Analog Modulation System (2)

5-3, 5-4, 5-5

Lecture 8, 2008-10-7

Contents

- Double-Sideband Suppressed Carrier (DSB-SC)
- Single Sideband (SSB)
- Vestigial Sideband (VSB)

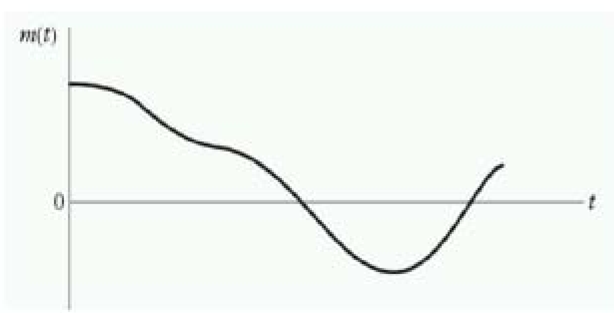
Double Sideband Suppressed Carrier

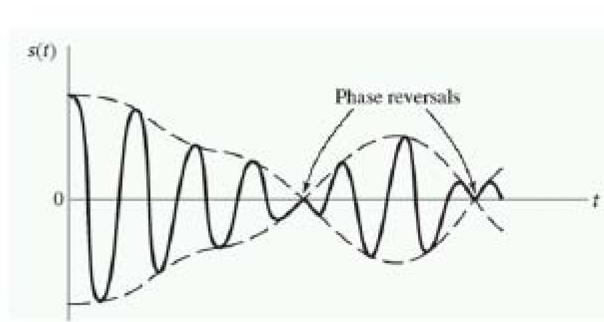
- The double-sideband suppressed carrier (DSB-SC) signal is an AM signal that has a suppressed discrete carrier.

AM $$s(t) = [A + m(t)]A'_c \cos\omega_c t = A_c[1 + am_n(t)]\cos\omega_c t$$

DSB-SC $$s(t) = \mathrm{Re}\{g(t)e^{j\omega_c t}\} = \mathrm{Re}\{A_c m(t)e^{j\omega_c t}\} = A_c m(t)\cos\omega_c t$$

No carrier component in the DSB signal!





- Envelope is no longer the input, cannot use envelope detector as demodulation.

Spectrum of DSB-SC

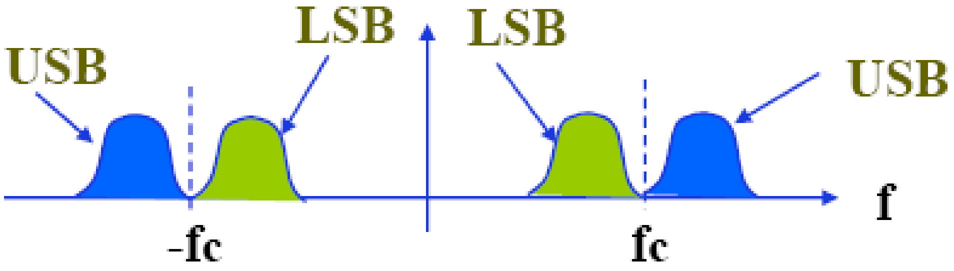
- The Fourier transform of DSB-SC signal is

$$S(f) = \frac{A_c}{2}[M(f - f_c) + M(f + f_c)]$$

- Upper Sideband and Lower Sideband

  Upper Sideband (USB): [fc, fc + W], [- fc – W, -fc].

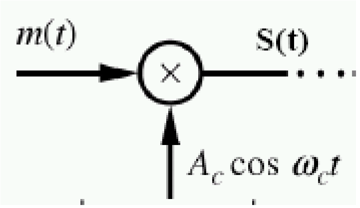
  Lower Sideband (LSB): [fc - W, fc], [- fc, -fc + W].



- Bandwidth? Modulation Efficiency?

DSB-SC Generation

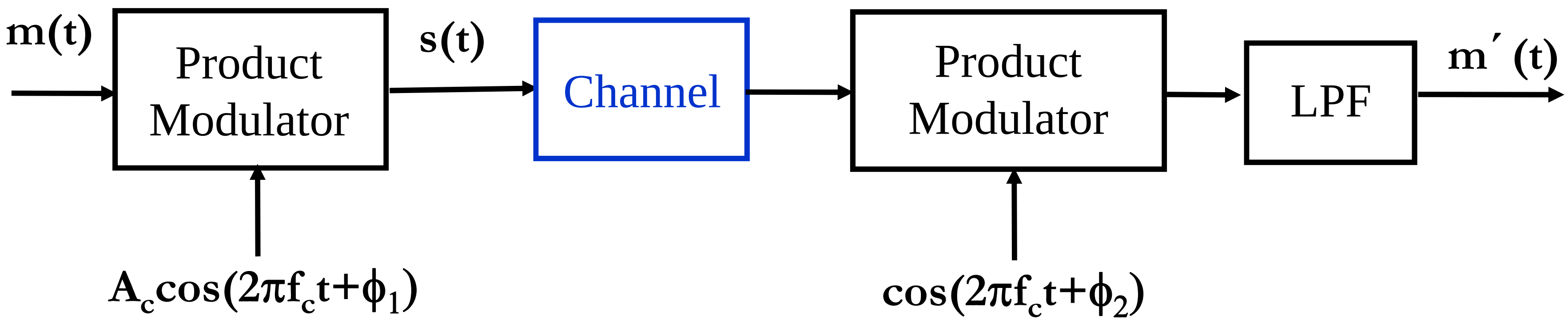
- Modulator: The generation of DSB signal is straightforward, Just multiply the message with the carrier.
- This is also called Product Modulator



