Baseband Digital System

- Analog to Digital Conversion
  - Sampling
  - Quantizing
  - Encoding
- Digital Signal Representation
  - Orthogonal condition
  - Bandwidth
- Line Codes and Spectrum
- ISI and Eye Pattern
- Matched Filter
- Performance

Bandpass Digital System (1)

LC 5-9

Lecture 19, 2008-11-25

Contents

- Review of Digital Baseband System
- OOK
- BPSK
- DPSK
- FSK

Binary Modulated Bandpass Signals

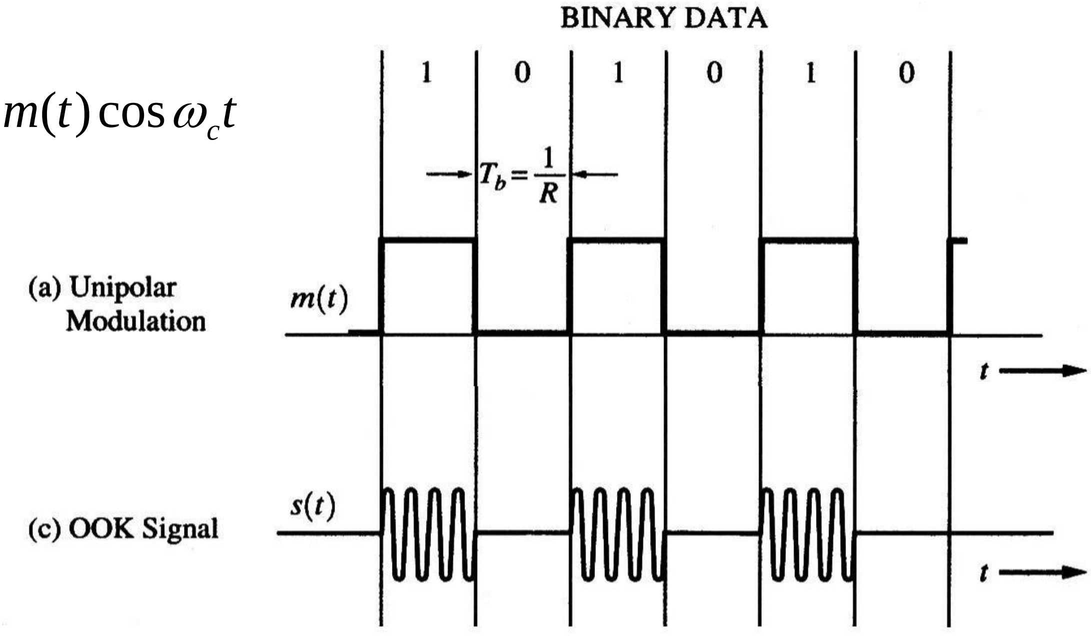
- Digitally modulated bandpass signals are generated by using the complex envelops for AM, PM, FM or QM (quadrature modulation) signaling.
- For digital modulated signals, the modulating signal $m(t)$ is a binary or multilevel digital signal.
- The most common binary signaling techniques are as follows:
  - On-off keying (OOK), also called amplitude shift keying (ASK).
  - Binary phase-shift keying (BPSK).
  - Frequency-shift keying (FSK)

On-Off Keying

- Also called amplitude shift keying (ASK), which consists of keying (switching) a carrier sinusoid on and off with a unipolar binary signal.
- OOK is identical to unipolar binary modulation on a DSB-SC signal.

$$s(t) = \mathrm{Re}\{g(t)e^{j\omega_c t}\} = A_c m(t)\cos\omega_c t$$

$$g(t) = A_c m(t)$$



OOK Spectrum

$$m(t) = \begin{cases} A \\ 0 \end{cases}, \qquad g(t) = A_c m(t) = \begin{cases} A_c A \\ 0 \end{cases}$$

