Analog Modulation System (1)

4-1, 4-2,4-3, 5-1

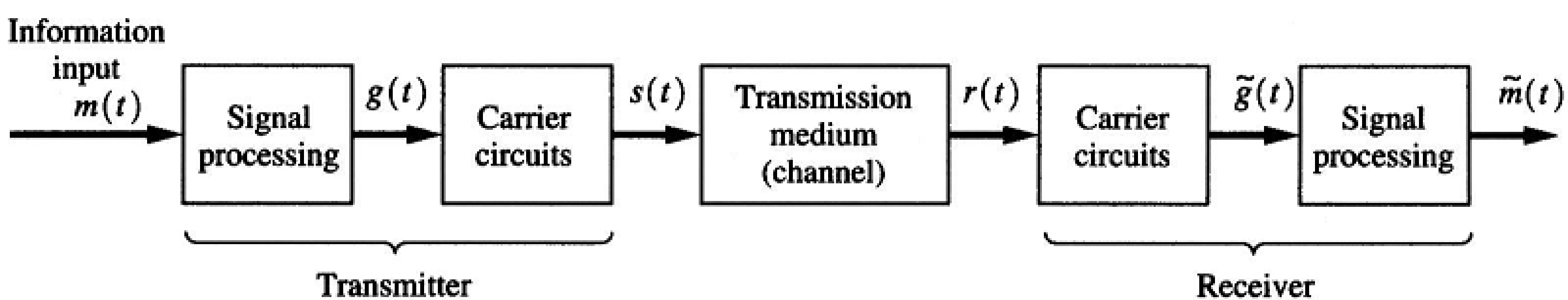
Lecture 7, 2008-9-26

Contents

- Baseband
- Bandpass
- Modulation
- Amplitude Modulation

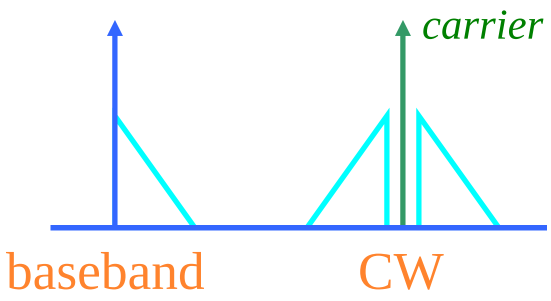
Baseband, Bandpass, and Modulation

- A baseband waveform has a spectral magnitude that is nonzero for frequencies in the vicinity of the origin (i.e., $f = 0$) and negligible elsewhere.
- A bandpass waveform has a spectral magnitude that is nonzero for frequencies in some band concentrated about a frequency $f = \pm f_c$, where $f_c \gg 0$. The spectral magnitude is negligible elsewhere. f_c is called the carrier frequency.
- Modulation is the process of imparting the source information onto a bandpass signal with a carrier frequency f_c by the introduction of amplitude or phase perturbations or both. This bandpass signal is called the modulated signal $s(t)$, and the baseband source signal is called the modulating signal $m(t)$.

