Bandpass Digital System (3)

LC 7-3,7-4

Lecture 21, 2007-12-1

Contents

- Coherent Detection
  - OOK
  - BPSK
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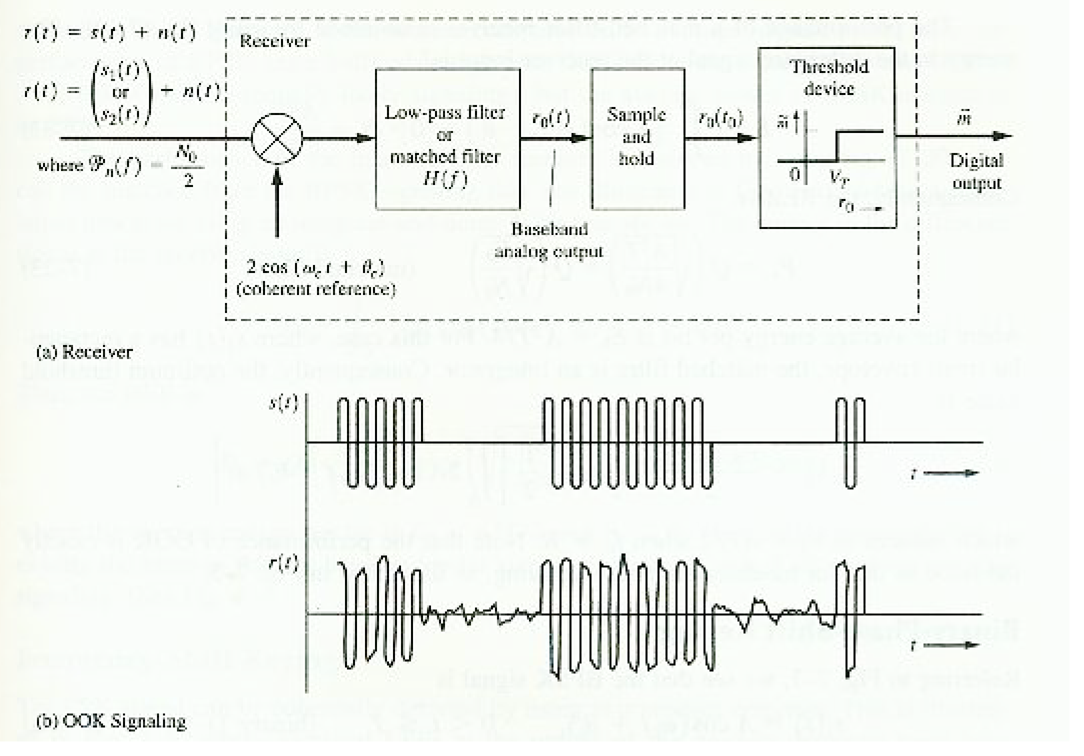
  - OOK
  - FSK
  - DPSK

On-Off Keying

- An OOK signal is represented by

$$s_1(t) = A\cos(\omega_c t + \theta_c), \qquad 0<t \leq T \quad \text{(binary 1)}$$
$$s_2(t) = 0, \qquad 0<t \leq T \quad \text{(binary 0)}$$



- The bandpass noise is represented by

$$n(t) = x(t)\cos(\omega_c t + \theta_n) - y(t)\sin(\omega_c t + \theta_n)$$

Where the PSD of n(t) is Pn(f) = N0/2 and θ_n is a uniformly distributed random variable that is independent of θ_c

- In the case of LPF, assume that the equivalent bandwidth of the filter is B>2/T.

The baseband analog output will be

$$r_0(t) = \begin{Bmatrix} A, & 0<\mathrm{t} \le \mathrm{T}, & \text{(binary 1)} \\ 0, & 0<\mathrm{t} \le \mathrm{T}, & \text{(binary 0)} \end{Bmatrix} + x(t)$$

Where x(t) is the baseband noise. The noise power

$$\overline{x^2(t)} = \overline{n^2(t)} = 2(N_0/2)(2B) = 2N_0B$$

Because s_{01}=A and s_{02}=0, the optimum threshold setting is V_T=A/2.

The BER is $Pe = Q\left(\sqrt{\dfrac{A^2}{8N_0B}}\right)$

$$P_e = Q\left(\sqrt{\frac{(s_{01} - s_{02})^2}{4\sigma_0^2}}\right)$$

- In the case of matched filter, the energy of the difference signal at the receiver input is

$$E_d = \int_0^T \left[A\cos(\omega_c t + \theta_c) - 0 \right]^2 dt = A^2 T / 2$$

$$P_e = Q\left(\sqrt{\frac{E_d}{2N_0}} \right)$$

The BER is

$$Pe = Q\left(\sqrt{\frac{A^2 T}{4N_0}} \right) = Q\left(\sqrt{\frac{E_b}{N_0}} \right)$$

Where the average energy per bit is $E_b = A^2T/4$.