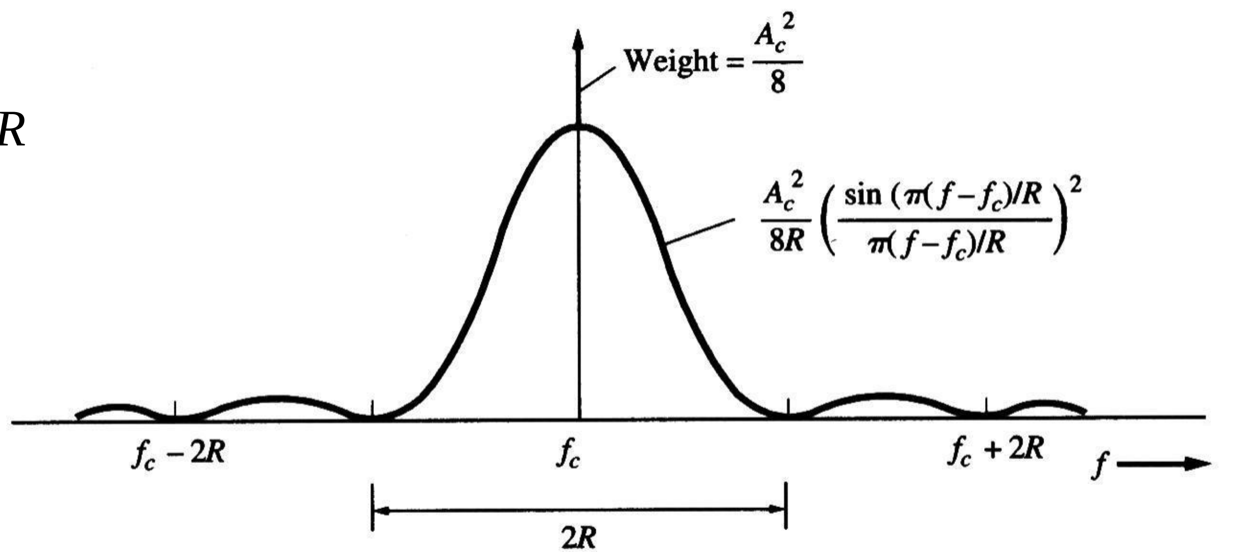
$$P_g(f) = \frac{(A_c A)^2}{4}[\delta(f) + T_b Sa^2(\pi f T_b)] = \frac{A_c^{\ 2}}{2}[\delta(f) + T_b Sa^2(\pi f T_b)]$$

$$P_s(f) = \frac{1}{4}[P_g(f - f_c) + P_g(f + f_c)]$$

$$= \frac{A_c^{\ 2}}{8}[\delta(f - f_c) + \delta(f + f_c)] + \frac{A_c^{\ 2} T_b}{8}\left\{Sa^2[\pi(f - f_c)T_b] + Sa^2[\pi(f + f_c)T_b]\right\}$$