$$s(t) = A_c m(t) \cos(2\pi f_c t)$$

Coherent Detection of DSB-SC

- Detector uses another product modulator



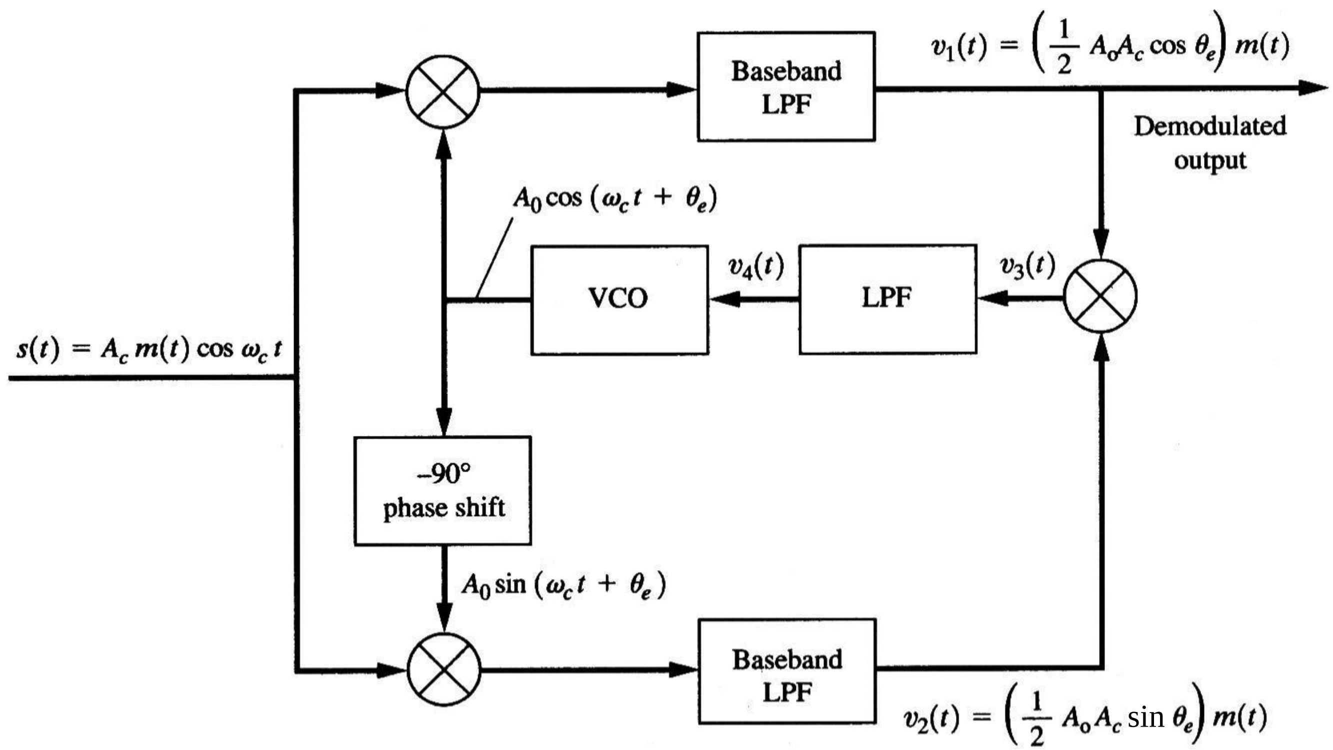
- Demodulated signal: $m'(t)=.5A_c\cos(\phi_2-\phi_1)m(t)$
  - Phase offset: if $\phi_2-\phi_1=\pm\pi/2$, $m'(t)=0$
- Coherent detection ($\phi_2\approx\phi_1$) required
- Synchronization is important. How to ensure it?

Methods to generate a phase coherent carrier:

- Method 1. Costas receiver
  - Invented by John Costas at General Electric in the 1950s. Also known as Costas Phase Locked Loop (PLL):
  - A negative feedback system that generates a signal, whose phase is locked to the phase of an input or "reference" signal.
  - Accomplished by a voltage controlled oscillator (VCO)
- Method 2. Squaring the received signal
- Method 3. Transmitting pilot signal
  - Used in stereo FM

Costas Loop

- The coherent reference for product detection cannot be obtained by the use of an ordinary PPL since there are no spectral line components at $\pm f_c$. Instead, the Costas PLL may be used to demodulated the DSB-SC signal.



(a) Costas Phase-Locked Loop

Costas Loop (Cont'd)

$$s(t) = A_c m(t) \cos \omega_c t$$

$$s(t) \cdot A_0 \cos(\omega_c t + \theta_e) = A_c m(t) \cos \omega_c t \cdot A_0 \cos(\omega_c t + \theta_e)$$

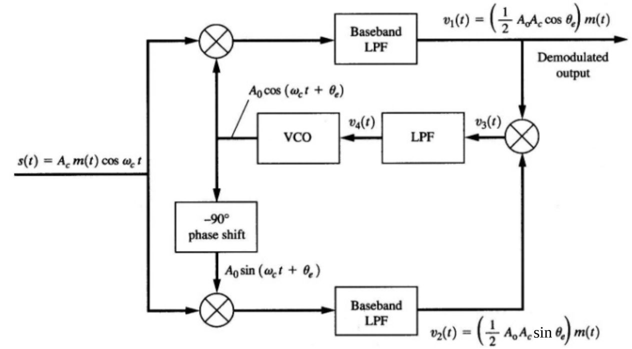
$$= \frac{1}{2} A_0 A_c m(t)[\cos \theta_e + \cos(2\omega_c t + \theta_e)]$$

$$v_1(t) = \frac{1}{2} A_0 A_c m(t) \cos \theta_e$$

$$s(t) \cdot A_0 \sin(\omega_c t + \theta_e) = A_c m(t) \cos \omega_c t \cdot A_0 \sin(\omega_c t + \theta_e)$$

