Bandpass Digital System (2)

LC 5-10

Lecture 20, 2008-11-28

Contents

- MASK
- MPSK, QPSK
- QAM
- OQPSK

Multilevel Modulated Bandpass Signaling

- With multilevel signaling, digital inputs with more than two levels are allowed on the transmitter input.

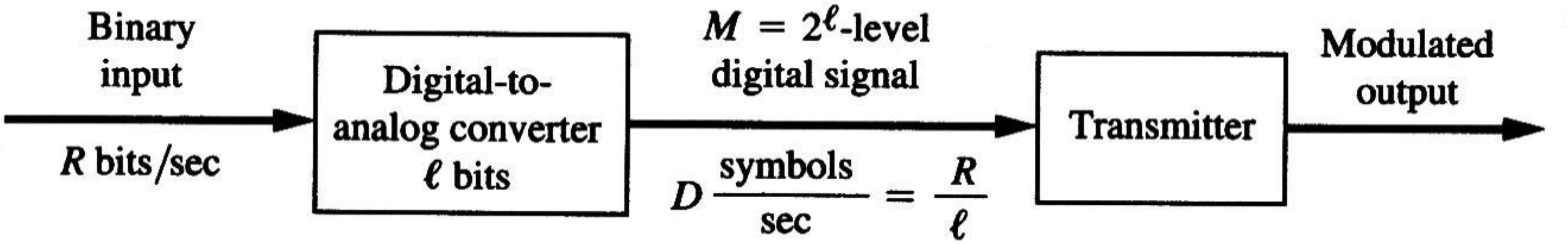
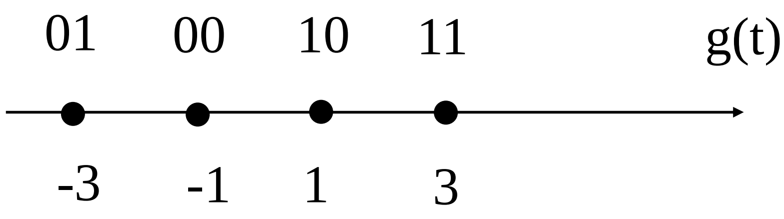


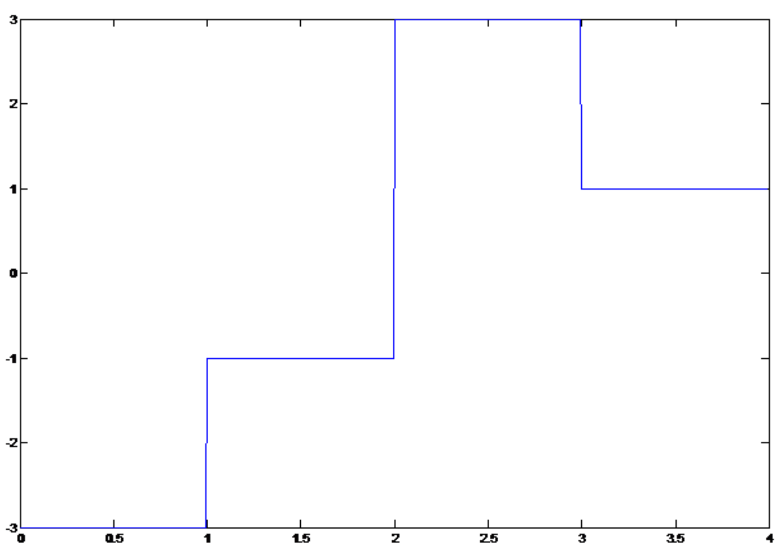
Figure 5–29 Multilevel digital transmission system.

Multiple Amplitude Shift Keying (MASK)

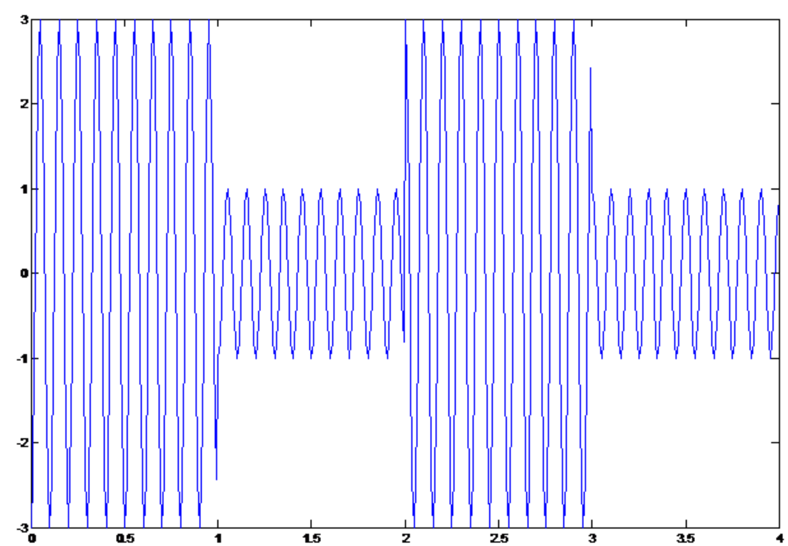
$$e_{MASK}(t) = \sum_n a_n h(t - nT_S)\cos\omega_c t$$



b_{1n}	0	0	1	1
b_{2n}	1	0	0	1
a_n	-3	-1	1	3



g(t)

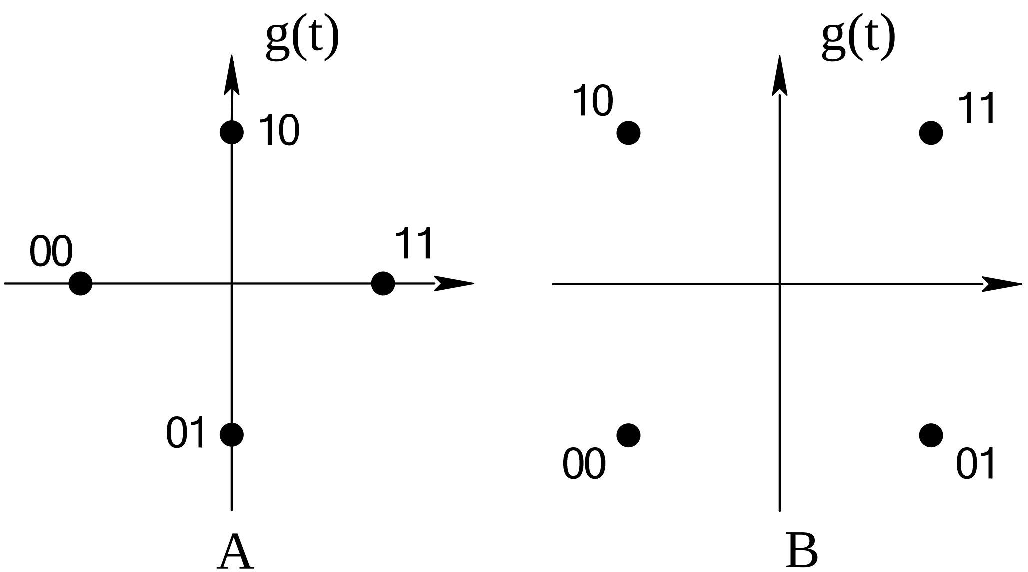


$e_{MASK}(t)$

M-ary Phase-Shift Keying (MPSK)

- If the transmitter is a PM transmitter with an $M = 4$-level digital modulation signal, M-ary phase-shift keying (MPSK) is generated at the transmitter output. The 4PSK is called quadrature phase-shift keyed (QPSK) signaling.