The optimum threshold value is

Ex 6.12

$$V_T = \frac{s_{01} + s_{02}}{2} = \frac{s_{01}}{2} = \frac{1}{2}\int_0^T 2A\cos^2(\omega_c t + \theta_c)dt = AT/2$$

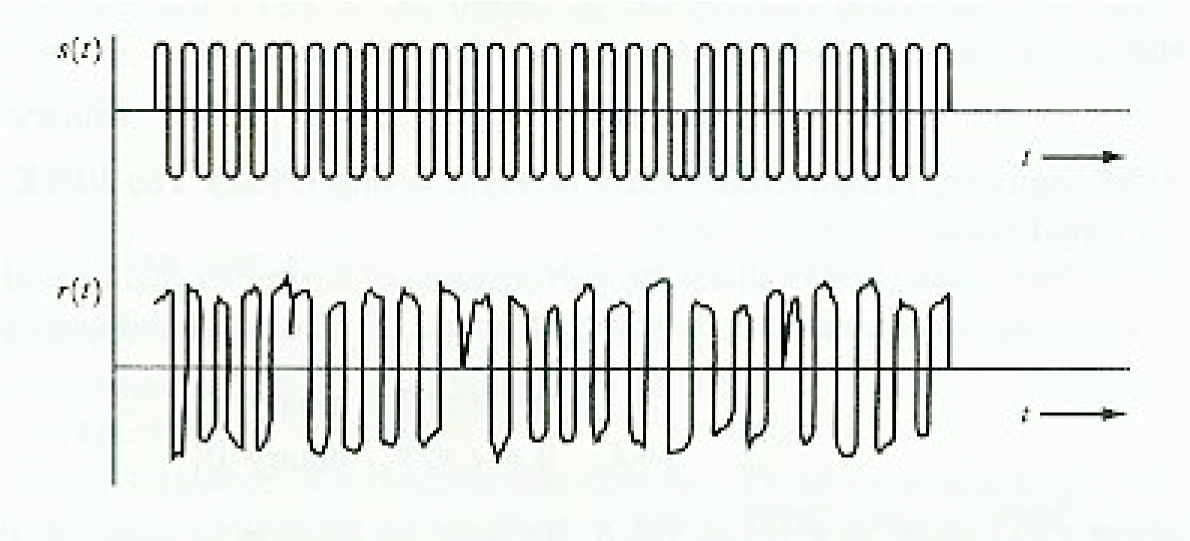
Note that the performance of OOK is exactly the same as that of basedband unipolar signaling.

Binary-Phase-Shift Keying

- An BPSK signal is

$$s_1(t) = A\cos(\omega_c t + \theta_c), \quad 0<t \le T \ \text{(binary 1)}$$

$$s_2(t) = -A\cos(\omega_c t + \theta_c), \quad 0<t \le T \ \text{(binary 0)}$$



- In the case of LPF, assume that the equivalent bandwidth of the filter is B>2/T.

The baseband analog output will be

$$r_0(t) = \begin{cases} A, & 0<\mathrm{t} \le \mathrm{T}, \quad \text{(binary 1)} \\ -A, & 0<\mathrm{t} \le \mathrm{T}, \quad \text{(binary 0)} \end{cases} + x(t)$$

Where x(t) is the baseband noise. The noise power

The optimum threshold is V_T=0

The BER is

$$Pe = Q\left(\sqrt{\frac{A^2}{2N_0B}}\right)$$

- In the case of matched filter, the energy of the difference signal at the receiver input is

$$E_d = \int_0^T \left[2A\cos(\omega_c t + \theta_c)\right]^2 dt = 2A^2 T$$

The BER is

$$Pe = Q\left(\sqrt{\frac{A^2 T}{N_0}}\right) = Q\left(\sqrt{2\left(\frac{E_b}{N_0}\right)}\right)$$

Where the average energy per bit is $E_b = A^2T/2$, $V_T = 0$

Note that the performance of BPSK is exactly the same as that of basedband polar signaling.