$$= \frac{1}{2} A_0 A_c m(t)[\sin \theta_e + \sin(2\omega_c t + \theta_e)]$$

$$v_2(t) = \frac{1}{2} A_0 A_c m(t) \sin \theta_e$$

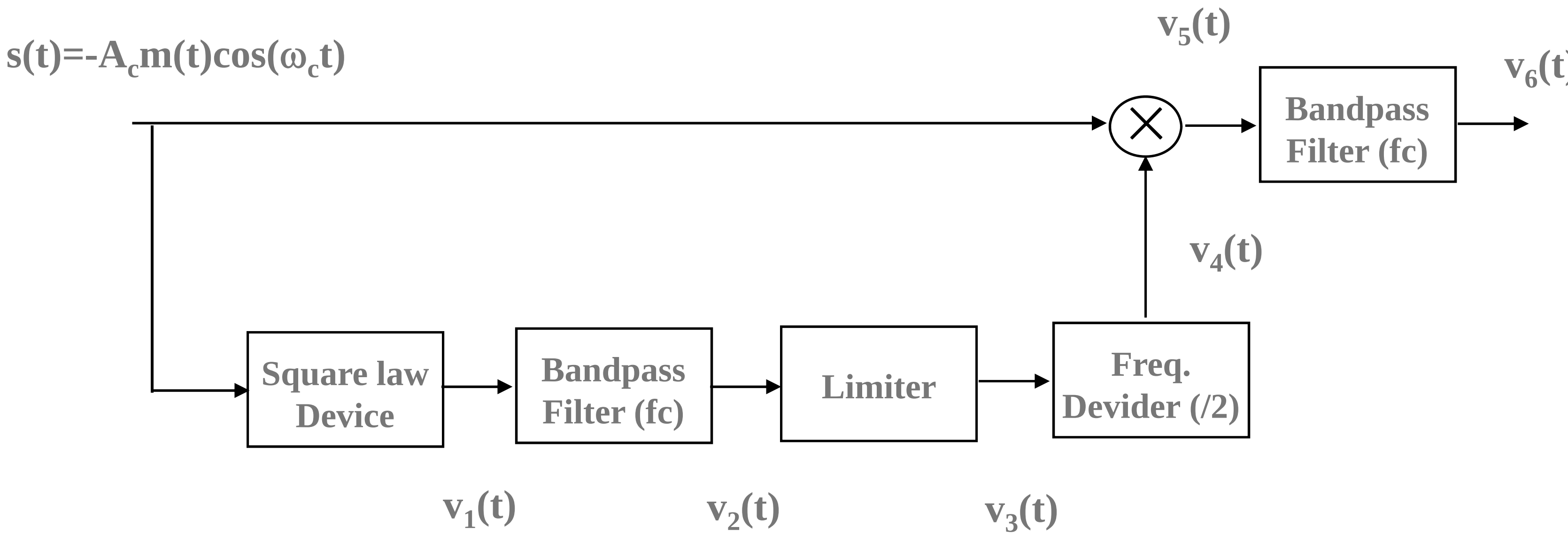


(a) Costas Phase-Locked Loop

$$v_3(t) = v_1(t) v_2(t) = \frac{1}{8} (A_0 A_c)^2 m^2(t) \sin 2\theta_e$$

$$v_4(t) = < v_3(t) > = < \frac{1}{8} (A_0 A_c)^2 m^2(t) \sin 2\theta_e > = \frac{1}{8} (A_0 A_c)^2 < m^2(t) > \sin 2\theta_e = K \sin 2\theta_e$$

Squaring Loop and Phase Ambiguity



Main Points of DSB-SC

- The phase of DSB-SC modulated carrier is reversed when modulating signal $m(t) = 0$.
- DSB-SC eliminates the power inefficiency of standard AM, the required bandwidth is the same as standard AM with 2 times of message signal bandwidth.
- Coherent detection is needed for DSB-SC: obtaining the carrier phase is one of biggest challenges in all demodulators.

USSB

- An upper single sideband (USSB) signal has a zero-valued spectrum for $|f| < f_c$, where f_c is the carrier frequency.

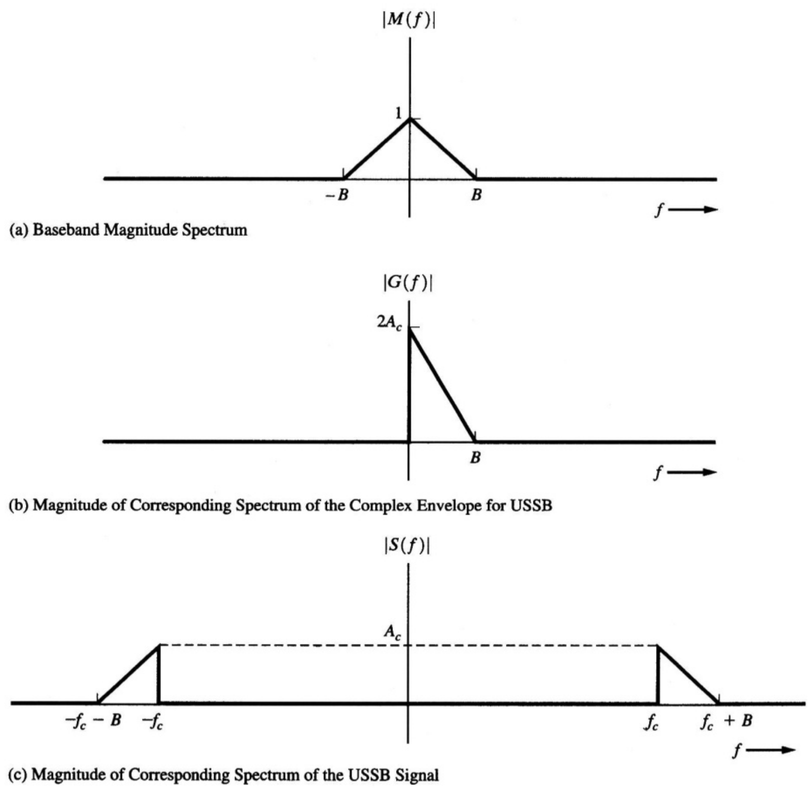
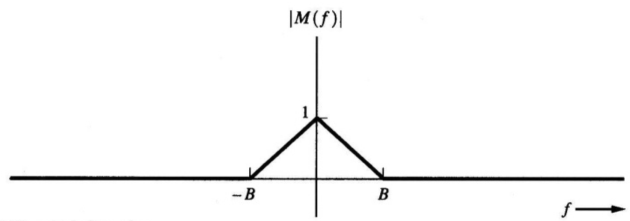


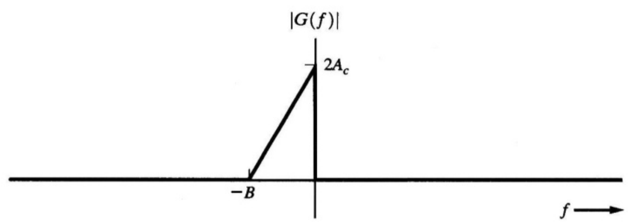
Figure 5–4 Spectrum for a USSB signal.

LSSB

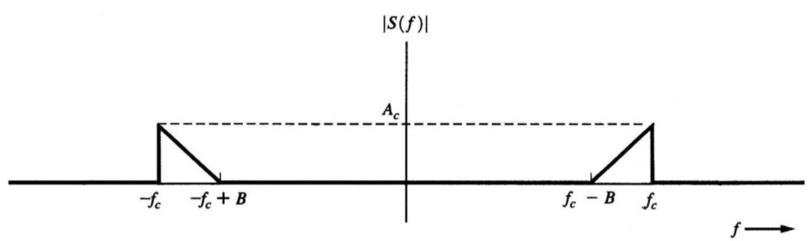
- A lower single sideband (LSSB) signal has a zero-valued spectrum for $|f| > f_c$, where f_c is the carrier frequency.