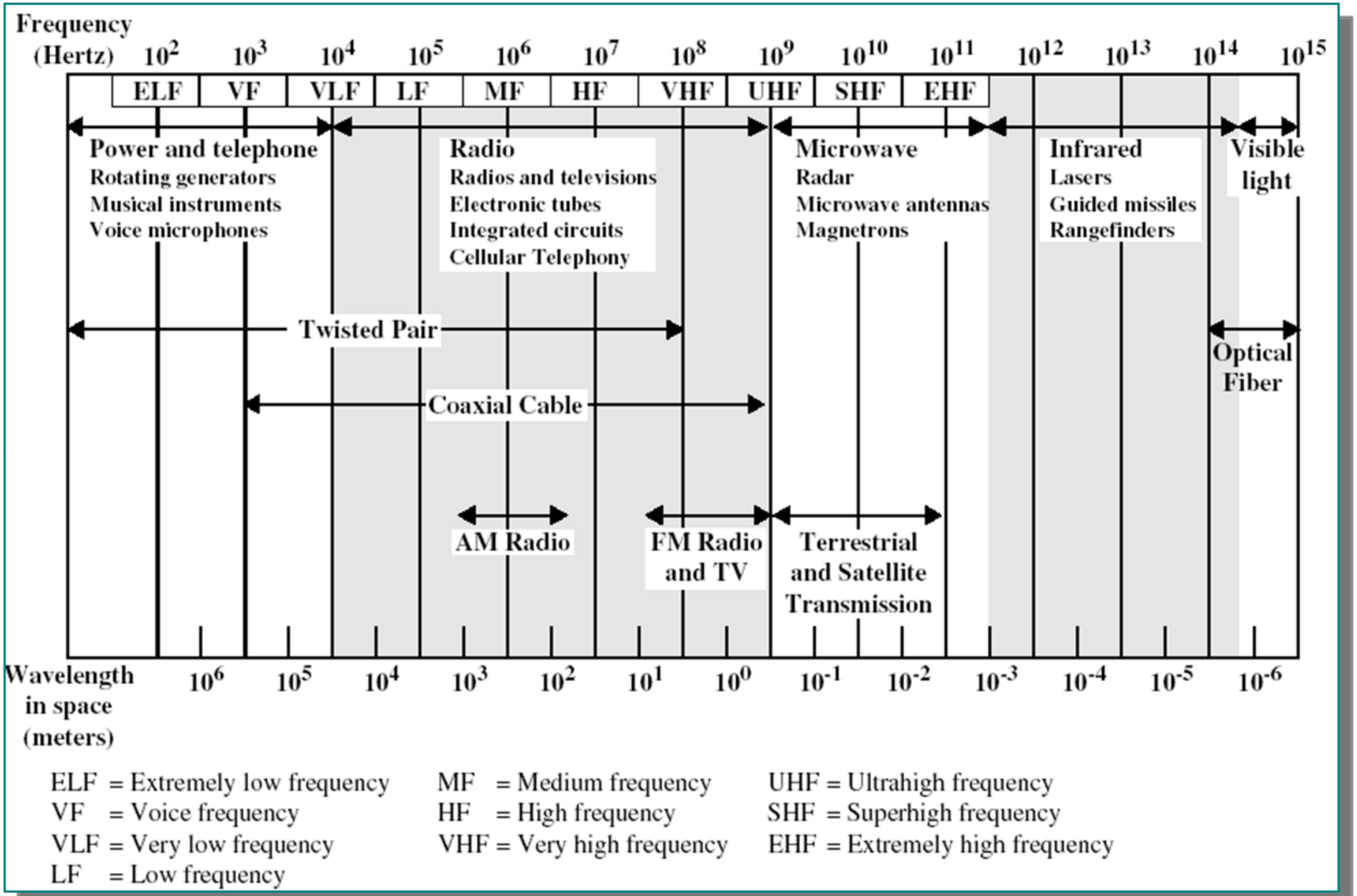


Why Modulation is Used?

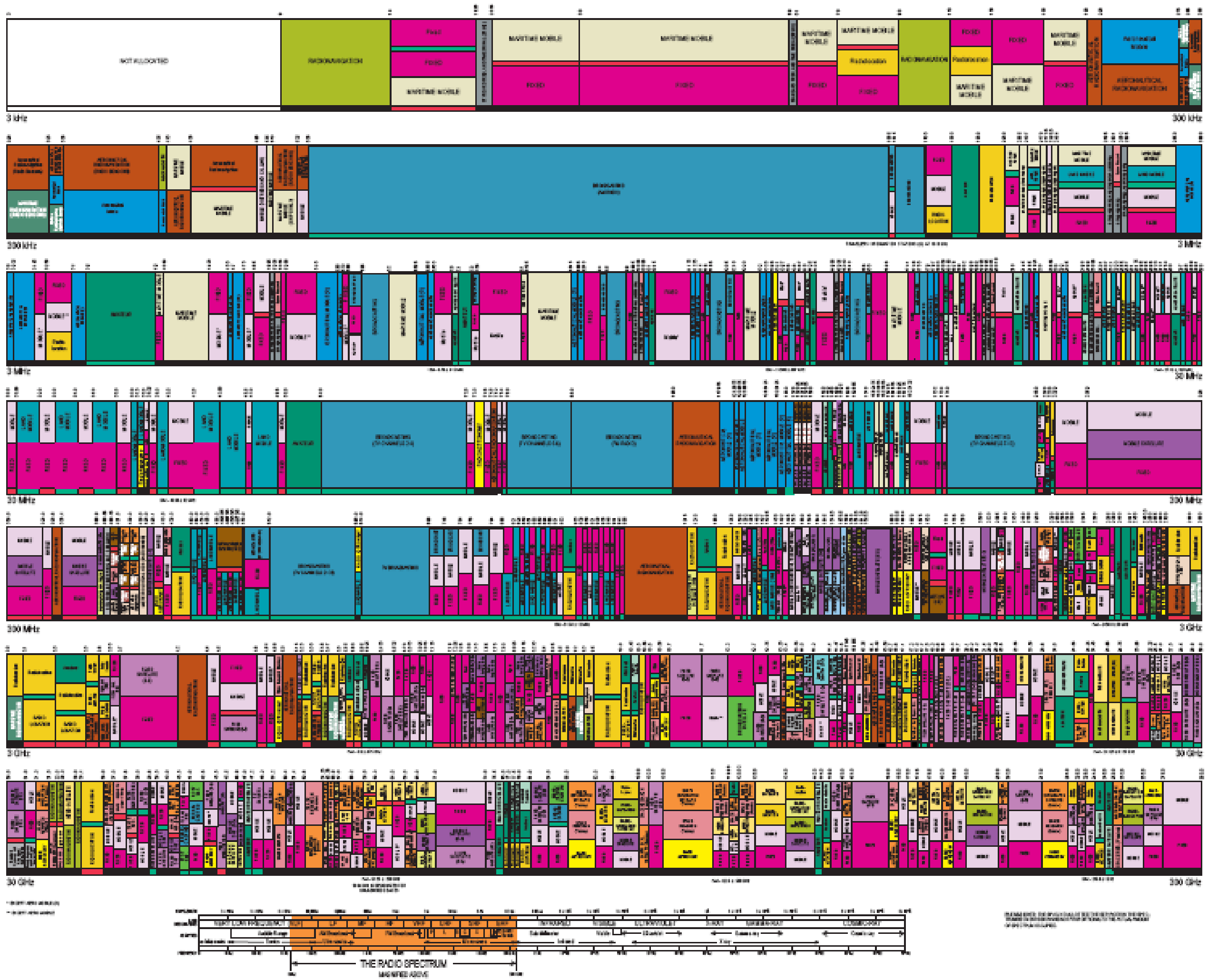
- Baseband communications is used in
  - PSTN local loop
  - PCM communications for instance between exchanges
- Using carrier to shape and shift the frequency spectrum (eg CW techniques) enable modulation by which several advantages are obtained
  - different **radio bands** can be used for communications
  - **wireless** communications
  - **multiplexing** techniques become applicable