Frequency-Shift Keying

- An FSK signal is

$$s_1(t) = A\cos(\omega_1 t + \theta_c), \quad 0<t \le T \text{ (binary 1)}$$
$$s_2(t) = A\cos(\omega_2 t + \theta_c), \quad 0<t \le T \text{ (binary 0)}$$

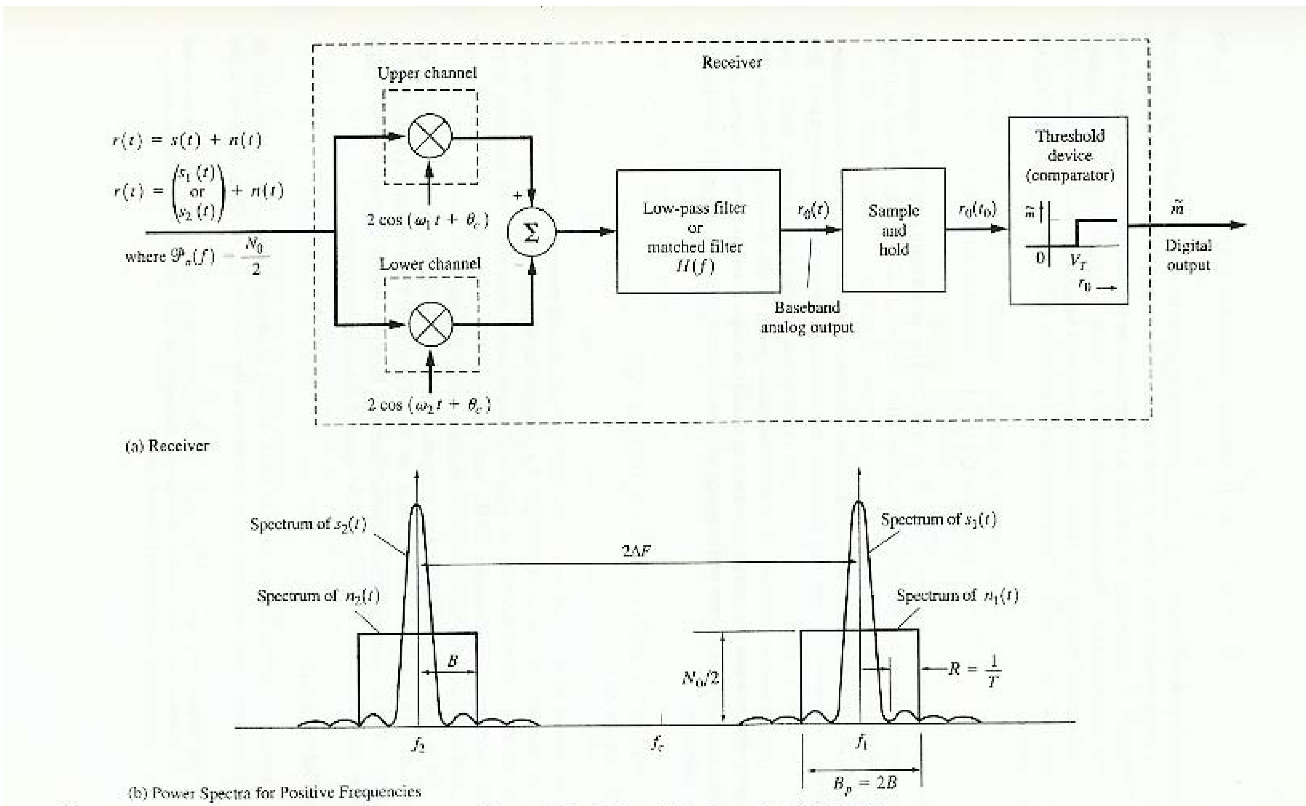


Figure 7–8 Coherent detection of an FSK signal.

- In the case of LPF, it is equivalent that there is dual bandpass filters – one centered at f1 and the other centered at f2, where each has an equivalent bandwidth Bp=2B

The input noise that effects the output consists of two narrowband components $n_1(t)$ and $n_2(t)$.

$$n(t) = n_1(t) + n_2(t)$$

$$n_1(t) = x_1(t)\cos(\omega_1 t + \theta_1) - y_1(t)\sin(\omega_1 t + \theta_1)$$

$$n_2(t) = x_2(t)\cos(\omega_2 t + \theta_2) - y_2(t)\sin(\omega_2 t + \theta_2)$$

The input signal and noise that pass through the upper channel is

$$r_1(t) = \begin{Bmatrix} s_1(t), & \text{(binary 1)} \\ 0, & \text{(binary 0)} \end{Bmatrix} + n_1(t)$$