(a) Baseband Magnitude Spectrum



(b) Magnitude of Corresponding Spectrum of the Complex Envelope for USSB



(c) Magnitude of Corresponding Spectrum of the LSSB Signal

Theorem

An SSB signal is obtained by using the complex envelope

$$g(t) = A_c[m(t) \pm j\hat{m}(t)]$$

which results in the SSB signal

$$s(t) = A_c[m(t)\cos\omega_c t \mp \hat{m}(t)\sin\omega_c t]$$

where the upper sign is used for USSB and the lower sign is used for LSSB. $\hat{m}(t)$ denotes the Hilbert transform of $m(t)$, which is given by

$$\hat{m}(t) = m(t) * h(t)$$

where

$$h(t) = \frac{1}{\pi t} \leftrightarrow H(f) = \begin{cases} -j, & f > 0 \\ j, & f < 0 \end{cases}$$

Proof

$$g(t) = A_c[m(t) \pm j\hat{m}(t)] = A_c[m(t) \pm jm(t) * h(t)]$$

$$G(f) = A_c[M(f) \pm j\hat{M}(f)] = A_c[M(f) \pm jM(f)H(f)] = A_c M(f)[1 \pm jH(f)]$$

For the USSB case

$$G(f) = A_c M(f)[1 + jH(f)] = \begin{cases} 2A_c M(f), & f > 0 \\ 0, & f < 0 \end{cases}$$

$$S(f) = \begin{cases} A_c M(f - f_c), & f > f_c \\ 0, & -f_c < f < f_c \\ A_c M(f + f_c), & f < -f_c \end{cases}$$

For the LSSB case

$$G(f) = A_c M(f)[1 - jH(f)] = \begin{cases} 0, & f > 0 \\ 2A_c M(f), & f < 0 \end{cases}$$

$$S(f) = S(f) = \begin{cases} 0, & f > f_c \\ A_c M(f - f_c) + A_c M(f + f_c), & -f_c < f < f_c \\ 0, & f < -f_c \end{cases}$$

Power of SSB signal

The normalized average power of the SSB signal is

$$P_s = \langle s^2(t) \rangle = \tfrac{1}{2} \langle \left| g(t) \right|^2 \rangle = \tfrac{1}{2} \langle \left| A_c [m(t) + j\hat{m}(t)] \right|^2 \rangle$$

$$= \tfrac{1}{2} A_c^2 \langle m^2(t) + [\hat{m}(t)]^2 \rangle$$

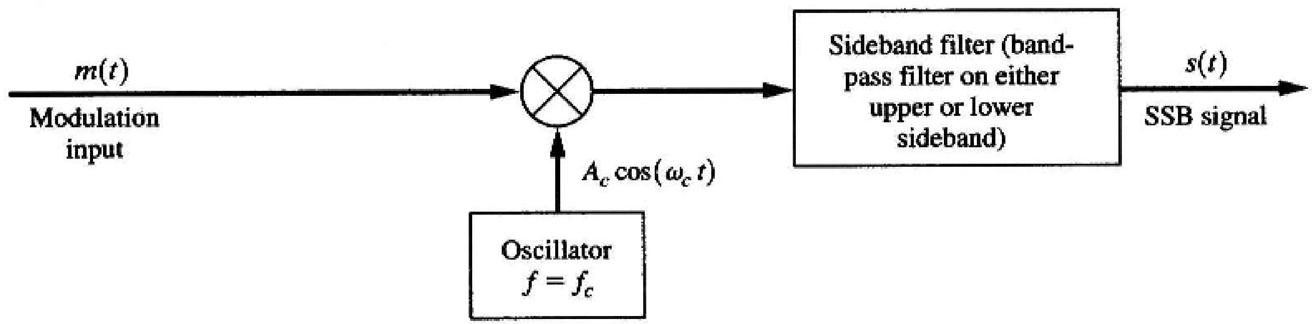
$$= \tfrac{1}{2} A_c^2 \langle m^2(t) \rangle + \tfrac{1}{2} A_c^2 \langle [\hat{m}(t)]^2 \rangle$$

$$\langle [\hat{m}(t)]^2 \rangle = \int_{-\infty}^{\infty} P_{\hat{m}}(f)df = \int_{-\infty}^{\infty} \left| H(f) \right|^2 P_m(f)df$$

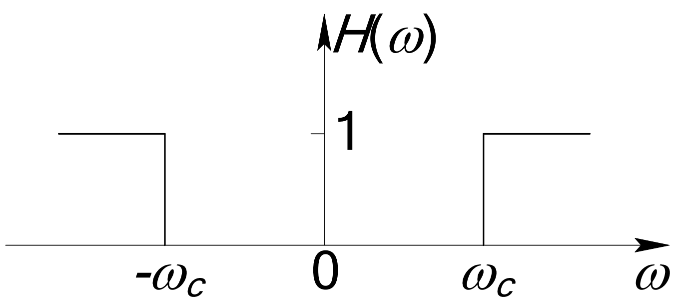
$$= \int_{-\infty}^{\infty} P_m(f)df = \langle m^2(t) \rangle$$