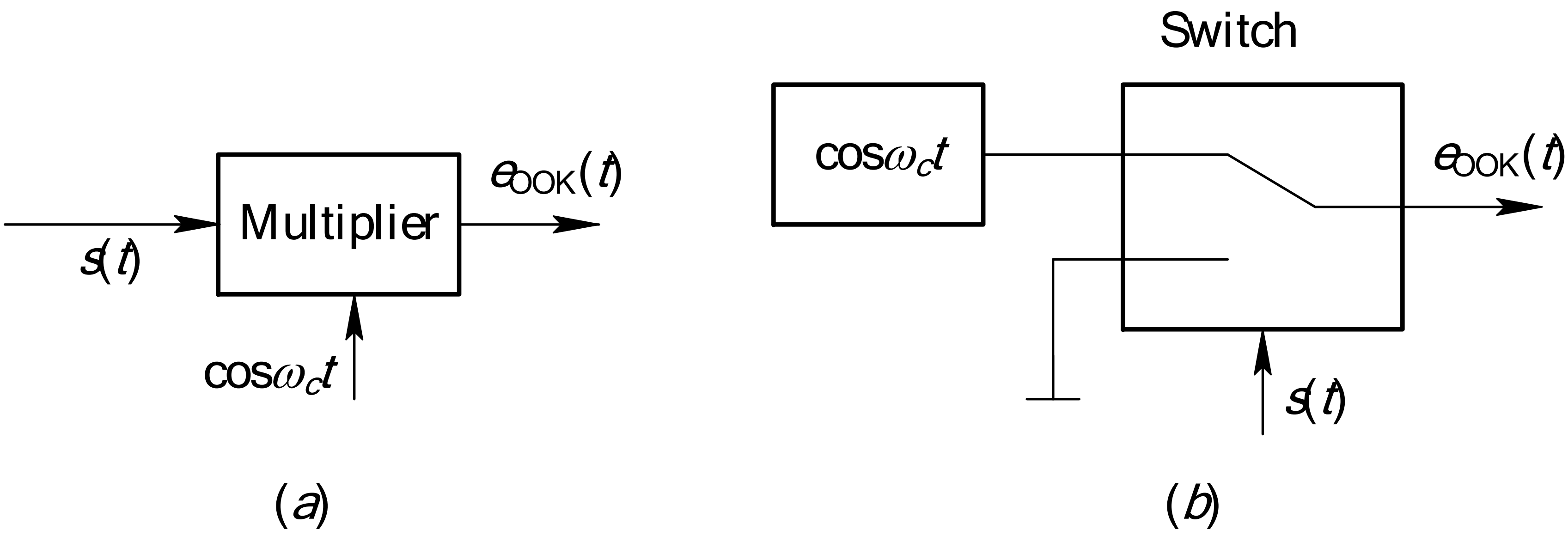
$$B_{T(abs)} = \infty$$

$$B_{T(null)} = 2R$$



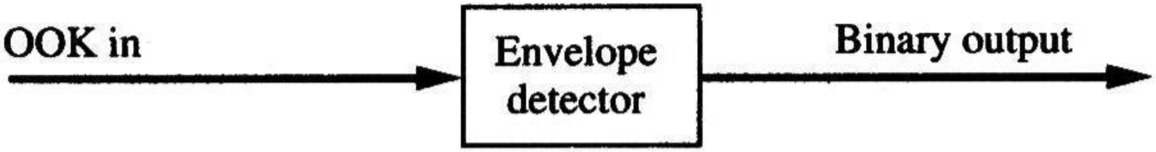
(a) OOK

OOK Generation

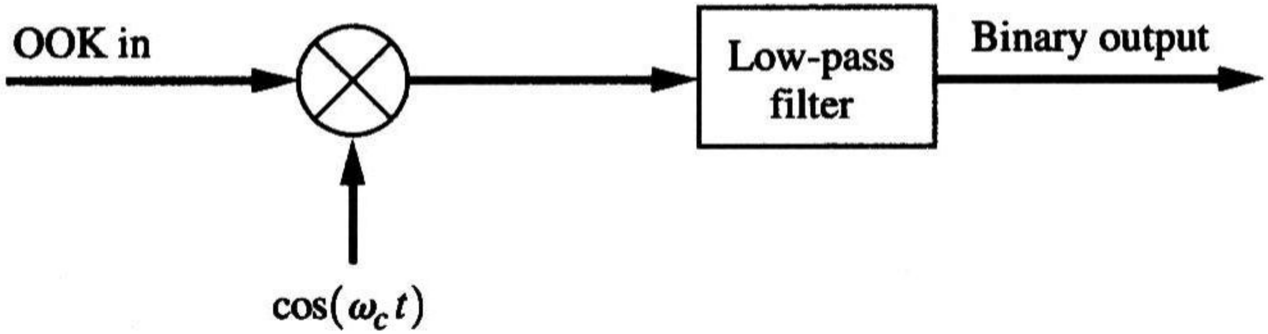


OOK Demodulation

- OOK may be detected by using either an envelope detector (noncoherent detection) or a product detector (coherent detection).