Radio Spectrum



United States Frequency Allocation



Representation of a Physical Bandpass Waveform

- I-Q (in-phase-quadrature) description for bandpass signals

$$v(t) = \mathrm{Re}\{g(t)e^{j\omega_c t}\} = R(t)\cos[\omega_c t + \theta(t)] = x(t)\cos\omega_c t - y(t)\sin\omega_c t$$

$$g(t) = x(t) + jy(t) = |g(t)|e^{j\angle g(t)} = R(t)e^{j\theta(t)}$$

$$x(t) = \mathrm{Re}\{g(t)\} = R(t)\cos\theta(t)$$

$$y(t) = \mathrm{Im}\{g(t)\} = R(t)\sin\theta(t)$$

$v(t)$: signal $s(t)$, noise $n(t)$, or signal plus noise $r(t)$

$g(t)$: complex envelope

$x(t)$: in-phase modulation associated with $v(t)$

$y(t)$: quadrature modulation associated with $v(t)$

Complex Envelope Representation of Bandpass Waveform

Proof

$$v(t)=\sum_{n=-\infty}^{\infty}c_n e^{jn\omega_0 t}=\sum_{n=-\infty}^{-1}c_n e^{jn\omega_0 t}+c_0+\sum_{n=1}^{\infty}c_n e^{jn\omega_0 t}$$

$$=\sum_{n=1}^{\infty}c_{-n}e^{-jn\omega_0 t}+\sum_{n=1}^{\infty}c_n e^{jn\omega_0 t}=\sum_{n=1}^{\infty}(c_{-n}e^{-jn\omega_0 t}+c_n e^{jn\omega_0 t})$$

$$=\sum_{n=1}^{\infty}(c_n^* e^{-jn\omega_0 t}+c_n e^{jn\omega_0 t})$$

$$=\sum_{n=1}^{\infty}2\,\mathrm{Re}\{c_n e^{jn\omega_0 t}\}$$

$$=\mathrm{Re}\{2\sum_{n=1}^{\infty}c_n e^{jn\omega_0 t}\}$$

$$=\mathrm{Re}\{2\sum_{n=1}^{\infty}c_n e^{j(n\omega_0-\omega_c)t}e^{j\omega_c t}\}$$

$$=\mathrm{Re}\{g(t)e^{j\omega_c t}\}$$

$$g(t)=2\sum_{n=1}^{\infty}c_n e^{j(n\omega_0-\omega_c)t}$$

The Phasor Description of Bandpass Signal

$$v(t) = \mathrm{Re}\{g(t)e^{j\omega_c t}\} = R(t)\cos[\omega_c t + \theta(t)]$$

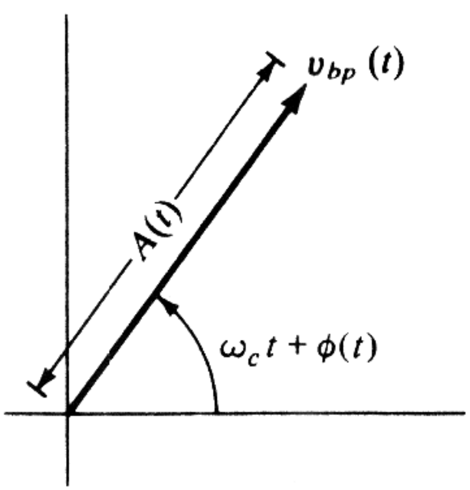
$$g(t) = x(t) + jy(t) = |g(t)|e^{j\angle g(t)} = R(t)e^{j\theta(t)}$$

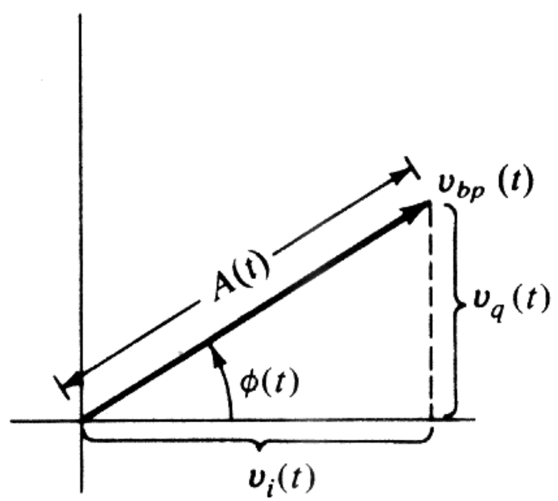
$$R(t) = |g(t)| = \sqrt{x^2(t) + y^2(t)}$$

$$\theta(t) = \angle g(t) = \tan^{-1}\frac{y(t)}{x(t)}$$

$R(t)$: amplitude modulation (AM) on $v(t)$

$\theta(t)$: phasee modulation (PM) on $v(t)$





Spectrum of a Bandpass Signal

If a bandpass waveform is represented by

$$v(t) = \text{Re}\{g(t)e^{j\omega_c t}\}$$

then the spectrum of the bandpass waveform is

$$V(f) = \tfrac{1}{2}[G(f - f_c) + G^*(-f - f_c)]$$

and the PSD of the waveform is

$$P_v(f) = \tfrac{1}{4}[P_g(f - f_c) + P_g(-f - f_c)]$$

where $G(f) = F[g(t)]$ and $P_g(f)$ is the PSD of $g(t)$.