$$P_s = A_c^2 \langle m^2(t) \rangle = A_c^2 P_m$$

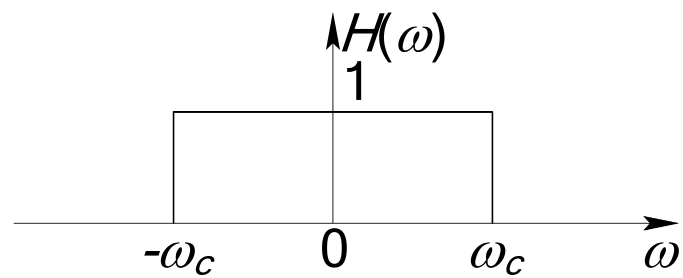
Generation SSB – Filtering method



(b) Filter Method



LSSB filter



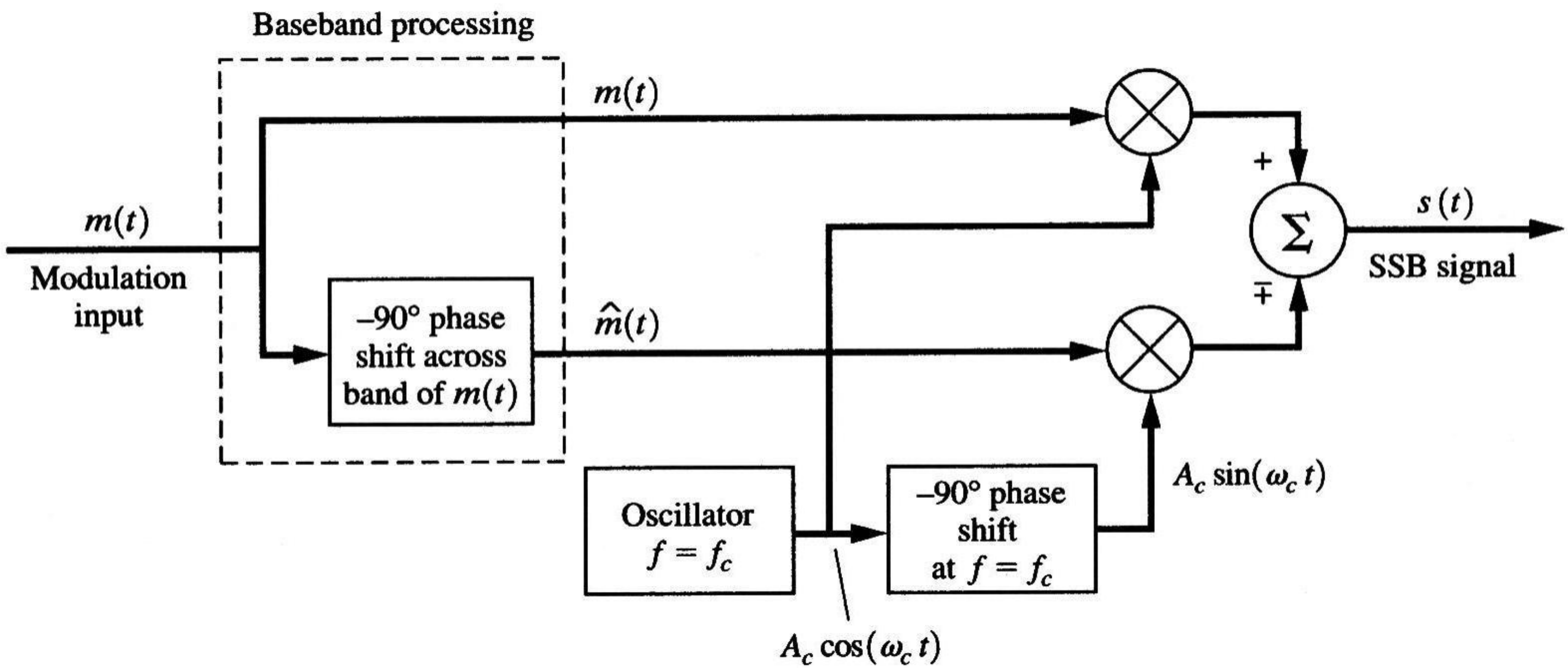
USSB filter

Problems:

- Only suitable for signals without low-freq contents (e.g., speech)
- For other signals, need nearly ideal filters around fc

  Difficult, especially if fc is tunable.

Generation SSB – Phasing Method



(a) Phasing Method

SSB have both AM and PM

- SSB signals have both AM and PM.

$$R(t) = \left|g(t)\right| = A_c\sqrt{m^2(t) + [\hat{m}(t)]^2}$$

$$\theta(t) = \angle g(t) = \tan^{-1}\left[\frac{\pm\hat{m}(t)}{m(t)}\right]$$

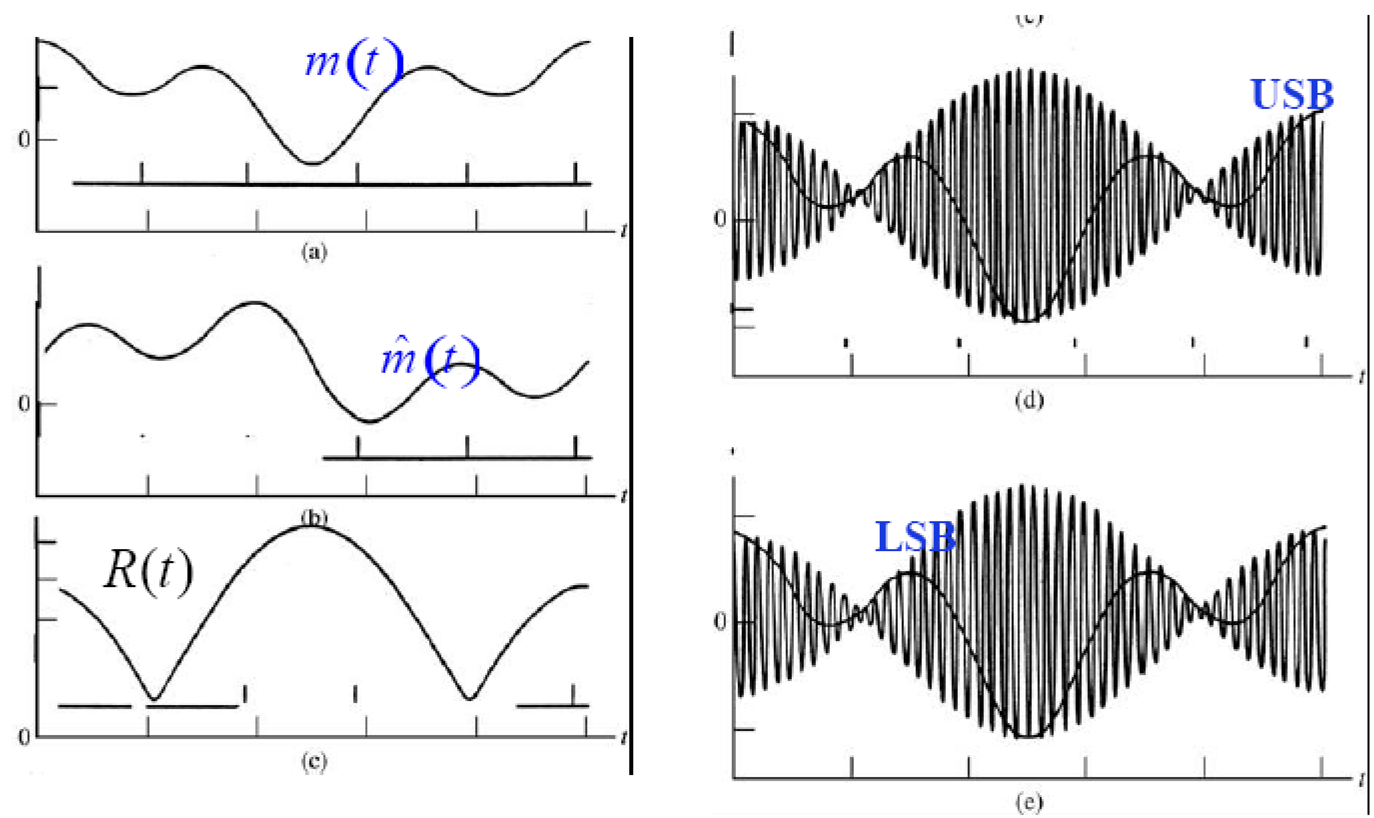
- SSB signals may be received by using a superheterodyne receiver that incorporates a product detector with $\theta_0 = 0$.