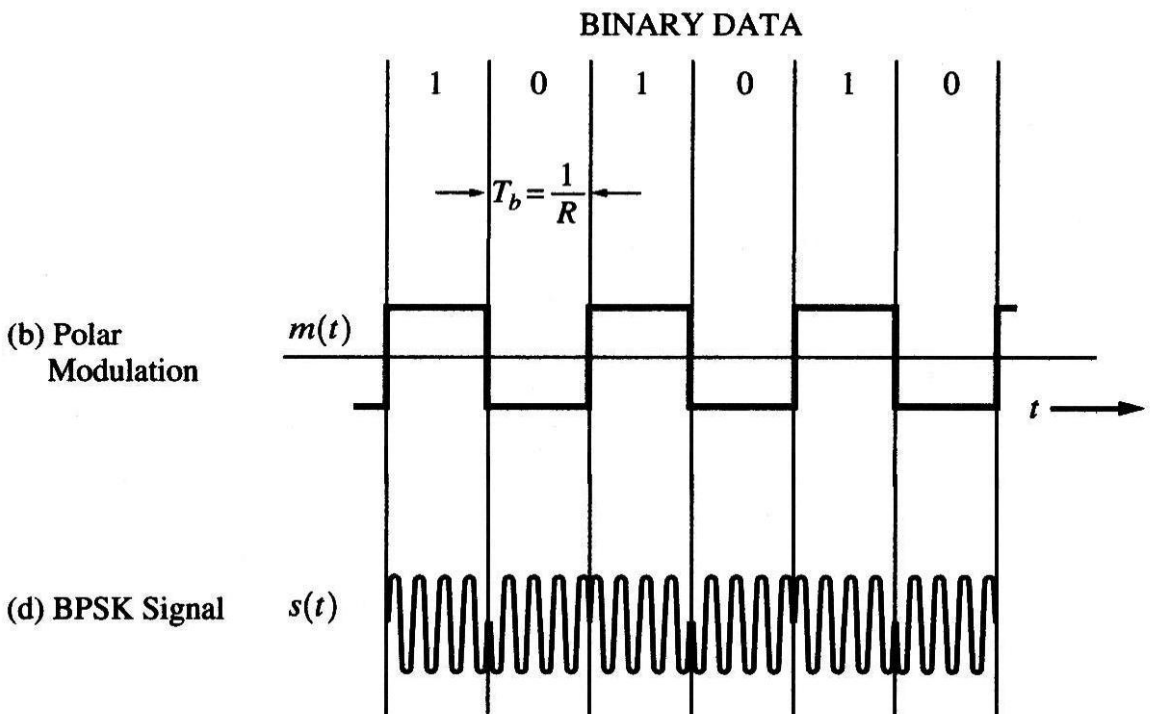
(a) Noncoherent Detection



(b) Coherent Detection with Low-Pass Filter Processing

Binary Phase-Shift Keying (BPSK)

- BPSK consists of shifting the phase of a carrier 0° or 180° with a unipolar binary signal. BPSK is equivalent to PM or DSB-SC with a polar binary modulation.



BPSK

$$m(t) = \pm 1 \quad \text{polar baseband data signal}$$

$$s(t) = \mathrm{Re}\{g(t)e^{j\omega_c t}\} = A_c \cos[\omega_c t + D_p m(t)]$$

$$= A_c \cos[D_p m(t)] \cos\omega_c t - A_c \sin[D_p m(t)] \sin\omega_c t$$

$$= \underbrace{A_c \cos(D_p)\cos\omega_c t}_{\textit{pilot carrier}} - \underbrace{A_c \sin(D_p) m(t) \sin\omega_c t}_{\textit{data}}$$

- The level of the pilot carrier term is set by the value of the peak deviation, $\Delta\theta = D_p$
- The digital modulation index h is defined by $h = \dfrac{2\Delta\theta}{\pi}$

The maximum signaling efficiency is obtained by letting

$$\Delta\theta = D_p = 90^\circ = \pi/2$$

$$s(t) = -A_c m(t) \sin\omega_c t$$

$$g(t) = jA_c m(t) = \pm jA_c$$

Spectrum of BPSK

$$P_g(f) = \frac{|F(f)|^2}{T_s} \sum_{n=-\infty}^{\infty} R(k) e^{j2\pi k f T_s}$$

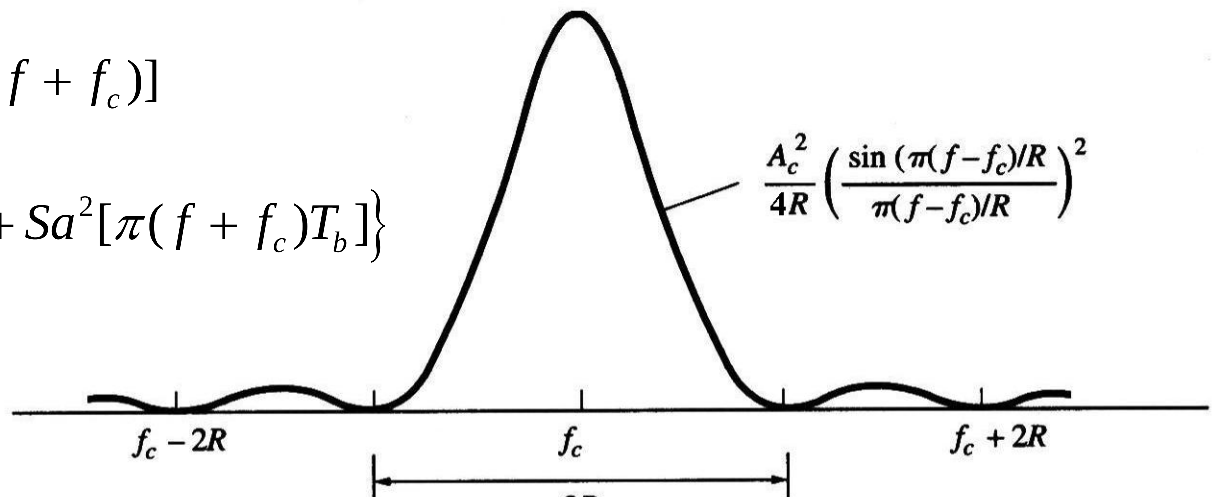
$$R(k) = \sum_{i=1}^{I} \left(g_n^* g_{n+k} \right)_i P_i = \cdots = \begin{cases} A_c^2, & k = 0 \\ 0, & k \neq 0 \end{cases}$$