Proof

Proof. $v(t) = \mathrm{Re}\{g(t)e^{j\omega_c t}\} = \frac{1}{2}[g(t)e^{j\omega_c t} + \left(g(t)e^{j\omega_c t}\right)^*]$

$w(t)e^{j\omega_c t} \leftrightarrow W(f - f_c) \Rightarrow F[g(t)e^{j\omega_c t}] = G(f - f_c)$

$w^*(t) \leftrightarrow W^*(-f) \Rightarrow F[\left(g(t)e^{j\omega_c t}\right)^*] = G^*(-f - f_c)$

$V(f) = F[\frac{1}{2}[g(t)e^{j\omega_c t} + \left(g(t)e^{j\omega_c t}\right)^*] = \frac{1}{2}[G(f - f_c) + G^*(-f - f_c)]$

$R_v(\tau) =< v(t)v(t+\tau) >=< \mathrm{Re}\{g(t)e^{j\omega_c t}\}\mathrm{Re}\{g(t+\tau)e^{j\omega_c (t+\tau)}\} >$

$\mathrm{Re}\{c_1 c_2\} = \mathrm{Re}\{(a_1 + jb_1)(a_2 + jb_2)\} = a_1 a_2 - b_1 b_2$

$\mathrm{Re}\{c_1^* c_2\} = \mathrm{Re}\{(a_1 - jb_1)(a_2 + jb_2)\} = a_1 a_2 + b_1 b_2$

$\mathrm{Re}\{c_1\}\mathrm{Re}\{c_2\} = a_1 a_2 = \frac{1}{2}[\mathrm{Re}\{c_1 c_2\} + \mathrm{Re}\{c_1^* c_2\}]$

$R_v(\tau) =< \frac{1}{2}[\mathrm{Re}\{g(t)e^{j\omega_c t} g(t+\tau)e^{j\omega_c (t+\tau)}\} + \mathrm{Re}\{g^*(t)e^{-j\omega_c t} g(t+\tau)e^{j\omega_c (t+\tau)}\}] >$

$= \frac{1}{2} < \mathrm{Re}\{g(t)g(t+\tau)e^{j2\omega_c t}e^{j\omega_c \tau}\} > + \frac{1}{2} < \mathrm{Re}\{g^*(t)g(t+\tau)e^{j\omega_c \tau}\} >$

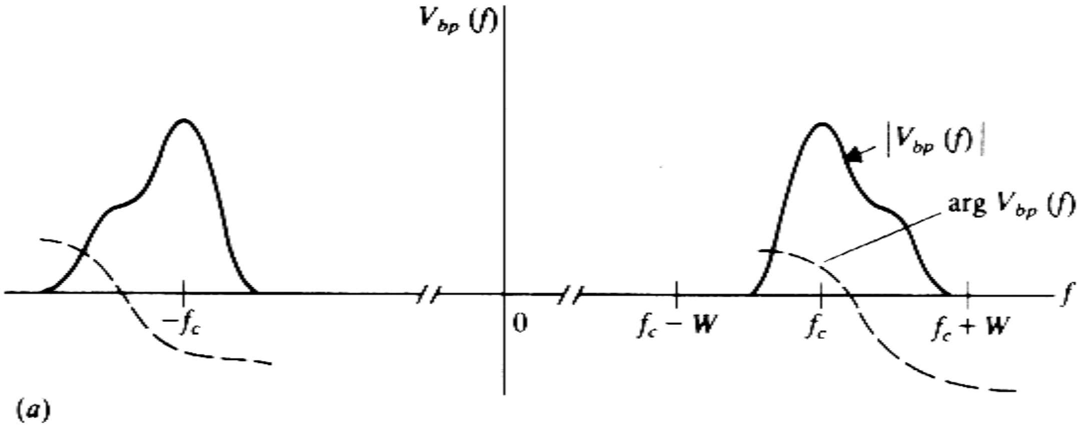
$= \frac{1}{2}\mathrm{Re}\{< g(t)g(t+\tau)e^{j2\omega_c t} > e^{j\omega_c \tau}\} + \frac{1}{2}\mathrm{Re}\{< g^*(t)g(t+\tau) > e^{j\omega_c \tau}\}$

$= \frac{1}{2}\mathrm{Re}\{< g^*(t)g(t+\tau) > e^{j\omega_c \tau}\} = \frac{1}{2}\mathrm{Re}\{R_g(\tau)e^{j\omega_c \tau}\}$

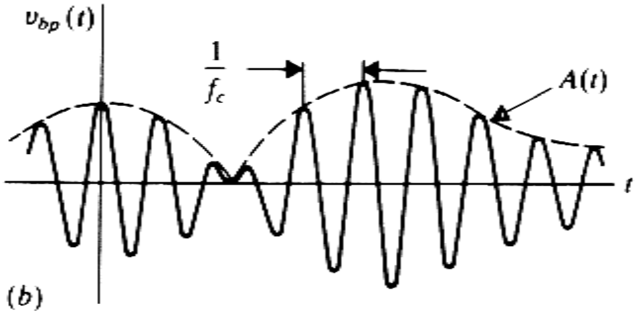
$P_v(f) = F[R_v(\tau)] = \frac{1}{2}F[\mathrm{Re}\{R_g(\tau)e^{j\omega_c \tau}\}] = \frac{1}{4}[P_g(f - f_c) + P_g(-f - f_c)]$