$$e_{MPSK}(t) = \sum_n h(t - nT_S)\cos\left(\omega_c t + \theta_n\right)$$



b_{1n}	b_{2n}	A	B
0	0	180^o	225^o
0	1	270^o	315^o
1	1	0^o	45^o
1	0	90^o	135^o

QPSK Generation

- MPSK can also be generated by using two quadrature carriers modulated by the x and y components of the complex envelope; in that case,

$$g(t) = A_c e^{j\theta(t)} = x(t) + jy(t)$$

$$x(t) = A_c \cos\theta(t) \Rightarrow x_i = A_c \cos\theta_i$$

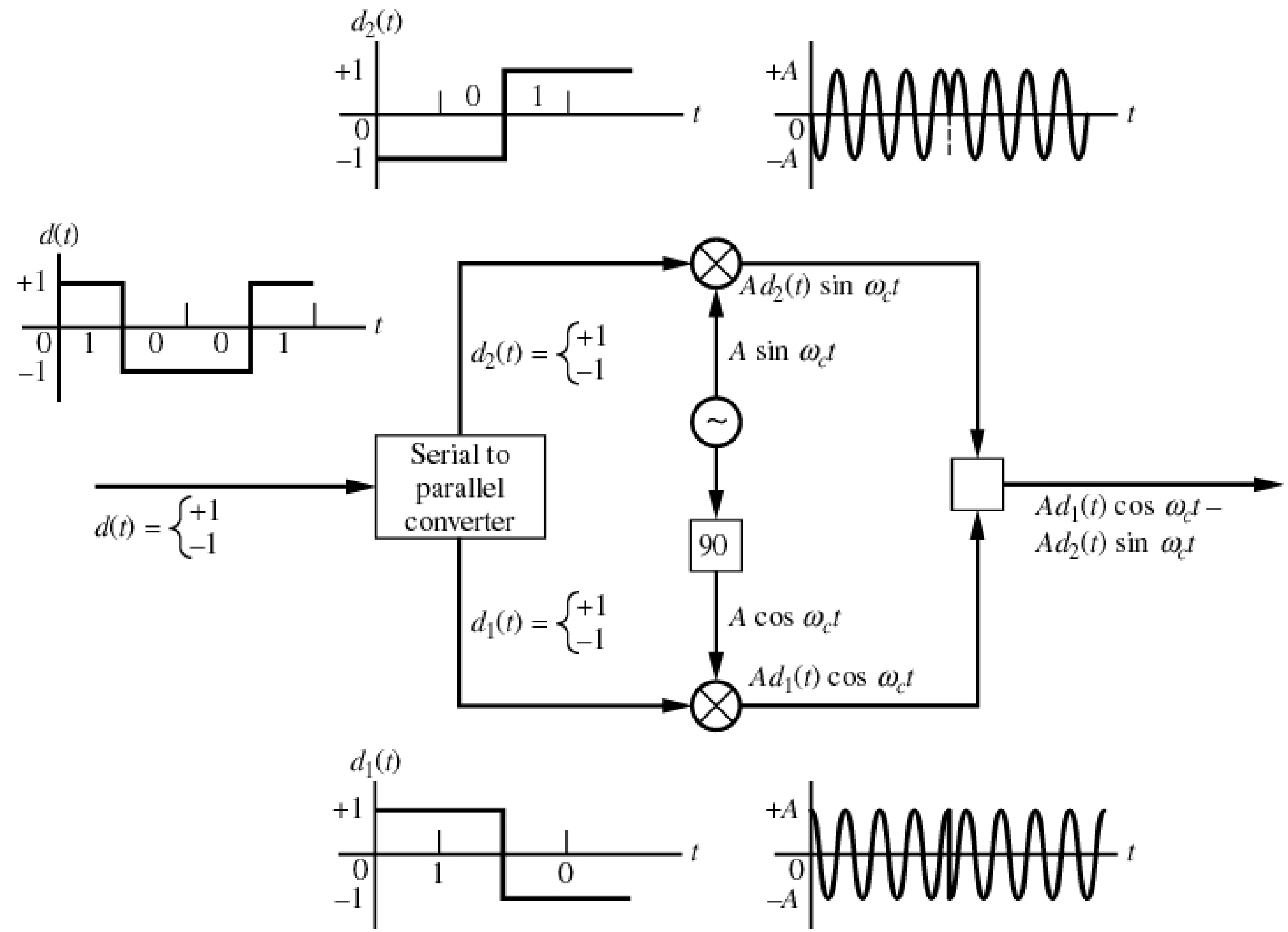
$$y(t) = A_c \sin\theta(t) \Rightarrow y_i = A_c \sin\theta_i$$

$$i = 1, 2, \ldots, M$$

$$\text{For QPSK}$$

$$(x_i, y_i) = (0, 0), (0, A_c), (-A_c, 0), (0, -A_c) \text{ or}$$

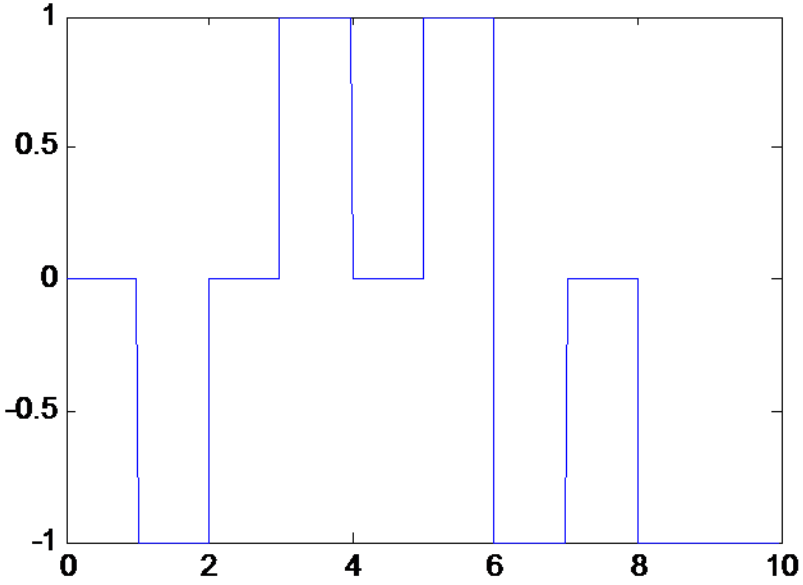
$$(x_i, y_i) = \left(\frac{A_c}{\sqrt{2}}, \frac{A_c}{\sqrt{2}}\right), \left(\frac{A_c}{\sqrt{2}}, -\frac{A_c}{\sqrt{2}}\right), \left(-\frac{A_c}{\sqrt{2}}, -\frac{A_c}{\sqrt{2}}\right), \left(\frac{A_c}{\sqrt{2}}, -\frac{A_c}{\sqrt{2}}\right)$$



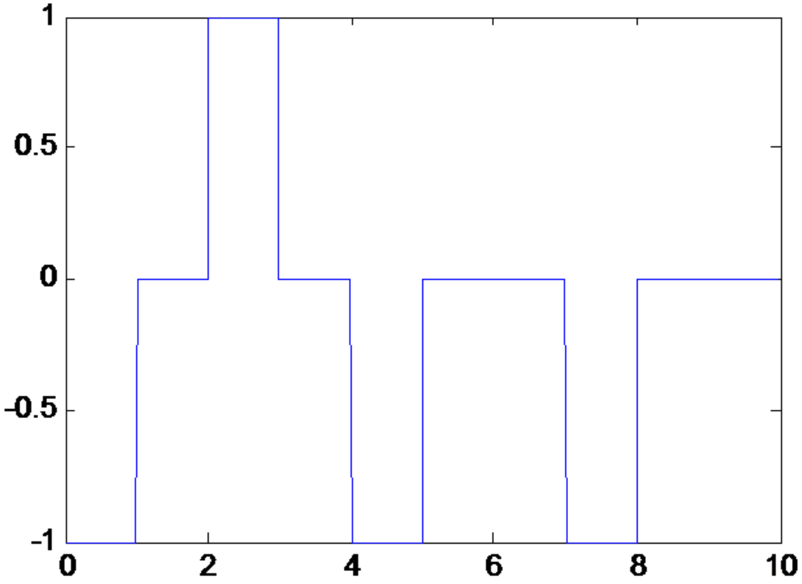
$d_2(t)$
+1
0
1
0
–1
t
+A
0
–A
t
$d(t)$
+1
0
1
0
0
1
–1
t
$Ad_2(t)$ sin $\omega_c t$
$d_2(t) = \begin{cases}+1\\-1\end{cases}$
A sin $\omega_c t$
Serial to parallel converter
$d(t) = \begin{cases}+1\\-1\end{cases}$
90
$Ad_1(t)$ cos $\omega_c t$ –
$Ad_2(t)$ sin $\omega_c t$
$d_1(t) = \begin{cases}+1\\-1\end{cases}$
A cos $\omega_c t$
$Ad_1(t)$ cos $\omega_c t$
$d_1(t)$
+1
0
1
0
–1
t
+A
0
–A
t

QPSK Waveform (Complex Envelope)