$$P_g(f) = A_c^2 T_b Sa(\pi f T_b)$$

$$P_s(f) = \frac{1}{4}[P_g(f - f_c) + P_g(f + f_c)]$$

$$= \frac{A_c^2}{4} T_b \left\{ Sa^2[\pi(f - f_c)T_b] + Sa^2[\pi(f + f_c)T_b] \right\}$$

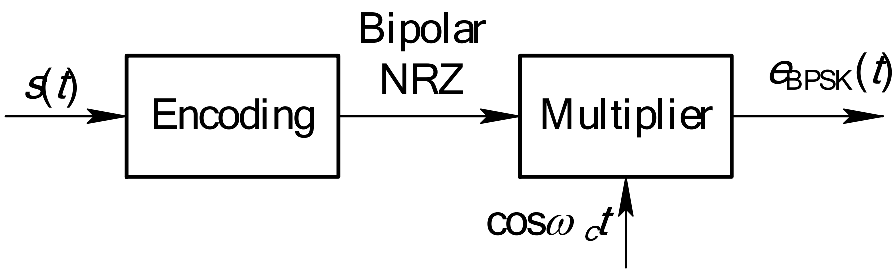
$$B_{T(null)} = 2R$$



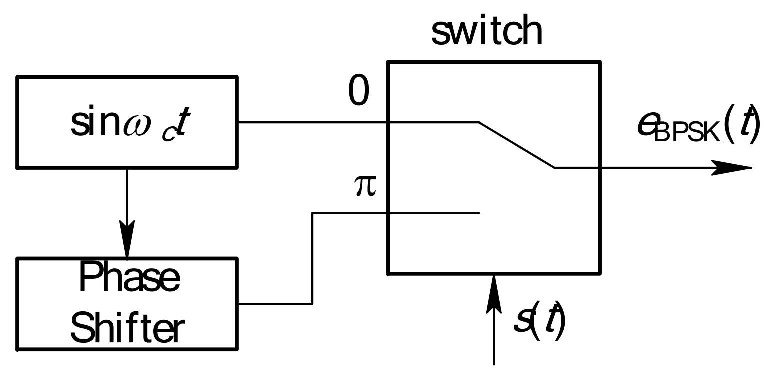
(b) BPSK (See Fig 5–15 for a more detailed spectral plot)

Figure 5–20 PSD of bandpass digital signals (positive frequencies shown).

BPSK Generation

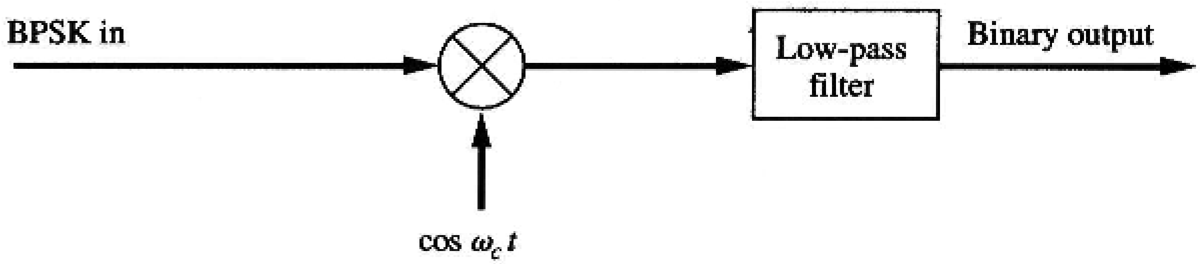


(a)



(b)