$$v_{in}(t) = \mathrm{Re}\{g(t)e^{j\omega_c t}\}$$

$$v_0(t) = A_0\cos[\omega_c t + \theta_0]$$

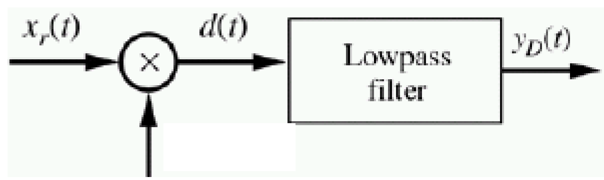
$$v_{out}(t) = \tfrac{1}{2}A_0\,\mathrm{Re}\{g(t)e^{-j\theta_0}\} = \tfrac{1}{2}A_0\,\mathrm{Re}\{g(t)\} = \tfrac{1}{2}A_0A_c m(t)$$

SSB Waveform



SSB Demodulation

- The coherent detection is still applicable to SSB:
  - multiply with carrier, then LPF
- Assume demodulation carrier has a phase error:



$$d(t) = \frac{A_c}{2}\left(m(t)\cos(2\pi f_c t) \mp \hat{m}(t)\sin(2\pi f_c t)\right)A_c'\cos(2\pi f_c t + \phi)$$

$$= \frac{A_c A_c'}{4}\left\{m(t)\left[\cos\phi + \cos(4\pi f_c t + \phi)\right] \mp \hat{m}(t)\left[\sin(4\pi f_c t + \phi) - \sin\phi\right]\right\}$$

$$y_D(t) = \frac{A_c A_c'}{4}\left\{m(t)\cos\phi \pm \hat{m}(t)\sin\phi\right\}$$

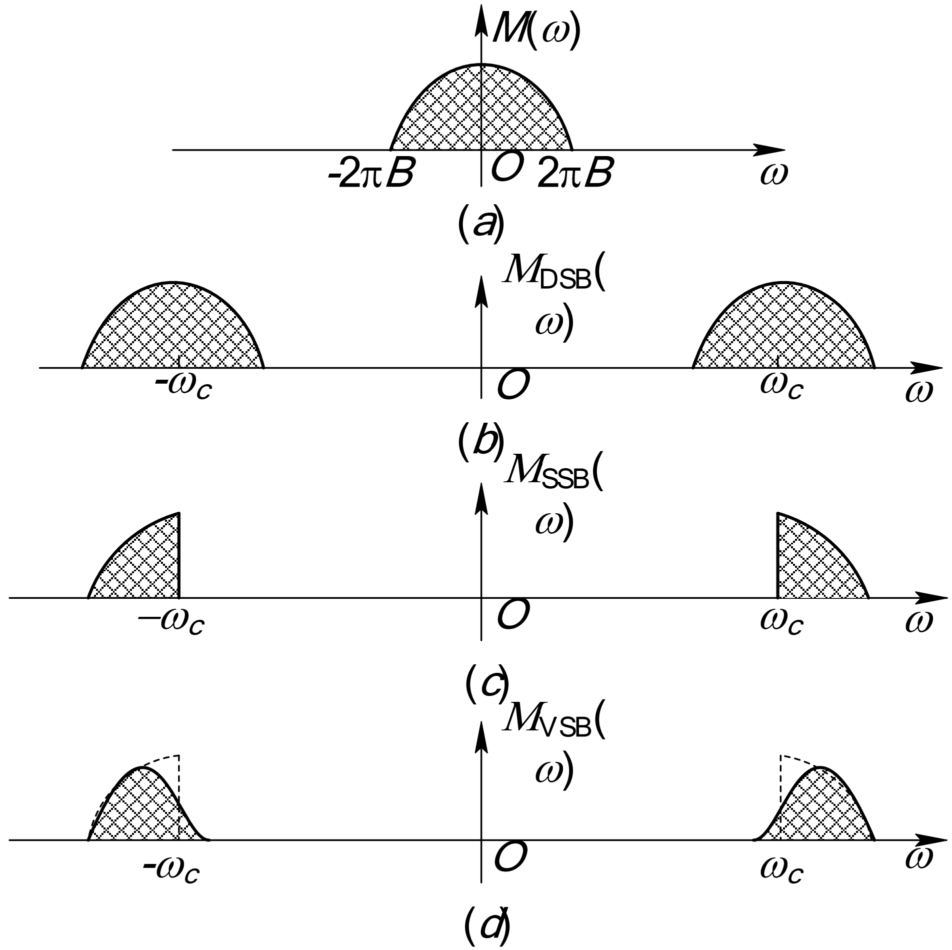
If $\phi = 0$: $y_D(t) = \frac{A_c A_c'}{4} m(t)$: No distortion.

Main Points of SSB

- Advantage:
  - More bandwidth-efficient than DSB
- Disadvantages:
  - Need near-ideal filters in implementations
  - Cannot be used in signals with DC
- Solution:
  - VSB: Vestigial sideband modulation

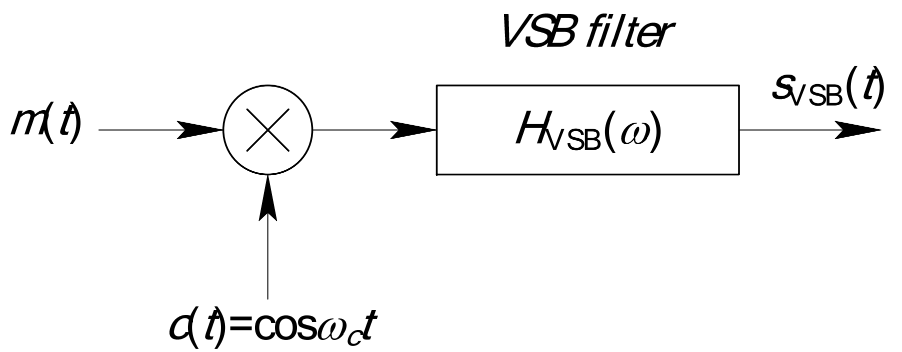
Vestigial Sibeband (VSB)

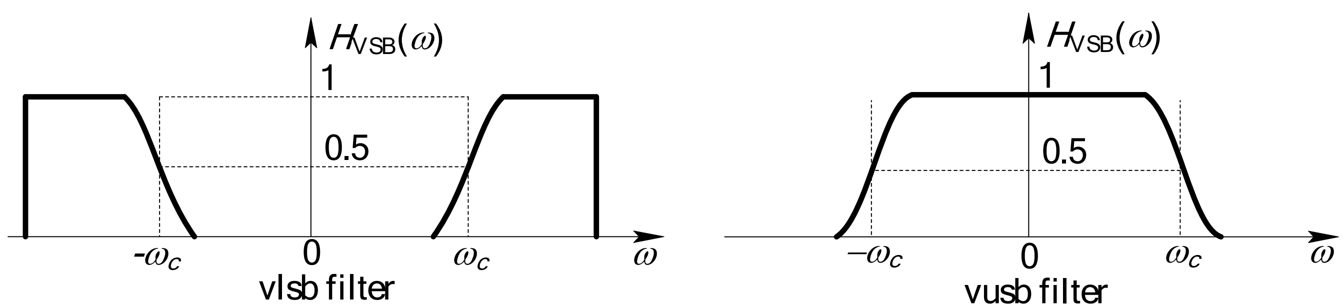
- Transmits USB or LSB and vestige of other sideband