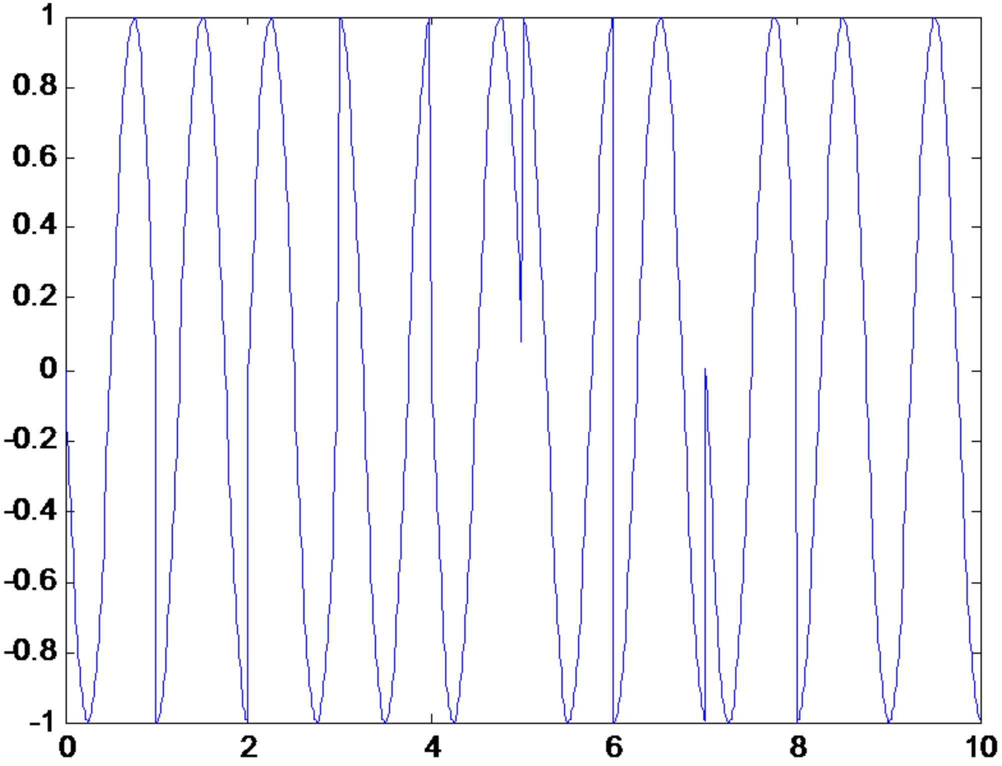
In phase



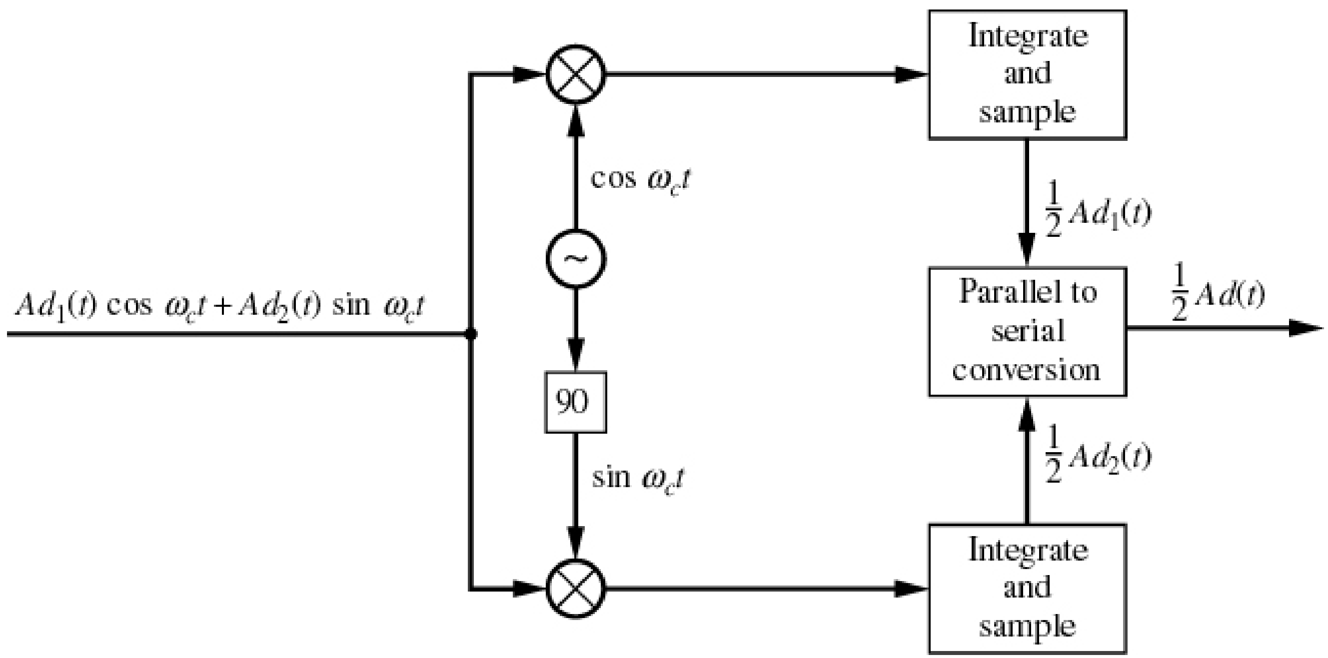
Quadrature



QPSK Waveform



QPSK Demodulation



Quadrature Amplitude Modulation (QAM)

- Quadrature carrier signaling is called quadrature amplitude modulation (QAM). In general QAM signal constellations are not restricted to having permitted signaling points only on a circle (of radius A_c, as was the case for MPSK). For example, a popular 16QAM constellation is shown below.

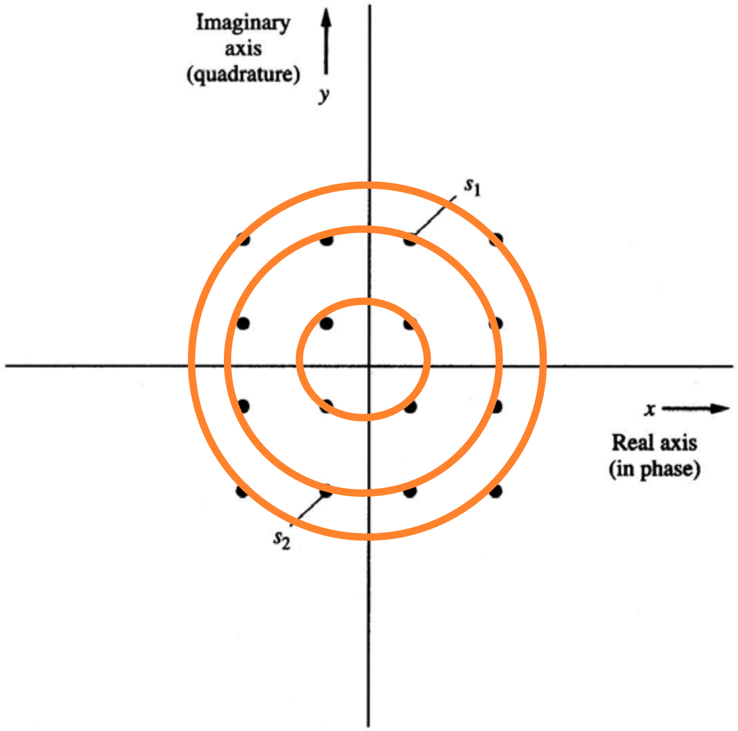


Figure 5–32 16-symbol QAM constellation (four levels per dimension).

- Quadrature Amplitude Modulation is really Quadrature Phase Amplitude Modulation

$$s(t) = \mathrm{Re}\{g(t)e^{j\omega_c t}\} = x(t)\cos\omega_c t - y(t)\sin\omega_c t$$

