = \frac{D_f \max[m(t)]}{2\pi B} > 1$$

and B is the bandwidth of m(t), the normalized PSD of the WBFM signal is approximated by

$$\mathcal{P}(f) = \frac{\pi A_c^2}{2D_f}\left[f_m\left(\frac{2\pi}{D_f}(f - f_c)\right) + f_m\left(\frac{2\pi}{D_f}(-f - f_c)\right)\right]$$

where $f_m(\cdot)$ is the PDF of the modulating signal.

Example 5-3

Main Points of Angle Modulation

- An angle-modulation signal is a nonlinear function of the modulation, and consequently, the bandwidth of the signal increases as the modulation index increases.
- The discrete carrier level changes, depending on the modulating signal, and is zero for certain types of modulating waveforms.
- The bandwidth of a narrowband angle-modulated signal is twice the modulating signal bandwidth (the same as that for AM signaling).
- The real envelope of an angle-modulated signal is constant and consequently does not depend on the level of the modulating signal.

Homework

- LC 5-21, 5-24, 5-38, 5-40, 5-44