Bandpass Signal

Bandpass signal in frequency domain



Bandpass signal in time domain



dashed line denotes envelope

Evaluation of Power

- Example 4-1 Amplitude-modulated signal

$$g(t) = A_c[1+m(t)] \leftrightarrow G(f) = A_c[\delta(f) + M(f)]$$

$$s(t) = \mathrm{Re}\{g(t)e^{j\omega_c t}\} = \mathrm{Re}\{A_c[1+m(t)]e^{j\omega_c t}\} = A_c[1+m(t)]\cos\omega_c t$$

$$w(t)\cos(\omega_c t+\theta) \leftrightarrow \tfrac{1}{2}[e^{j\theta}W(f-f_c) + e^{-j\theta}W(f+f_c)] \Rightarrow$$

$$S(f) = \tfrac{1}{2}[A_c[\delta(f-f_c) + M(f-f_c)] + A_c[\delta(f+f_c) + M(f+f_c)]]$$

$$= \tfrac{1}{2}A_c[\delta(f-f_c) + M(f-f_c) + \delta(f+f_c) + M(f+f_c)]$$

$$P_s = \tfrac{1}{2} < \left|g(t)\right|^2 >= \tfrac{1}{2} < \left|A_c[1+m(t)]\right|^2 >= \tfrac{1}{2}A_c^2 < \left|[1+m(t)]\right|^2 >$$

$$= \tfrac{1}{2}A_c^2 < [1+m(t)]^2 >= \tfrac{1}{2}A_c^2 < 1+2m(t)+m^2(t) >$$

$$= \tfrac{1}{2}A_c^2[1+2 < m(t) > + < m^2(t) >] = \tfrac{1}{2}A_c^2[1+ < m^2(t) >]$$

$$= \tfrac{1}{2}A_c^2[1+P_m]$$

P_m : power in $m(t)$

$\tfrac{1}{2}A_c^2 P_m$: power in sideband

$\tfrac{1}{2}A_c^2$: carrier power

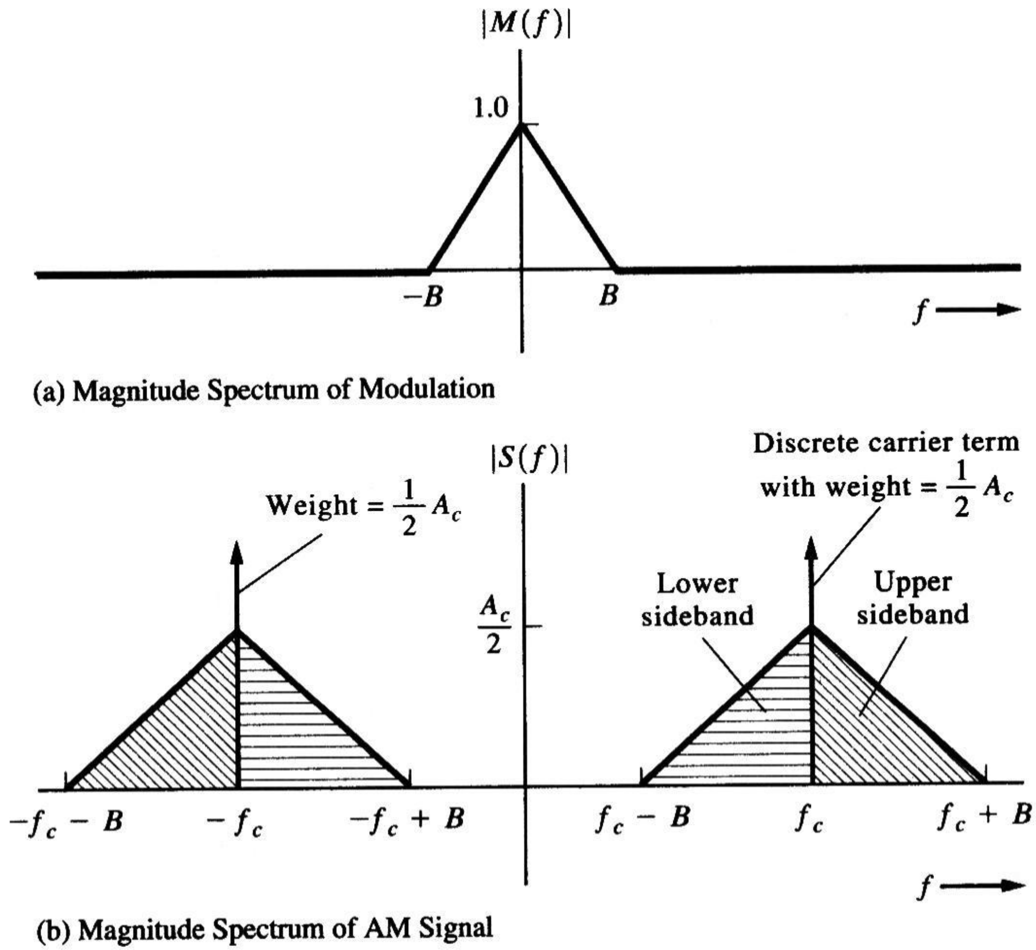


Figure 4–2 Spectrum of AM signal.