$$g(t) = x(t) + jy(t) = R(t)e^{j\theta(t)}$$

$$x(t) = \sum_n x_n h_1(t - nT_s) = \sum_n x_n h_1(t - \frac{n}{D})$$

$$y(t) = \sum_n y_n h_1(t - nT_s) = \sum_n y_n h_1(t - \frac{n}{D})$$

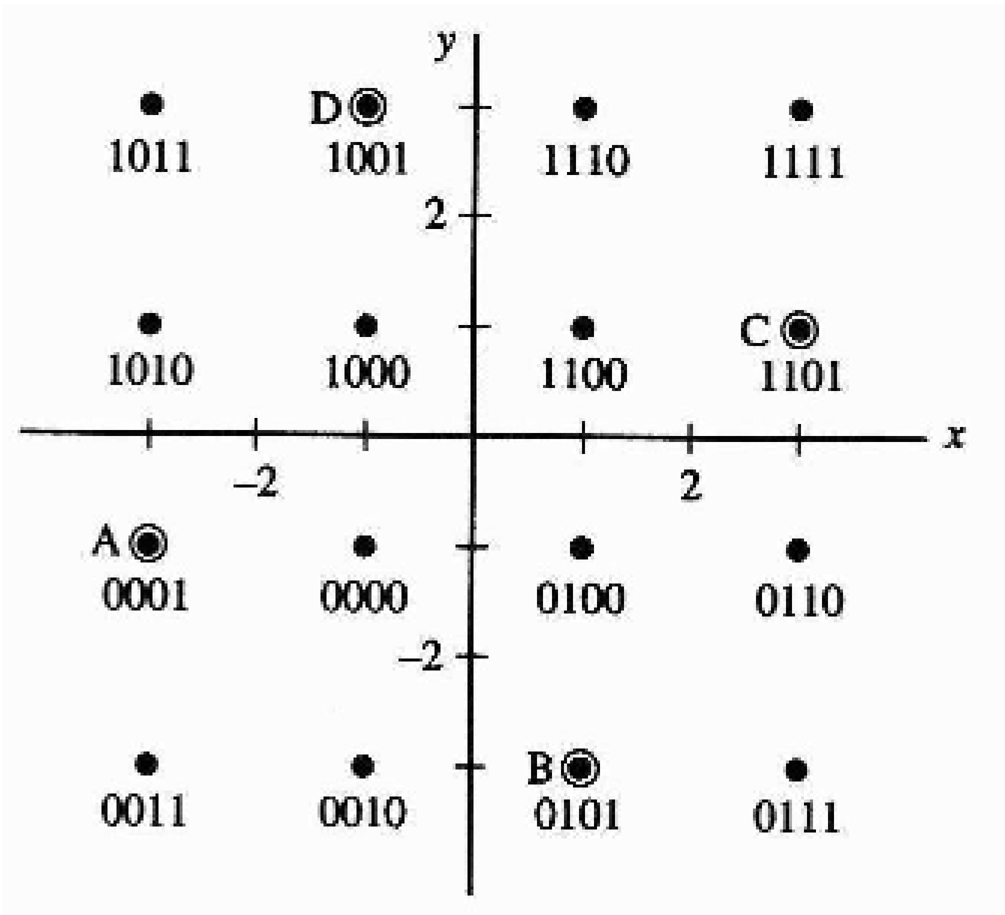
$h_1(t)$: pulse shape that is for each symbol

T_s : symbol interval

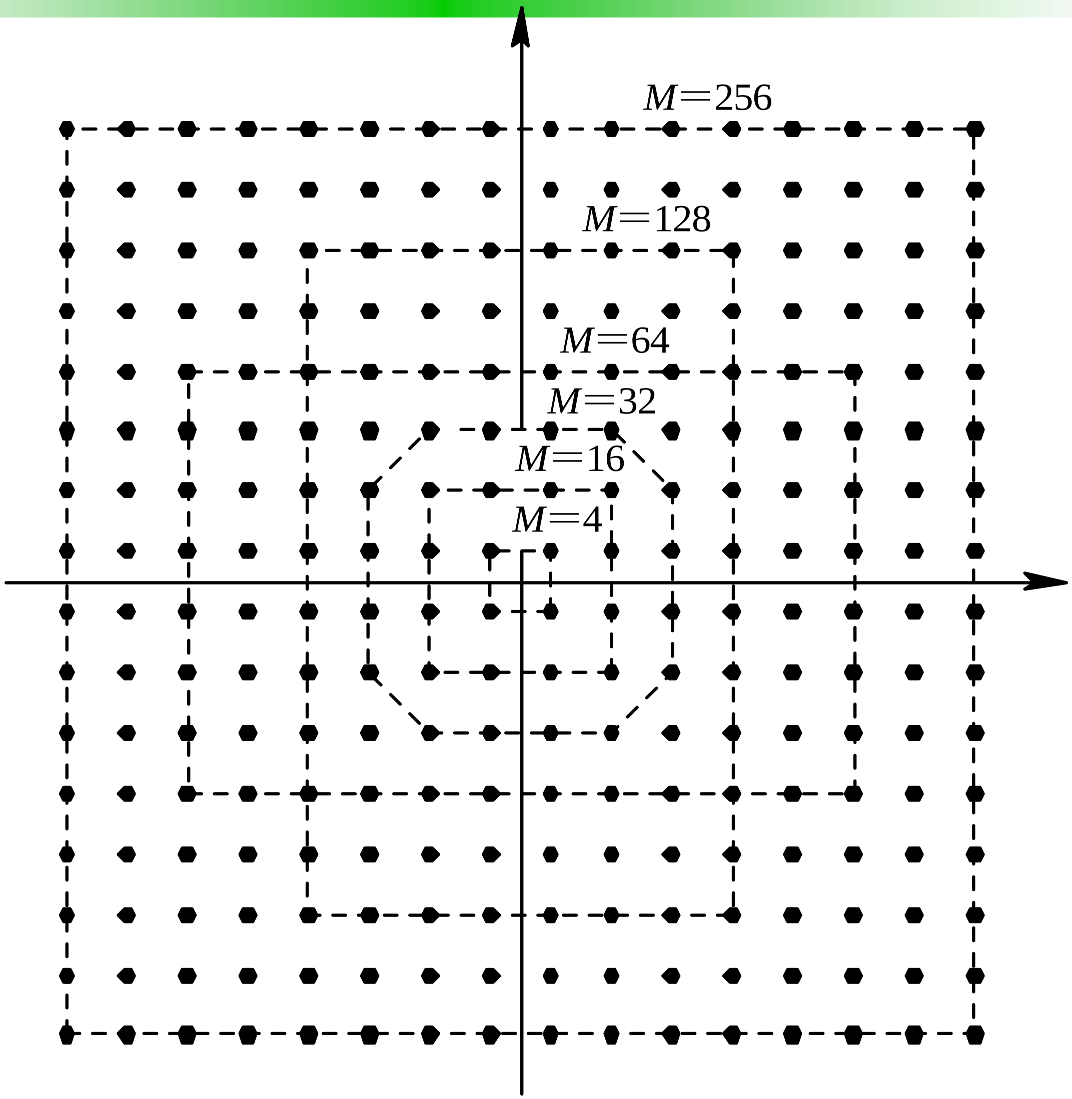
D : symbol rate (baud) $D = \frac{1}{T_s} = \frac{R}{l}$