The input signal and noise that pass through the lower channel is

$$r_2(t) = \begin{Bmatrix} 0, & \text{(binary 1)} \\ s_2(t), & \text{(binary 0)} \end{Bmatrix} + n_2(t)$$

$$r(t) = r_1(t) + r_2(t)$$

The noise power of n1(t) and n2(t) is $\overline{n_1^2(t)} = \overline{n_2^2(t)} = 2N_0B$

The baseband analog output is

$$r_0(t) = \begin{cases} A, & 0<\mathrm{t} \le \mathrm{T}, \quad \text{(binary 1)} \\ -A, & 0<\mathrm{t} \le \mathrm{T}, \quad \text{(binary 0)} \end{cases} + n_0(t)$$

Where s_{01}=+A, s_{02}=-A and $n_0(t) = x_1(t) - x_2(t)$. The optimum threshold setting is $V_T=0$

The resulting baseband noise processes x1(t) and x2(t) are independent, and the output noise power is

$$\overline{n_0^2(t)} = \sigma_0^2 = \overline{x_1^2(t)} + \overline{x_2^2(t)} = \overline{n_1^2(t)} + \overline{n_2^2(t)} = 4N_0B$$

The BER is

$$Pe = Q\left(\sqrt{\frac{A^2}{4N_0B}}\right) \qquad P_e = Q\left(\sqrt{\frac{(s_{01} - s_{02})^2}{4\sigma_0^2}}\right)$$

- In the case of matched filter, the energy in the difference signal is

$$E_d = \int_0^T \left[A\cos(\omega_1 t + \theta_1) - A\cos(\omega_2 t + \theta_2)\right]^2 dt$$

$$= \int_0^T \left[A^2 \cos^2(\omega_1 t + \theta_1) - 2A^2 \cos(\omega_1 t + \theta_1)\cos(\omega_2 t + \theta_2) + A^2 \cos^2(\omega_2 t + \theta_2)\right] dt$$

$$= \frac{1}{2}A^2 T - A^2 \int_0^T \cos^2\left[(\omega_1 - \omega_2)t + (\theta_1 - \theta_2)\right] dt + \frac{1}{2}A^2 T$$

Consider the case when 2ΔF=f1-f2=n/(2T)=nR/2. Under the condition the integral goes to zero.

The BER is

$$Pe = Q\left(\sqrt{\frac{A^2 T}{2N_0}}\right) = Q\left(\sqrt{\frac{E_b}{N_0}}\right)$$

Where the average energy per bit is $Eb=A^2T/2$

The performance of FSK signaling is equivalent to that of OOK.

On-Off Keying

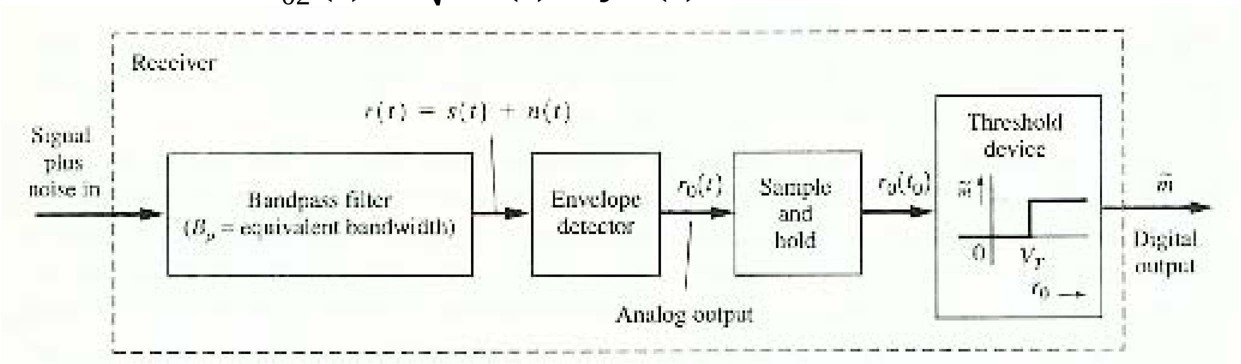
- The bandpass filter output is

$$r(t)=\begin{cases} r_1(t), & 0<\mathrm{t}\le \mathrm{T}\ \ \text{(binary 1)} \\ r_2(t), & 0<\mathrm{t}\le \mathrm{T}\ \ \text{(binary 0)} \end{cases}$$

$$r_1(t)=A\cos(\omega_c t+\theta_c)+n(t)$$
$$=[A+x(t)]\cos(\omega_c t+\theta_c)-y(t)\sin(\omega_c t+\theta_c)$$
$$r_2(t)=x(t)\cos(\omega_c t+\theta_c)-y(t)\sin(\omega_c t+\theta_c)$$