VSB Modulator

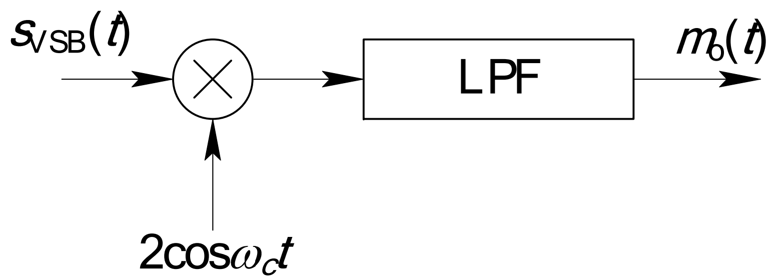
- VSB filter is odd symmetric at $\pm\omega_c$





$$S_{VSB}(\omega) = \frac{1}{2}[M(\omega+\omega_c) + M(\omega-\omega_c)]H_{VSB}(\omega)$$

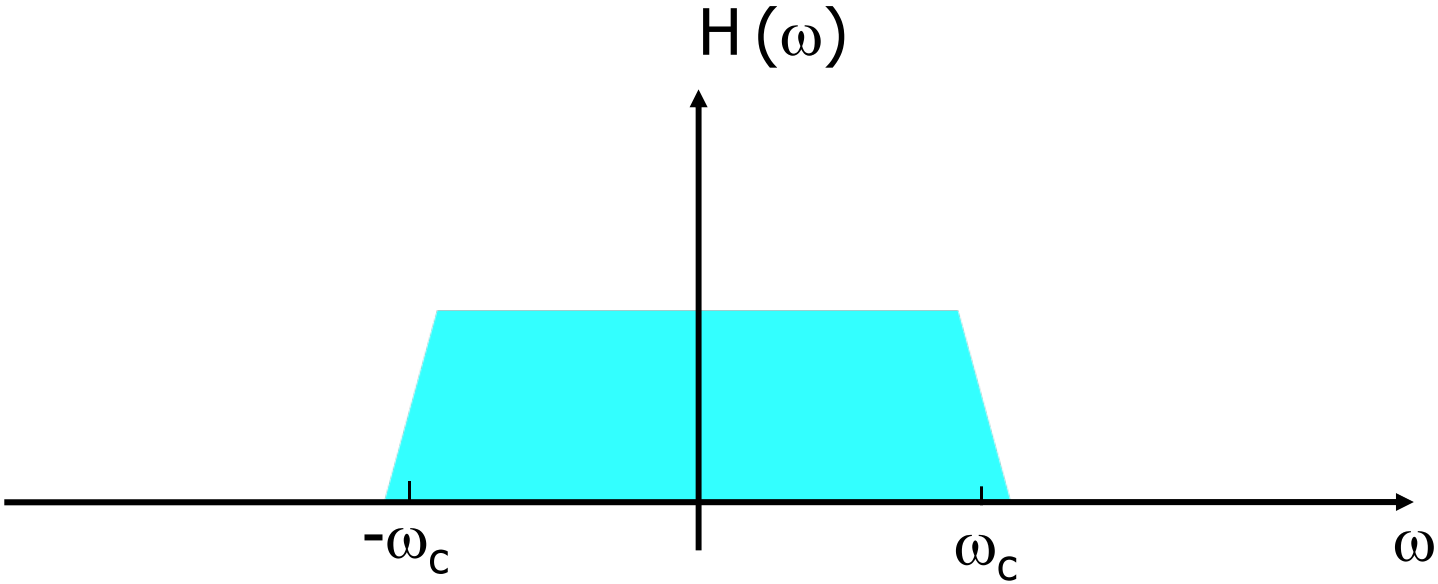
VSB Demodulator



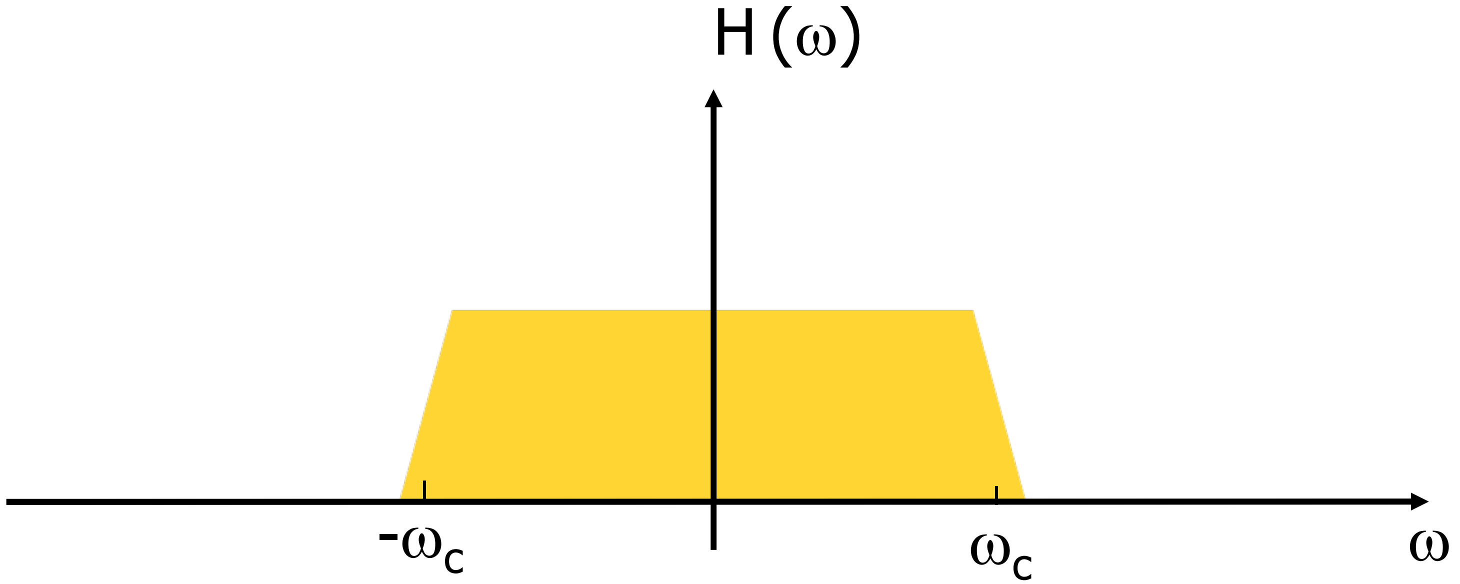
$$2s_{VSB}(t)\cos\omega_c t \Leftrightarrow [S_{VSB}(\omega+\omega_c)+S_{VSB}(\omega-\omega_c)]$$

$$M_o(\omega)=\frac{1}{2}M(\omega)[H_{VSB}(\omega+\omega_c)+H_{VSB}(\omega-\omega_c)]$$