R : bit rate

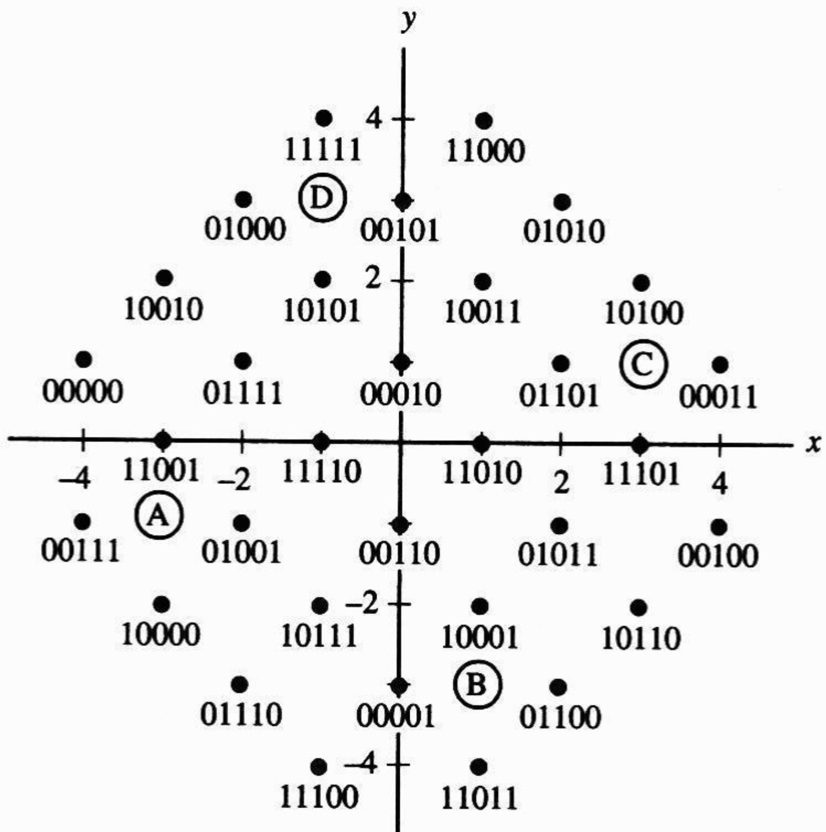
16 QAM Constellation



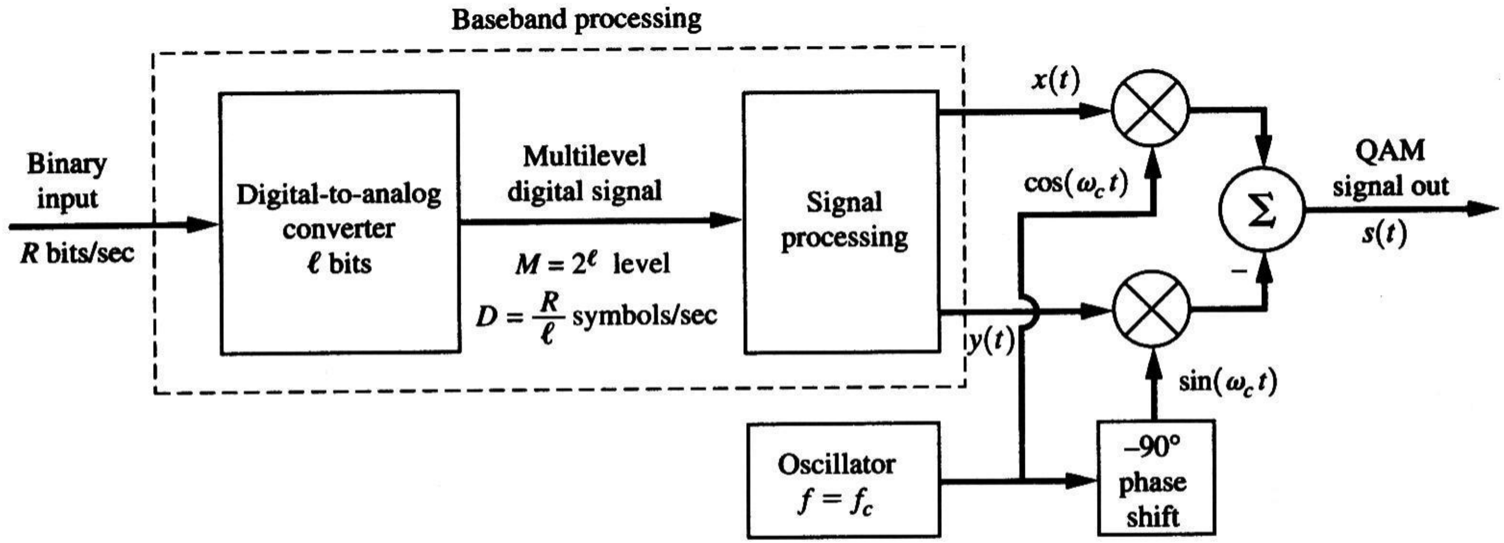
QAM Constellation



32 QAM Signal Constellation



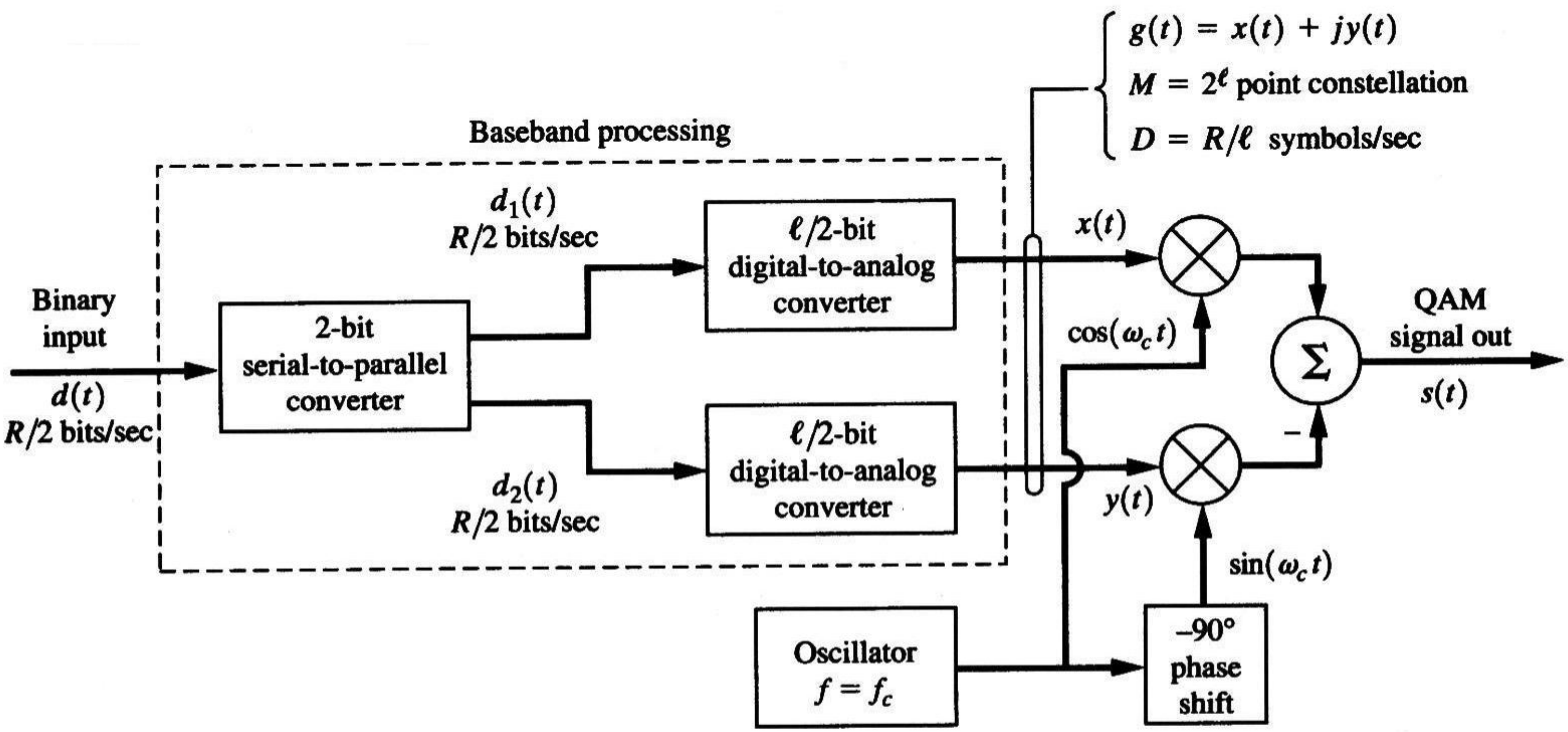
QAM Generation (1)



(a) Modulator for Generalized Signal Constellation

Figure 5–31 Generation of QAM signals.

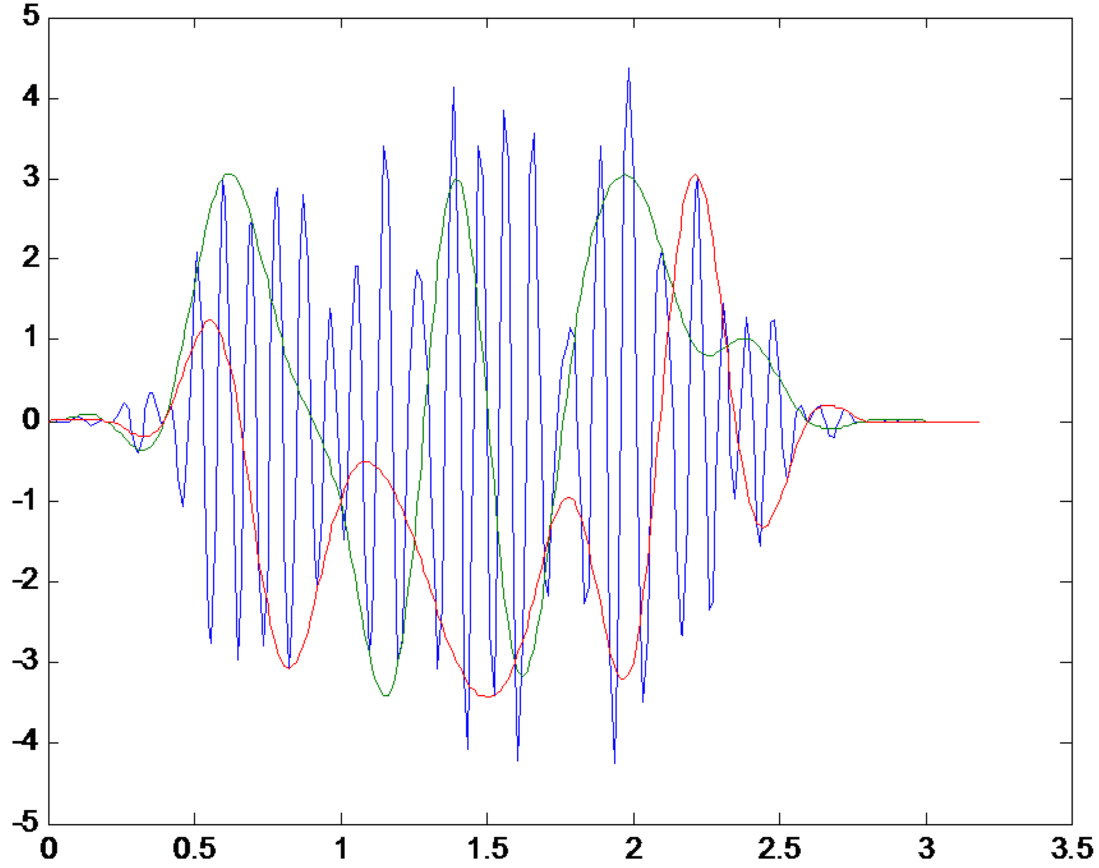
QAM Generation (2)