Amplitude Modulation

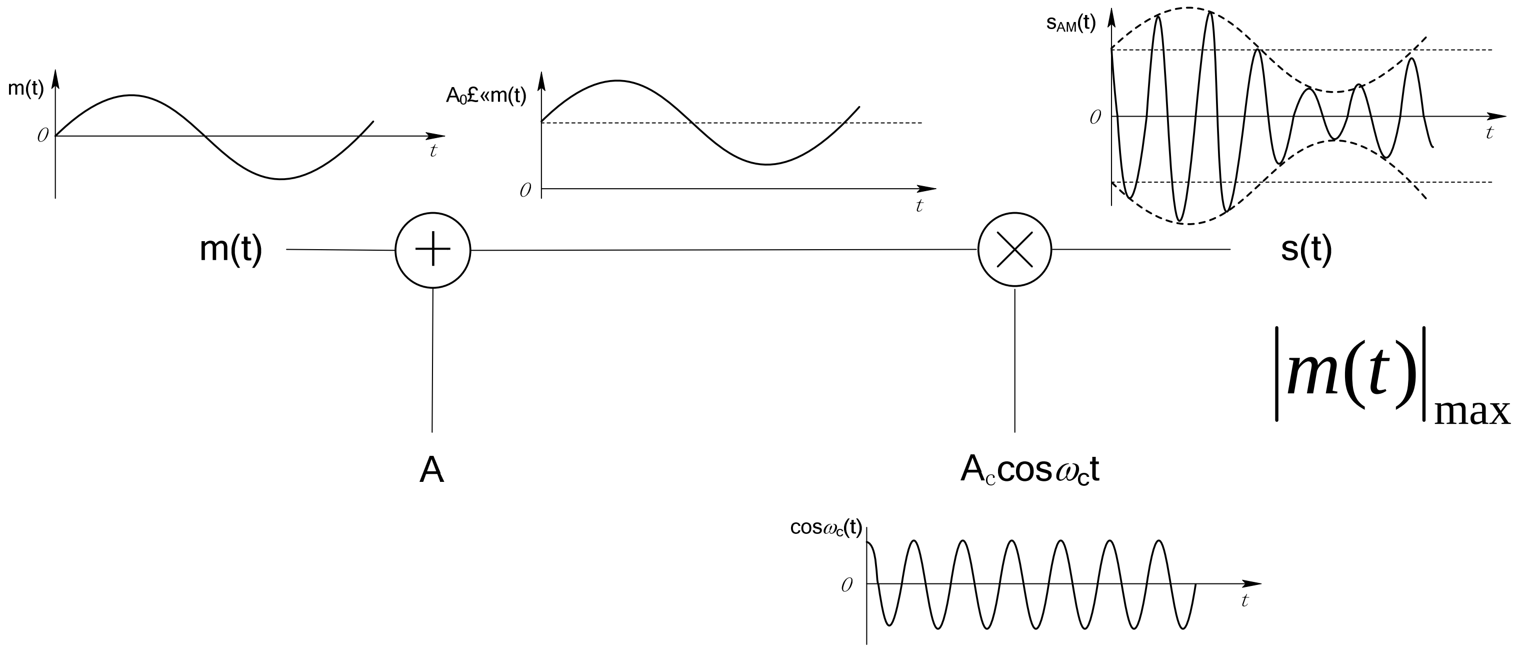
- Amplitude Modulation (AM) results when a dc bias is added to the signal m(t) before the modulation process.

$$s(t) = \left[A + m(t)\right] A_c' \cos \omega_c t = A_c \left[1 + a m_n(t)\right] \cos \omega_c t$$

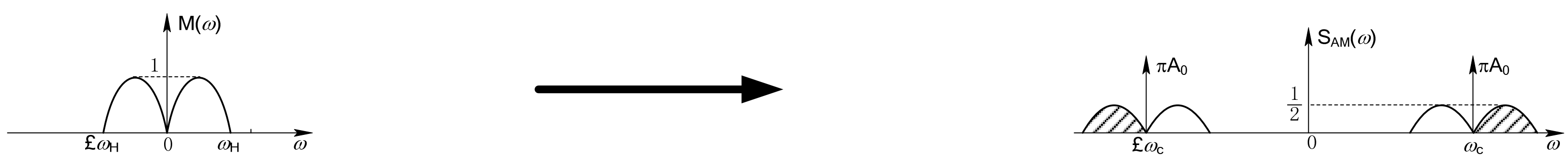
- $m_n(t)$ is the message signal normalized such that the minimum value of $m_n(t)$ is -1
- The parameter a is referred to as the modulation index.
- s(t) is said to be 100% modulated if a = 1.
- The spectrum of AM signal is given by:

$$S(f) = \frac{A_c}{2}\left[\delta(f - f_c) + aM_n(f - f_c) + \delta(f + f_c) + aM_n(f + f_c)\right]$$

Waveform and Spectrum of AM signal



$$\left|m(t)\right|_{\max} \leq A_0$$



$$B_{AM} = 2 f_H$$

Definition

- The normalized average power of the AM signal is

$$< s^2(t) > = < A_c^2[1+am_n(t)]^2 \cos^2 \omega_c t >$$
$$= \tfrac{1}{2} A_c^2 < 1 + 2am_n(t) + m_n^2(t) >$$
$$= \underbrace{\tfrac{1}{2} A_c^2}_{\text{discrete carrier power}} + \underbrace{\tfrac{1}{2} A_c^2 a^2 m_n^2(t)}_{\text{sideband power}}$$

- The modulation efficiency is the percentage of the total power of the modulated signal that conveys information.