- The envelope detector output is

$$r_{01}(t)=\sqrt{[A+x(t)]^2+y^2(t)}$$
$$r_{02}(t)=\sqrt{x^2(t)+y^2(t)}$$



- The BER is $P_e = \frac{1}{2}\int_{-\infty}^{V_T} f(r_0 \mid s_1)dr_0 + \frac{1}{2}\int_{-V_T}^{\infty} f(r_0 \mid s_2)dr_0$

$$f(r_0 \mid s_2) = \frac{r_0}{\sigma^2} e^{-r_0^2/(2\sigma^2)}$$ Rayleigh distribution

$$f(r_0 \mid s_1) = \frac{r_0}{\sigma^2} e^{-(r_0^2+A^2)/(2\sigma^2)} I_0\left(\frac{r_0 A}{\sigma^2}\right)$$ Racian distribution

$$I_0(z) = \frac{1}{2\pi}\int_0^{2\pi} e^{z\cos\theta} d\theta$$

the modified Bessel function of the first kind of zero order

$P_e = \frac{1}{2}\left[1 - Q(\sqrt{2r}, b)\right] + \frac{1}{2}e^{-b^2/2}$, $r = \frac{A^2}{2\sigma^2}$ is SNR, and $b = \frac{V_T}{\sigma}$ is normalized threshold

$$Q(\alpha, \beta) = \int_\beta^\infty x \exp\left(-\frac{x^2+\alpha^2}{2}\right) I_0(\alpha x) dx$$ Marcum Q function

- To obtain the optimum BER,

$$\frac{\partial P_e}{\partial b} = 0 \quad \rightarrow \quad r = \ln I_0\left(\sqrt{2r}b_{opt}\right)$$