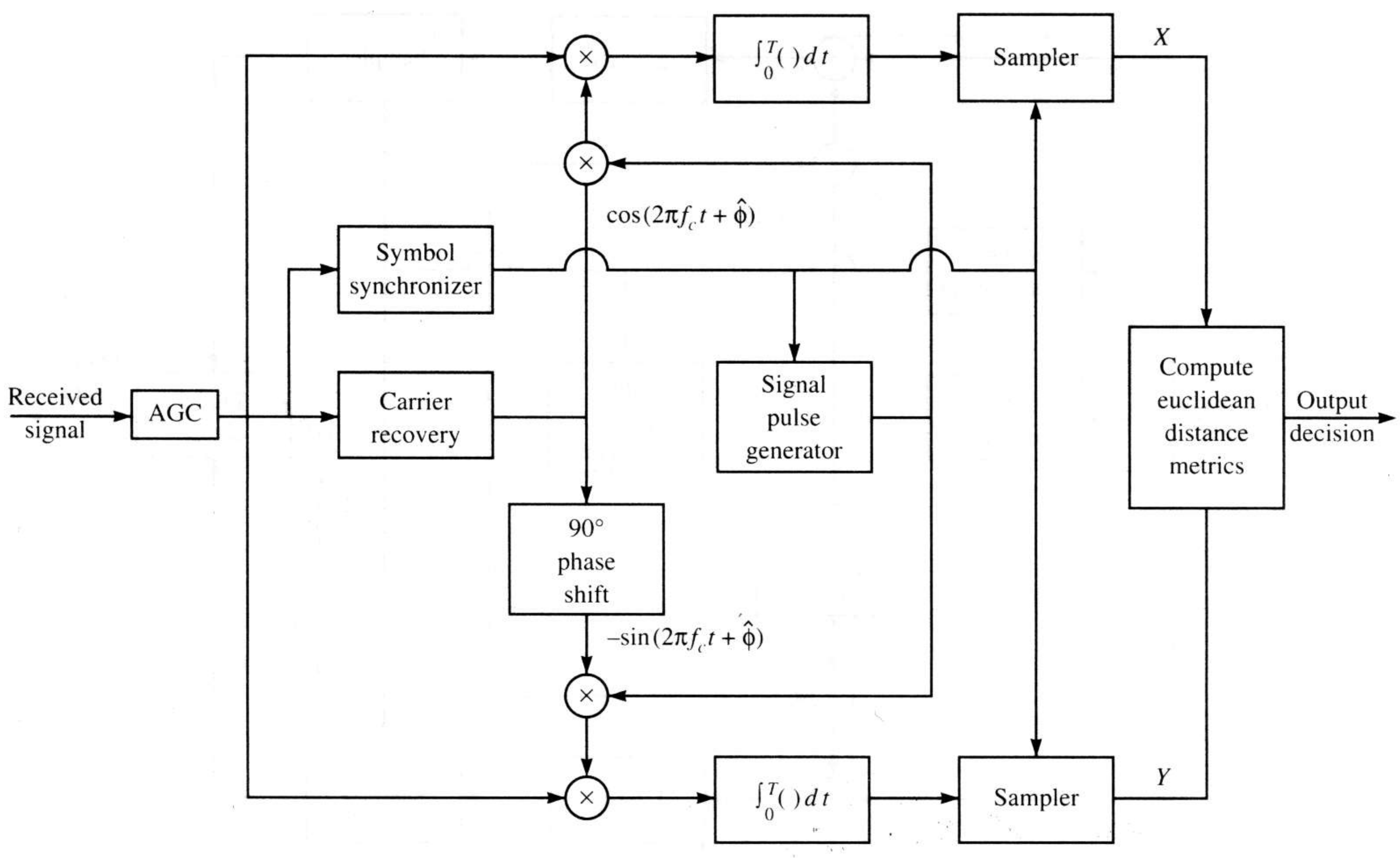
(b) Modulator for Rectangular Signal Constellation

Figure 5–31 Generation of QAM signals.

Waveform of QAM



QAM Demodulation



Offset QPSK

- In some applications, the timing between the x(t) and y(t) components is offset by

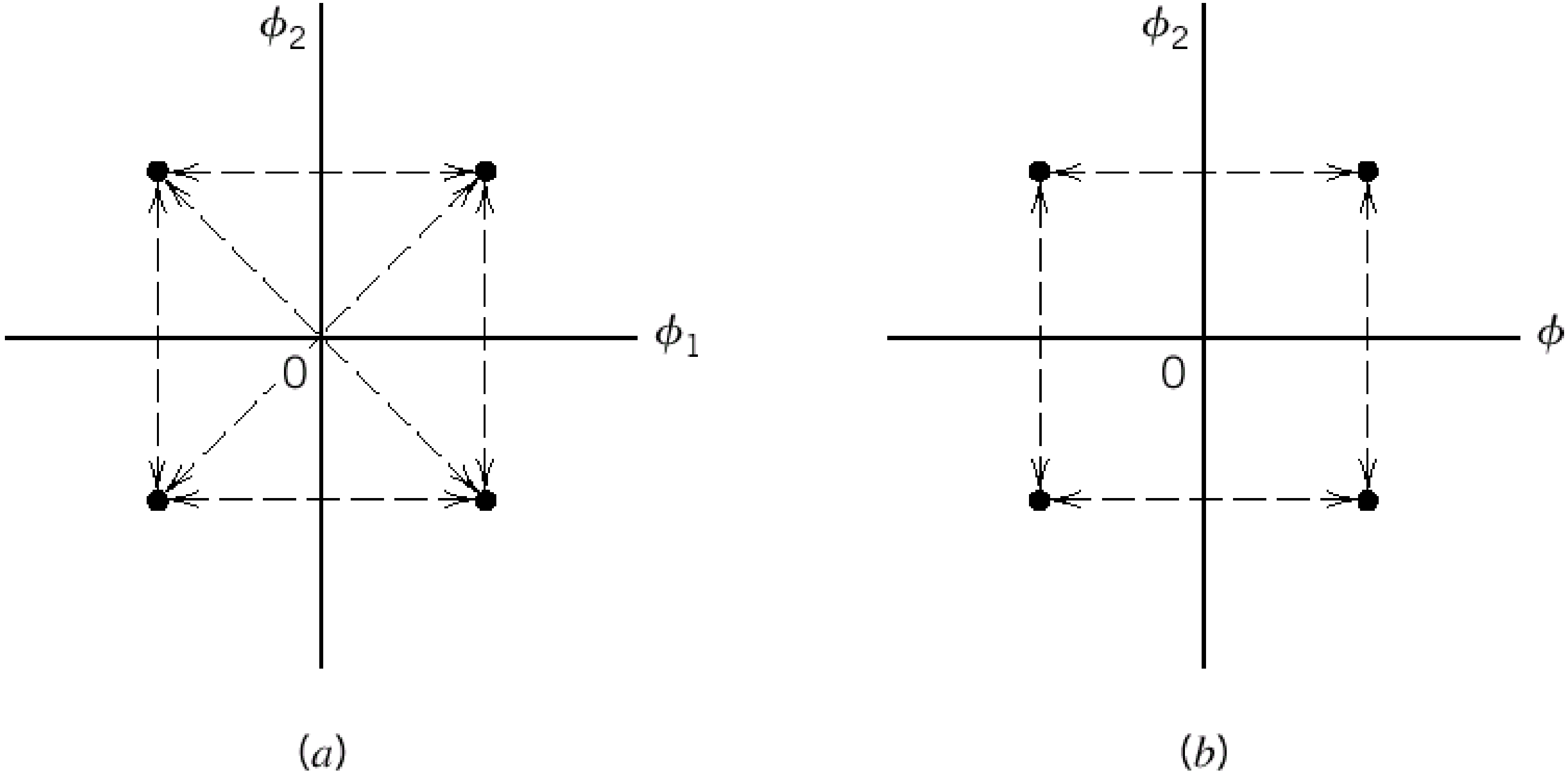
$$\frac{T_s}{2} = \frac{1}{2D}$$

$$x(t) = \sum_n x_n h_1(t - nT_s) = \sum_n x_n h_1(t - \frac{n}{D})$$

$$y(t) = \sum_n y_n h_1(t - nT_s - \frac{T_s}{2}) = \sum_n y_n h_1(t - \frac{n}{D} - \frac{1}{2D})$$