$$E = \frac{\tfrac{1}{2} A_c^2 a^2 < m_n^2(t) >}{\tfrac{1}{2} A_c^2 + \tfrac{1}{2} A_c^2 a^2 < m_n^2(t) >} \times 100\% = \frac{a^2 < m_n^2(t) >}{1 + a^2 < m_n^2(t) >} \times 100\%$$

- Max[E] = 50%

Example

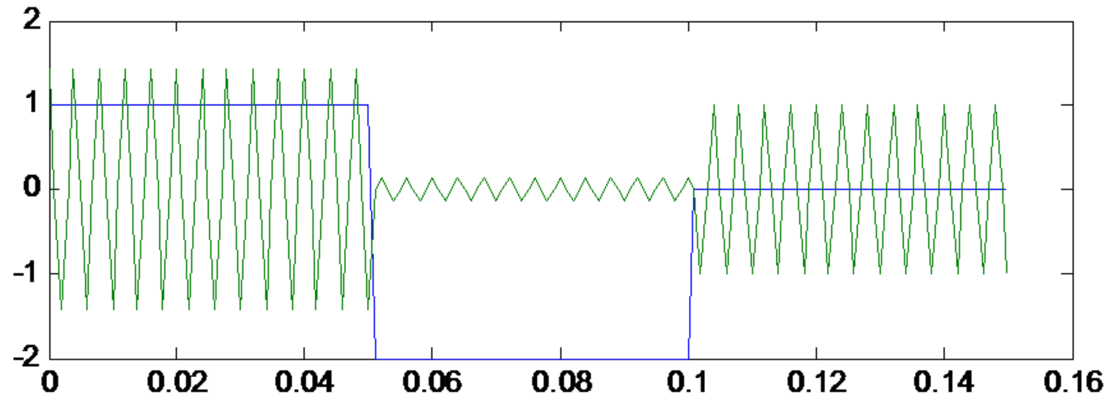
- The message signal is

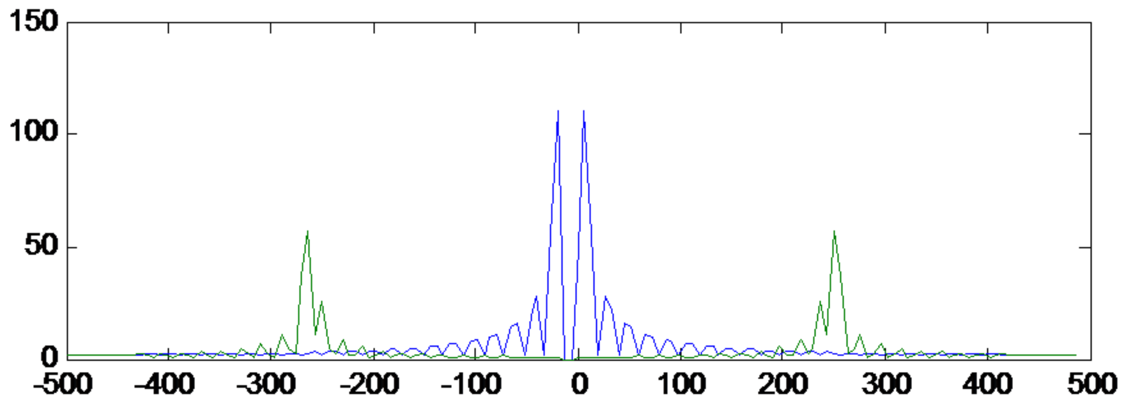
$$m(t) = \begin{cases} 1, & 0 \le t < \frac{t_0}{3} \\ -2, & \frac{t_0}{3} \le t < \frac{2t_0}{3} \\ 0, & \frac{2t_0}{3} \le t \end{cases}$$

- The carrier is $c(t) = \cos(2\pi f_c t)$

  Where fc = 250 Hz, t0 = 0.15s, a = 0.85
- Calculate
  - waveform and spectrum
  - Power of modulated signal and modulation efficiency