If SNR r >>1

$$b_{opt} = \sqrt{\frac{r}{2}}$$

If SNR r <<1

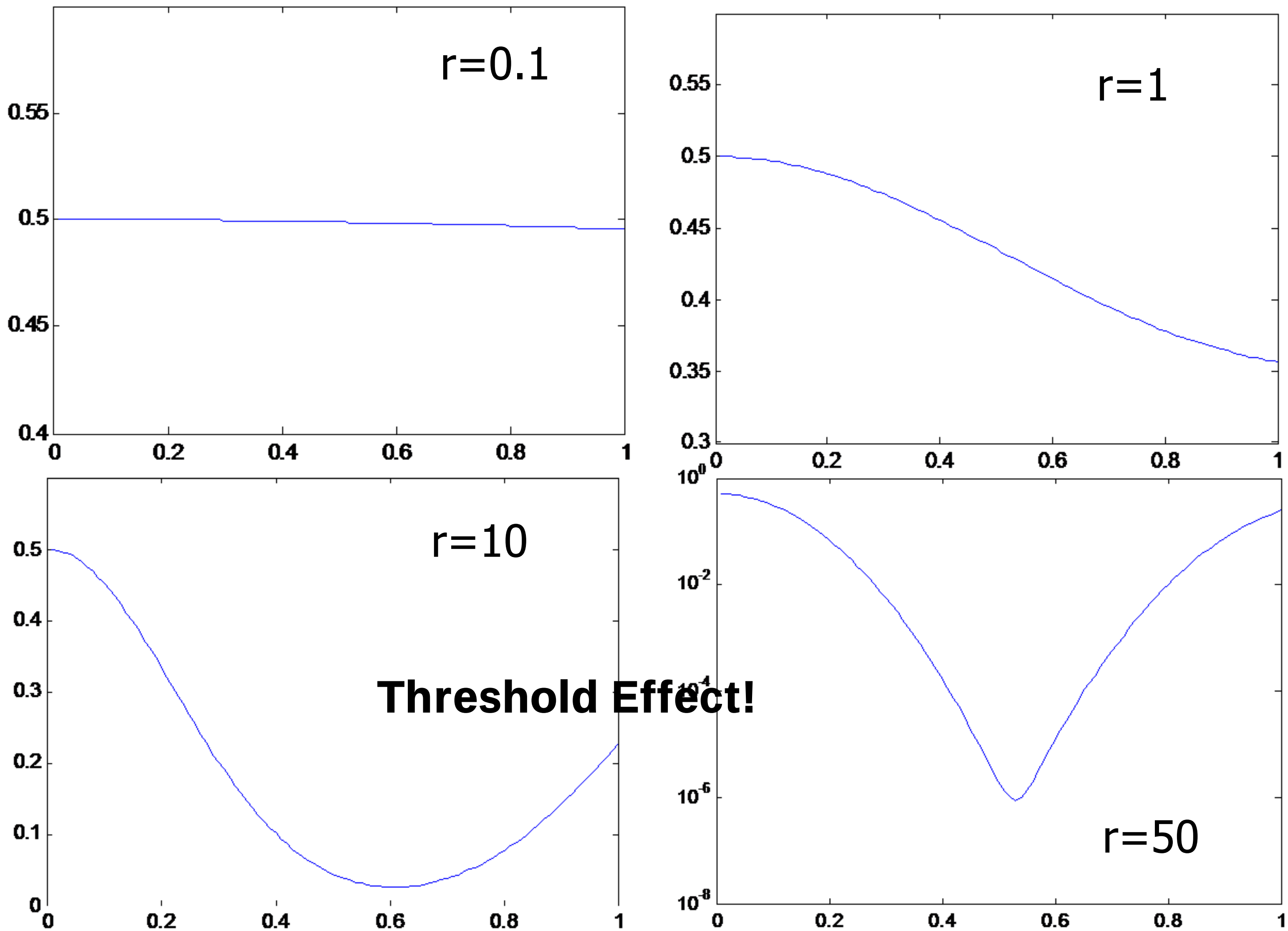
$$b_{opt} = \sqrt{2}$$

$$P_e = \frac{1}{4} erfc\left(\sqrt{\frac{r}{4}}\right) + \frac{1}{2} e^{-r/4}$$

$r \rightarrow \infty$, the lower bound

$$P_e = \frac{1}{2} e^{-r/4} = \frac{1}{2} e^{-[1/(2TB_p)]/(E_b/N_0)}$$

BER vs. Threshold



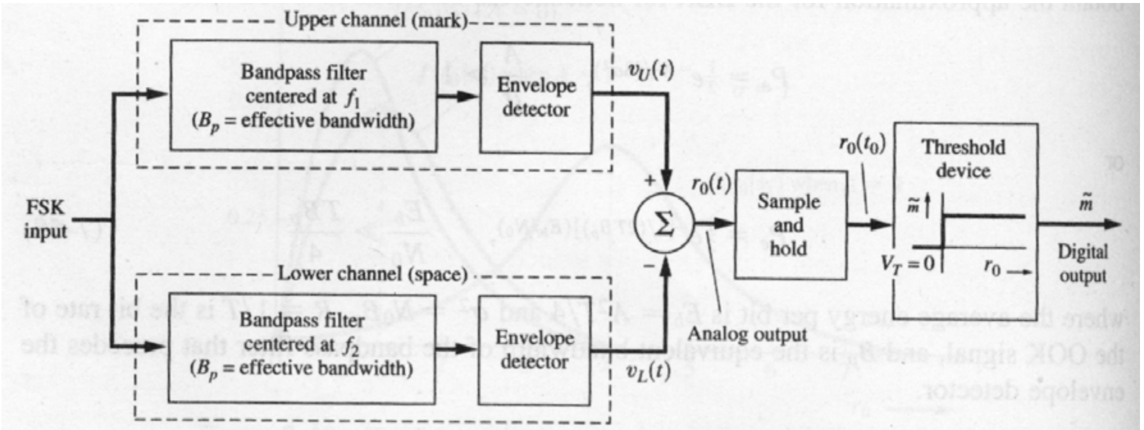
FSK

- For the signal alone at the receiver input, the output of the summing junction is r0(t) = +A when a mark (binary 1) is transmitted, and r0(t) = -A when a space (binary 0) is transmitted.
- Because of this symmetry and because the noise out of the upper and lower receiver channels is similar, the optimum threshold is V_T=0.

$$f(r_0 \mid s_1) = f(-r_0 \mid s_2)$$

- The BER is

$$P_e = \frac{1}{2}\int_{-\infty}^{0} f(r_0 \mid s_1)dr_0 + \frac{1}{2}\int_{0}^{\infty} f(r_0 \mid s_2)dr_0 = \int_{0}^{\infty} f(r_0 \mid s_2)dr_0$$



r0(t) is positive when the upper channel output v_U exceeds the lower channel output v_L

$$P_e = P(v_U > v_L \mid s_2)$$

$$f(v_U \mid s_2) = \frac{v_U}{\sigma^2} e^{-v_U^2/(2\sigma^2)}$$

$$f(v_L \mid s_2) = \frac{v_L}{\sigma^2} e^{-(v_L^2+A^2)/(2\sigma^2)} I_0\left(\frac{v_L A}{\sigma^2}\right)$$