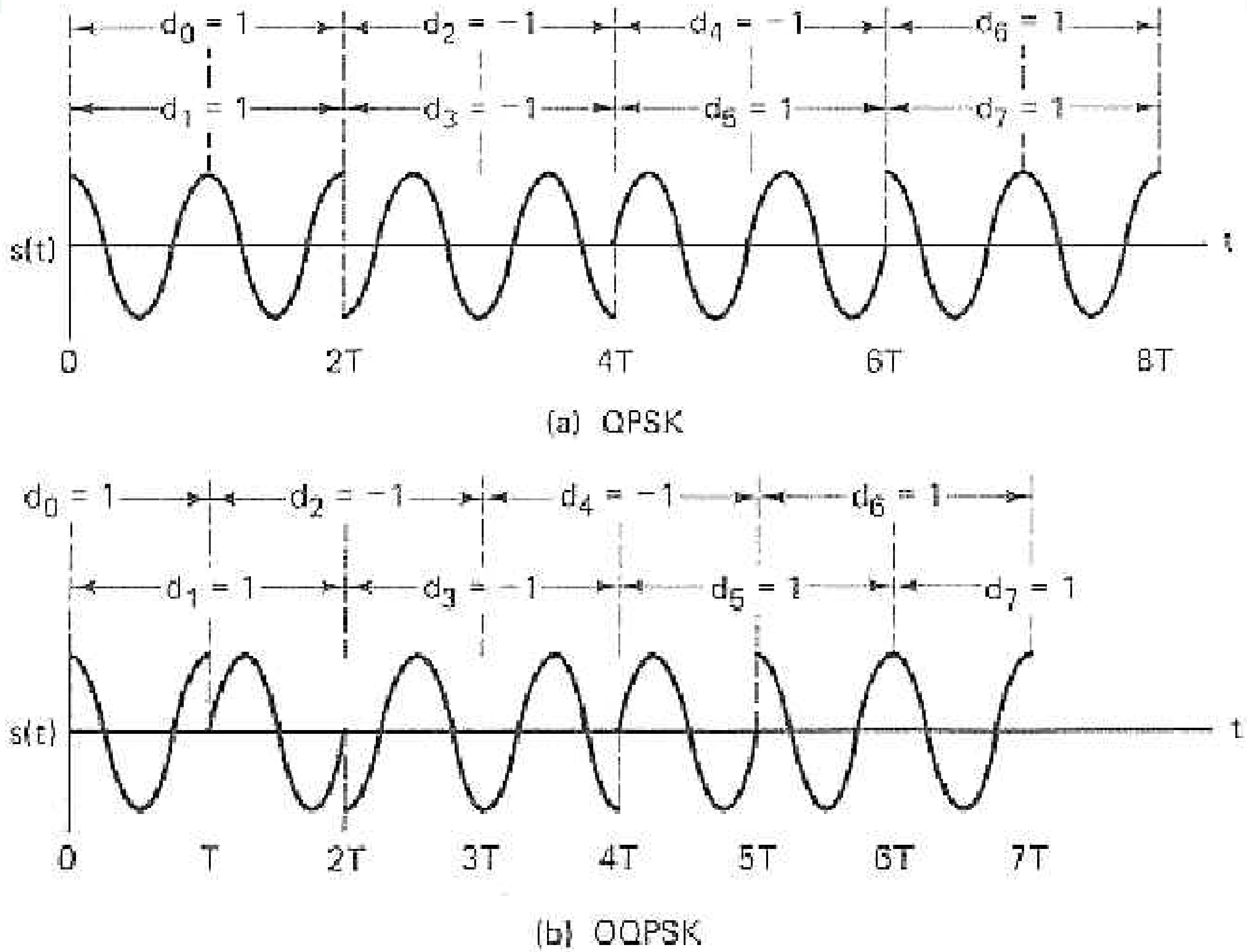
- One popular type of offset signaling is offset QPSK (OQPSK), which is identical to offset 4PSK or 4QAM.
- This offset greatly reduces the AM on the OQPSK signal compared with the AM on the corresponding QPSK signal.

Offset QPSK (Reducing Carrier Amplitude Change)

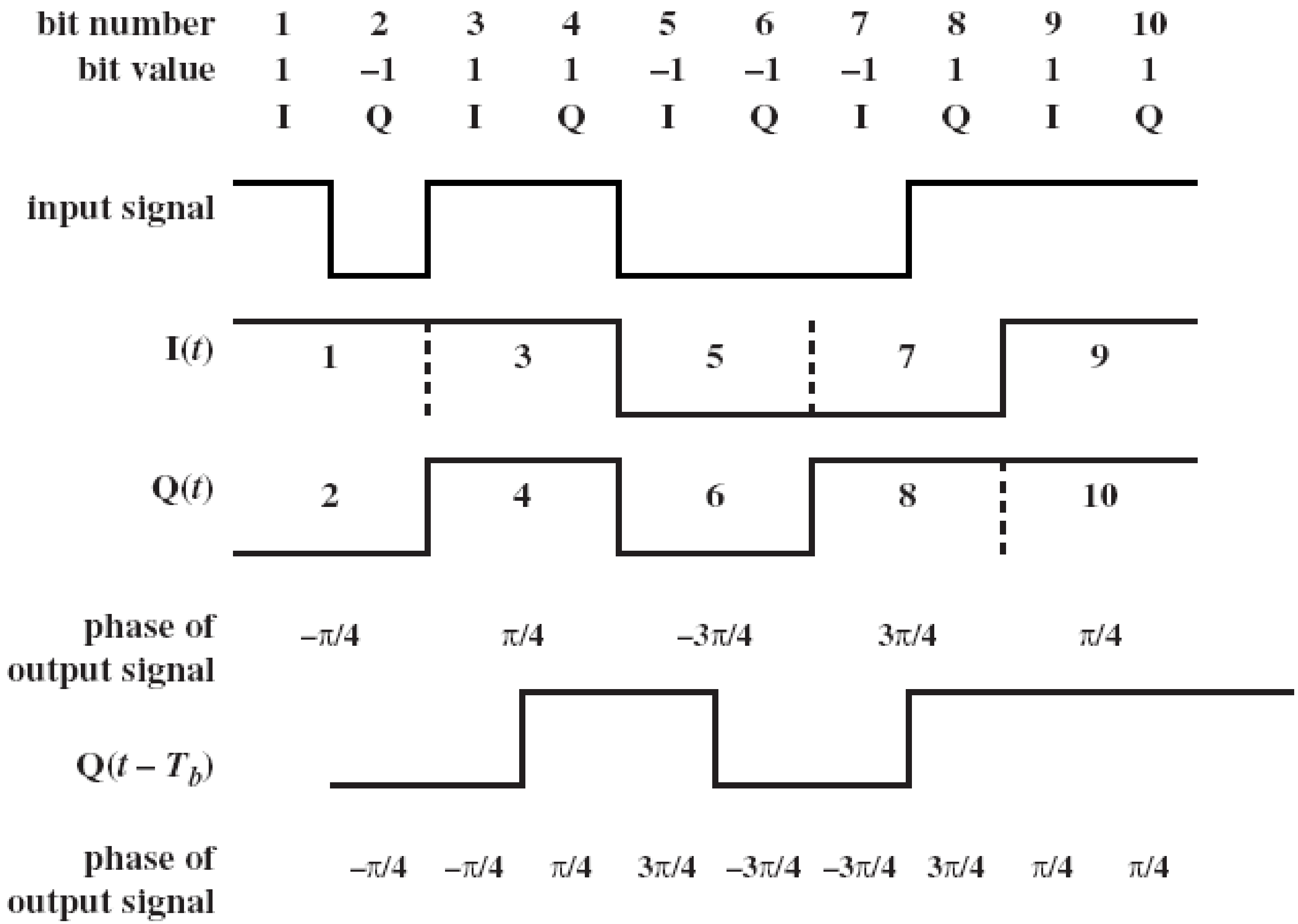


Possible paths for switching between the message points in (*a*) QPSK and (*b*) offset QPSK.