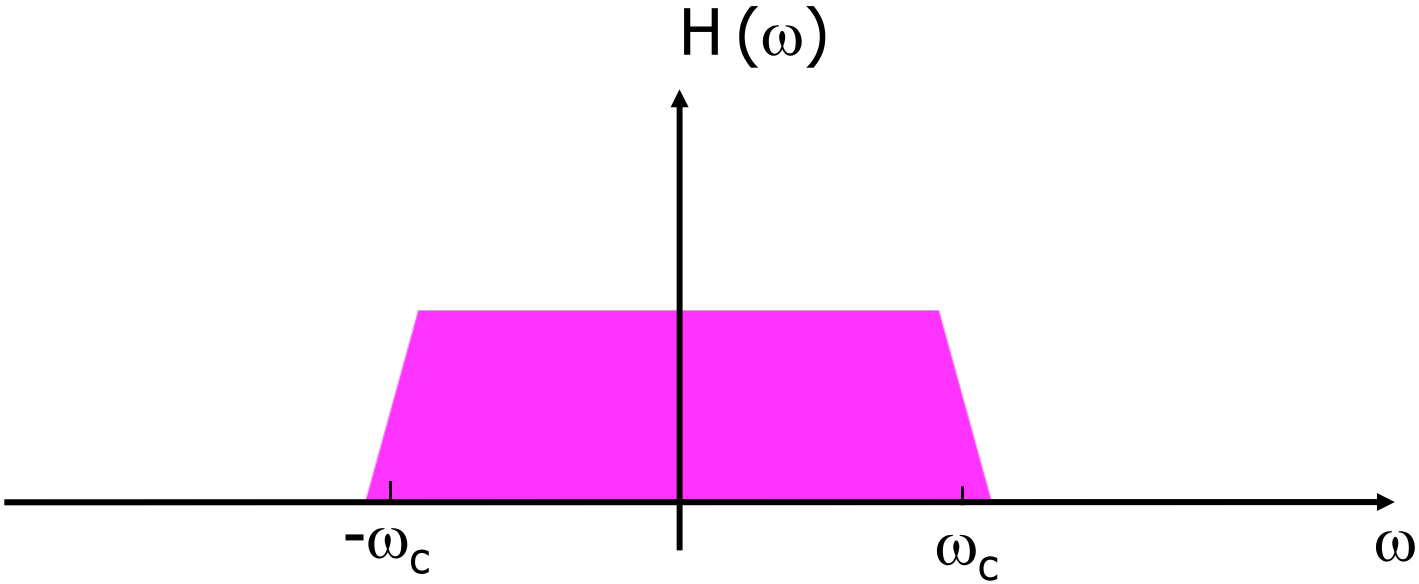
$H_{VSB}(\omega)$



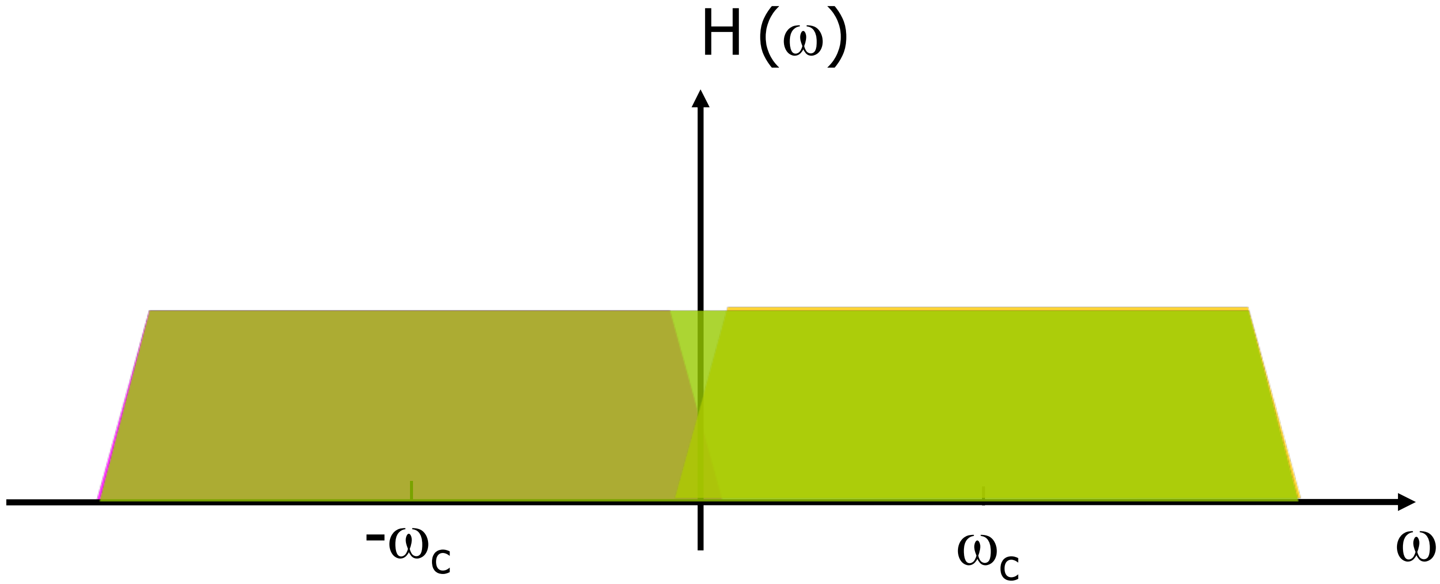
$$H_{VSB}(\omega-\omega_c)$$



$H_{VSB}(\omega+\omega_c)$



$$H_{VSB}(\omega + \omega_c) + H_{VSB}(\omega - \omega_c)$$



$$H_{VSB}(\omega + \omega_c) + H_{VSB}(\omega - \omega_c) = const. \quad |\omega| \le 2\omega_c$$

Main Points of VSB

- VSB is a tradeoff between DSB and SSB
- Use more bandwidth than SSB, but simplify the system.
  - The VSB filter is a lot easier to implement than the SSB filter, which requires near-ideal frequency response at 0 or fc
  - The VSB filter can be implemented at the receiver instead of at the transmitter, due to power constraints.
- Envelope detection is also possible for VSB:
  - Used in TV system.

Homework

- LC 5-7, 5-9, 5-12, 5-14, 5-15, 5-18