$$Pe = \int_0^\infty \frac{v_L}{\sigma^2} e^{-(v_L^2+A^2)/(2\sigma^2)} I_0\left(\frac{v_L A}{\sigma^2}\right) \int_{v_L}^\infty \frac{v_U}{\sigma^2} e^{-v_U^2/(2\sigma^2)} dv_U dv_L$$

$$= e^{-A^2/(2\sigma^2)} \int_0^\infty \frac{v_L}{\sigma^2} e^{-v_L^2/(2\sigma^2)} I_0\left(\frac{v_L A}{\sigma^2}\right) dv_L$$

$$= e^{-A^2/(2\sigma^2)} \cdot \frac{1}{2} e^{A^2/(4\sigma^2)} = \frac{1}{2} e^{-A^2/(4\sigma^2)} = \frac{1}{2} e^{-[1/(2TB_p)]/(E_b/N_0)}$$

DPSK

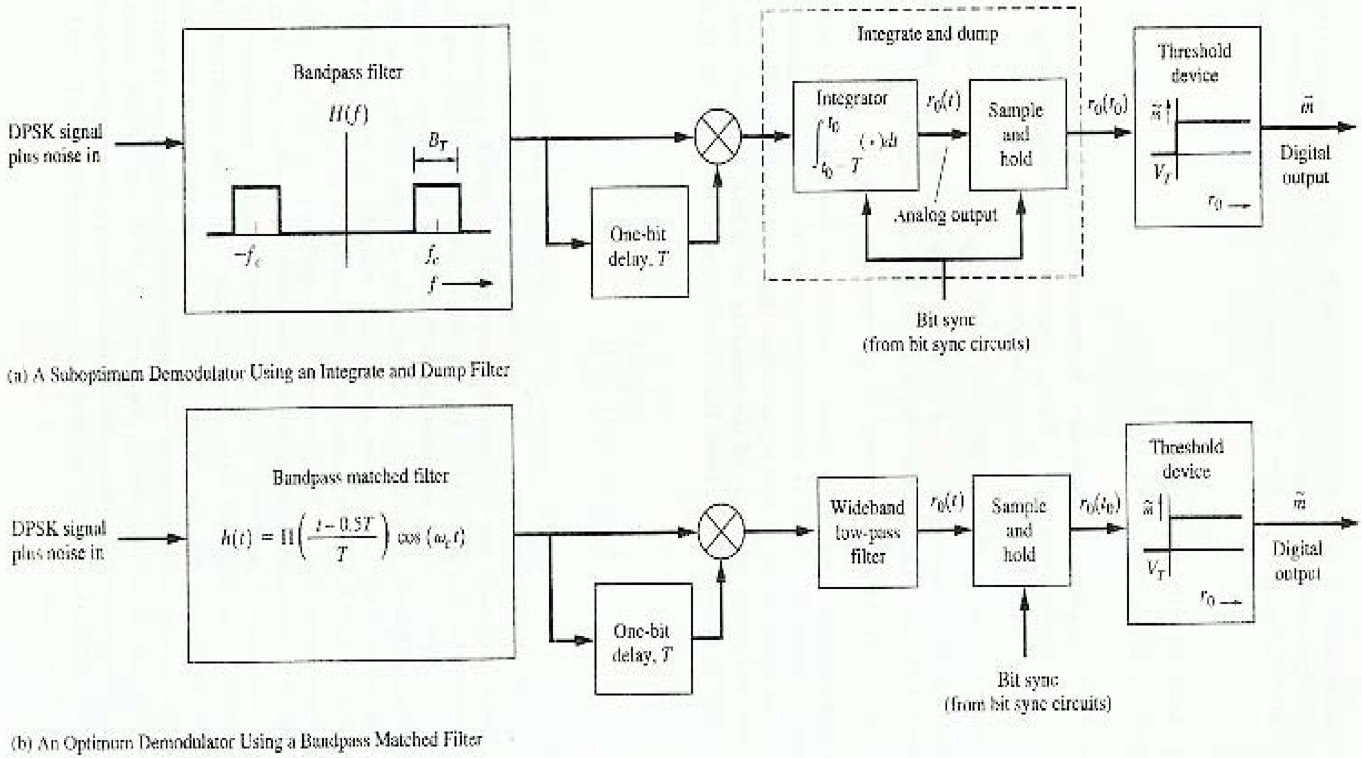


Figure 7–12 Demodulation of DPSK.