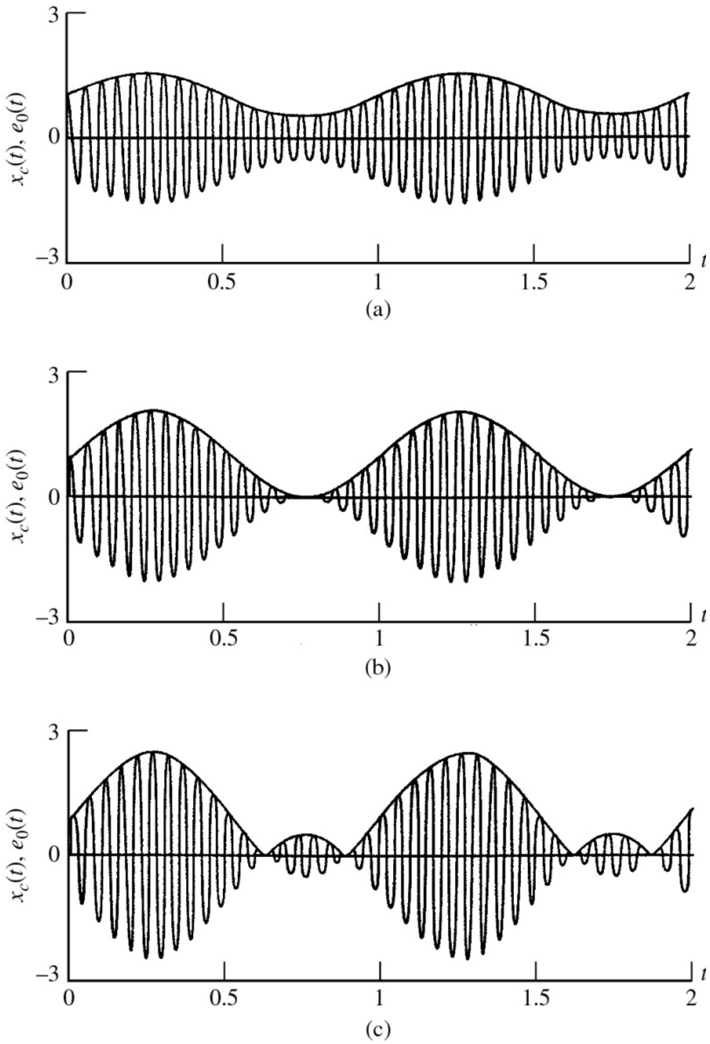
Waveform and Spectrum





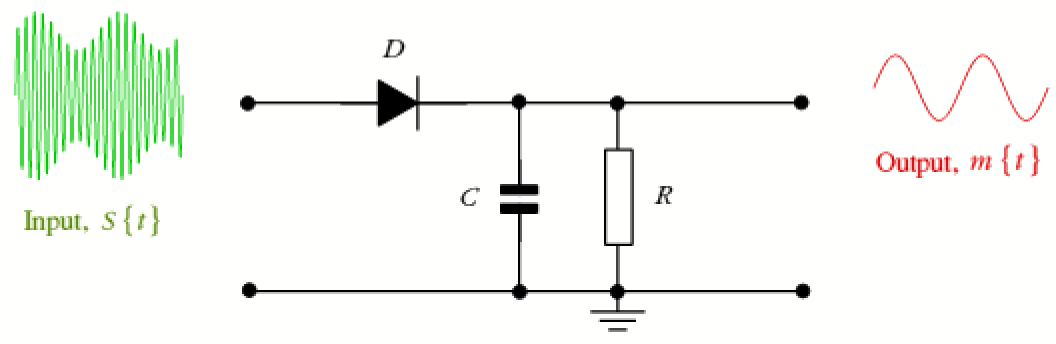
Overmodulation

- s(t) is said to be overmodulated if a > 1.
- Example, (a) a = 0.5, (b) a = 1, (c) a = 1.5
- Overmodulation results in spurious emissions by the modulated carrier, and distortion of the recovered modulating signal.
- Overmodulation is not allowed by FCC.



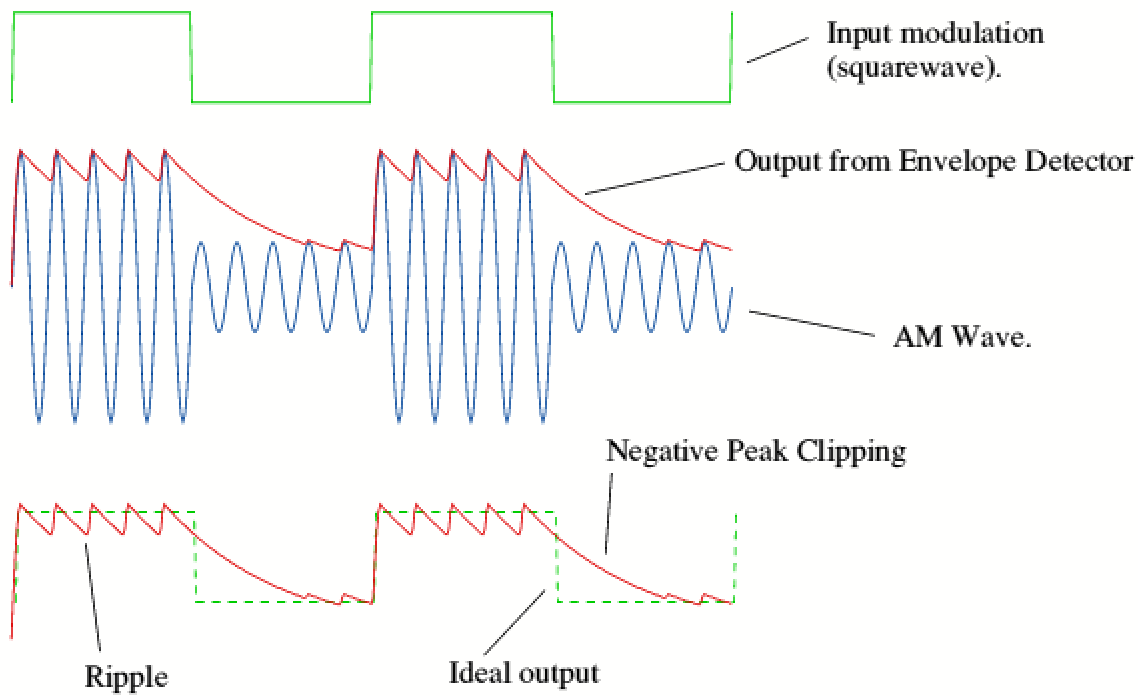
Envelope Detector

- The main advantage of AM is that, since a coherent reference is not needed for demodulation as long as a ≤ 1, the demodulator becomes simple and inexpensive.
- In many applications, such as commercial broadcast, AM is sufficient to justify its use.



Ripple and Negative Peak Clipping

- In order for the envelope-detection process to operate properly, the RC time constant of the demodulator, must be chosen carefully.



Homework

- LC 4-1, 4-9, 4-10, 5-2, 5-3, 5-5