QPSK vs. OQPSK



OQPSK Waveform



PSD for MPSK and QAM

$$P_x(f) = \frac{|F(f)|^2}{T_s} \sum_{k=-\infty}^{\infty} R(k) e^{jk\omega T_s}$$

$$R(k) = \sum_{i=1}^{I} (a_n^* a_{n+k})_i P_i = \overline{a_n^* a_{n+k}}$$

Assume the data symbols are uncorrelated

$$R(k) = \overline{a_n^* a_{n+k}} = \begin{cases} \overline{a_n^* a_n}, & k = 0 \\ \overline{a_n^*}\,\overline{a_{n+k}}, & k \neq 0 \end{cases} = \begin{cases} \sigma_a^2 + |m_a|^2, & k = 0 \\ |m_a|^2, & k \neq 0 \end{cases}$$

$$\sigma_a^2 = \overline{|a_n - m_a|^2} = \overline{(a_n - m_a)(a_n - m_a)^*}$$

$$= \overline{(a_n - m_a)(a_n^* - m_a^*)} = \overline{a_n a_n^* - m_a a_n^* - a_n m_a^* + m_a m_a^*}$$

$$= \overline{a_n a_n^*} - \overline{m_a a_n^*} - \overline{a_n m_a^*} + \overline{m_a m_a^*}$$

$$= \overline{a_n a_n^*} - m_a \overline{a_n^*} - \overline{a_n} m_a^* + m_a m_a^* = \overline{a_n a_n^*} - m_a m_a^* - m_a m_a^* + m_a m_a^*$$

$$= \overline{a_n a_n^*} - m_a m_a^* = \overline{a_n a_n^*} - \left| m_a \right|^2$$

$$P_x(f) = \frac{\left| F(f) \right|^2}{T_s} [\sigma_a^2 + \left| m_a \right|^2 + \sum_{\substack{k=-\infty \\ k \neq 0}}^{\infty} \left| m_a \right|^2 e^{jk\omega T_s}]$$

$$= \frac{\left| F(f) \right|^2}{T_s} [\sigma_a^2 + \left| m_a \right|^2 \sum_{k=-\infty}^{\infty} e^{jk\omega T_s}]$$

$$\sum_{k=-\infty}^{\infty} e^{jk\omega T_s} = \sum_{k=-\infty}^{\infty} e^{j2\pi k T_s f} = \frac{1}{T_s} \sum_{k=-\infty}^{\infty} \delta(f - k\frac{1}{T_s})$$

$$\left(\sum_{k=-\infty}^{\infty} \delta(t - kT_0) = f_0 \sum_{k=-\infty}^{\infty} e^{jk\omega_0 t} = f_0 \sum_{k=-\infty}^{\infty} e^{j2\pi k f_0 t} \right)$$

$$P_x(f) = \frac{|F(f)|^2}{T_s}[\sigma_a^2 + |m_a|^2 \frac{1}{T_s} \sum_{k=-\infty}^{\infty} \delta(f - k\frac{1}{T_s})]$$

$$= |F(f)|^2 D[\sigma_a^2 + |m_a|^2 D \sum_{k=-\infty}^{\infty} \delta(f - kD)]$$

$$= \underbrace{\sigma_a^2 D |F(f)|^2}_{\textit{Continuous spectrum}} + \underbrace{|m_a|^2 D^2 \sum_{n=-\infty}^{\infty} |F(nD)|^2 \delta(f - nD)}_{\textit{Discrete spectrum}}$$

$$g(t) = \sum_{n=-\infty}^{\infty} c_n f(t - nT_s)$$

$$f(t) = \Pi\left(\frac{t}{T_s}\right) \leftrightarrow F(f) = T_s Sa(\pi f T_s) = lT_b Sa(\pi f l T_b)$$

$$P_g(f) = \underbrace{\sigma_c^2 D |F(f)|^2}_{Continuous\ spectrum} + \underbrace{|m_c|^2 D^2 \sum_{n=-\infty}^{\infty} |F(nD)|^2 \delta(f - nD)}_{Discrete\ spectrum}$$

For symmetrical (polar type) signaling with equally likely multilevels

$$m_c = \overline{c_n} = 0$$

$$\sigma_c^2 = \overline{c_n c_n^*} - |m_c|^2 = \overline{c_n c_n^*} = \overline{|c_n|^2} = C$$

$$P_g(f) = CD[lT_b Sa(\pi f l T_b)]^2 = ClT_b Sa^2(\pi f l T_b) = KSa^2(\pi f l T_b)$$

$$B_T = \frac{2R}{l}$$

$$\eta = \frac{R}{B_T} = \frac{l}{2}\left(\frac{bits\ /s}{Hz}\right)$$

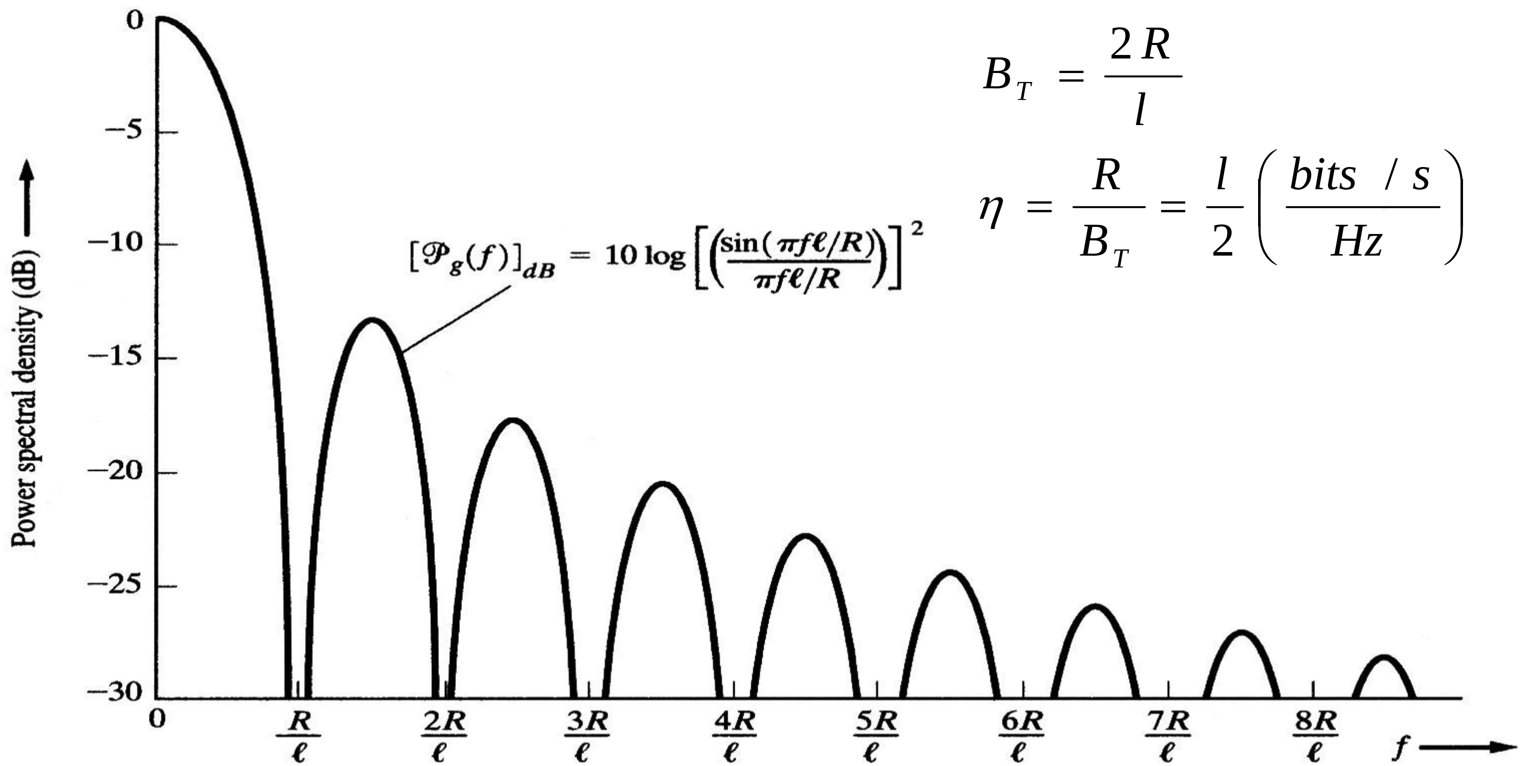


Figure 5–33 PSD for the complex envelope of MPSK and QAM with rectangular data pulses, where $M = 2^\ell$, R is the bit rate, and $R/\ell = D$ is the baud rate (positive frequencies shown). Use $\ell = 2$ for PSD of QPSK, OQPSK, and $\pi/4$ QPSK complex envelope.

M-ary FSK (?)

Homework

- LC 5-59, 5-62, 5-63, 5-66