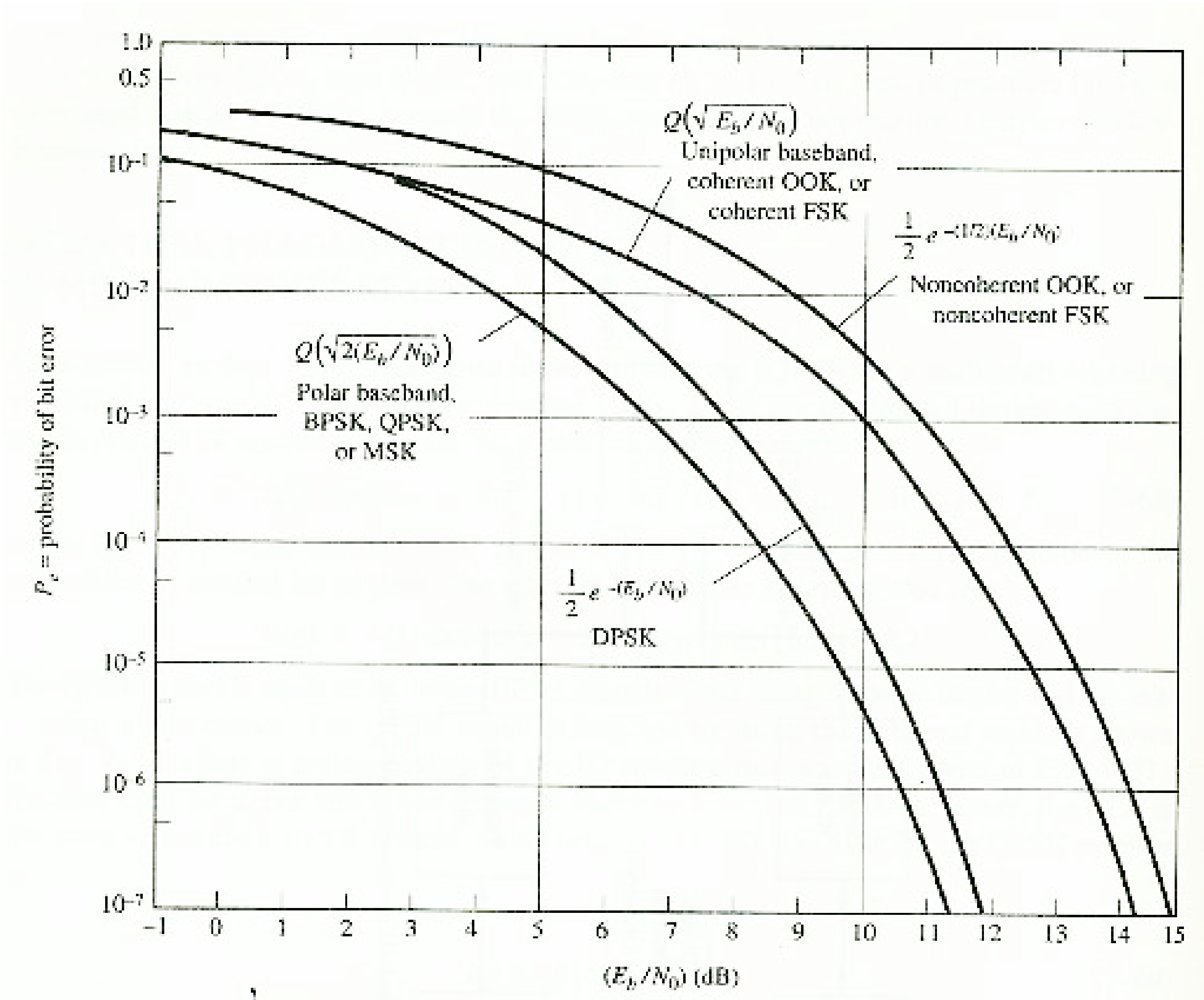
- The BER for these DPSK receivers can be derived under the following assumptions:
  - The additive input noise is white and Gaussian
  - The phase perturbation of the composite signal plus noise varies slowly so that the phase reference is essentially a constant from the past signaling interval to the present signaling interval.
  - The transmitter carrier oscillator is sufficiently stable so that the phase during the present signaling interval is the same as that from the past signaling interval.
- The BER of LPF is

$$P_e = Q\left(\sqrt{\frac{\left(E_b / N_0\right)}{1 + (BT/2)\left(E_b / N_0\right)}}\right)$$

- The BER of MF is

$$P_e = \frac{1}{2} e^{-E_b / N_0}$$

BER Comparison



Homework

- LC 7-18, 7-20, 7-26