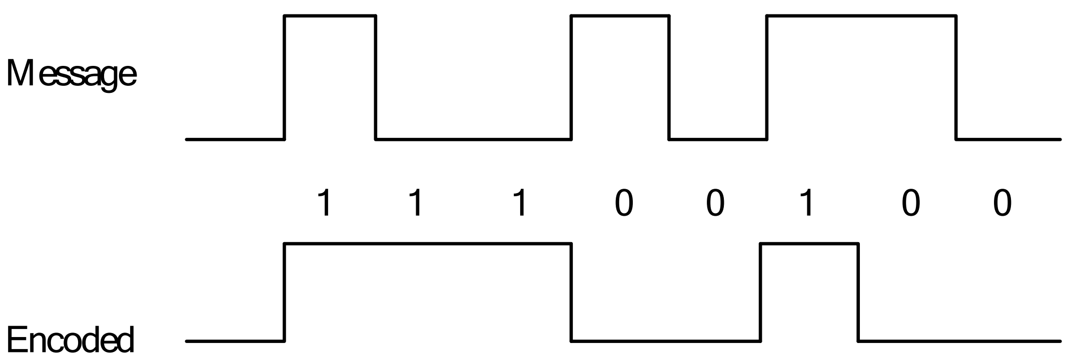
BPSK Demodulation



(a) Detection of BPSK (Coherent Detection)

Differential Phase-Shift Keying (DPSK)

- PSK signals cannot be detected incoherently.
- In differential encoding, the information is conveyed by phase shifts between any two successive signal intervals.



DPSK Generation

- Differential encoding and decoding:

Input message		1	0	1	1	0	1	0	0
Encoded message	1	1 →	0	0	0 →	1	1 →	0 →	1
Transmitted phase	π	π	0	0	0	π	π	0	π
Phase-comparison sign		+	−	+	+	−	+	−	−
Regenerated message		1	0	1	1	0	1	0	0

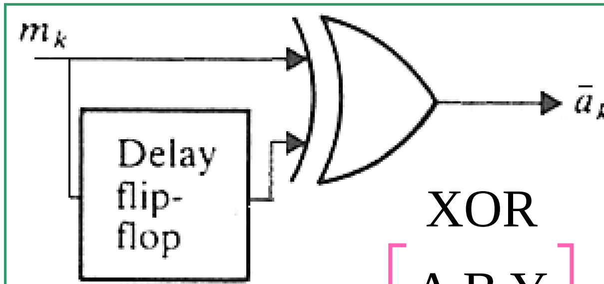
start, say with $a_k = 1$

$$\begin{cases} \text{if } m_k = 1, \text{set } a_k = a_{k-1} \\ \text{if } m_k = 0, \text{set } a_k \neq a_{k-1} \end{cases}$$

- Decoding is obtained by the simple rule:

$$d_k = a_{k-1} \oplus a_k$$

that is realized by the circuit shown right.



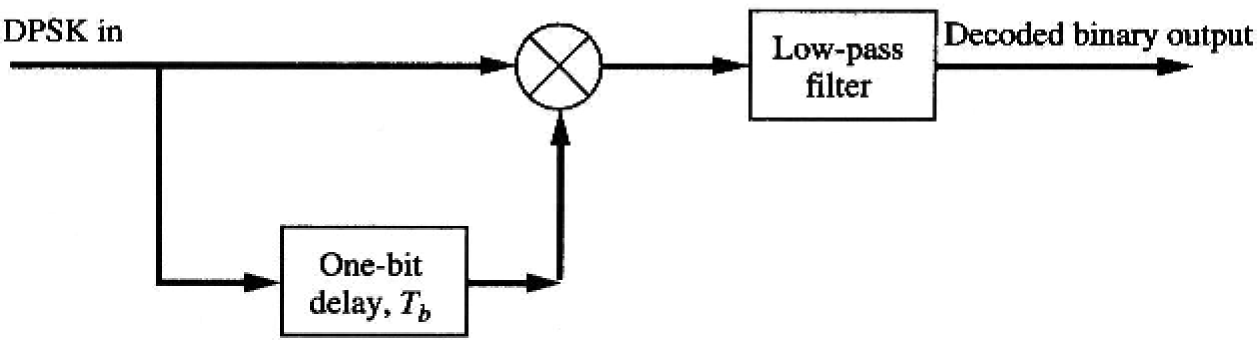
XOR

A	B	Y
0	0	1
0	1	0
1	0	0
1	1	1

- Note that no local oscillator is required
- How would you construct the encoder?

DPSK Demodulation

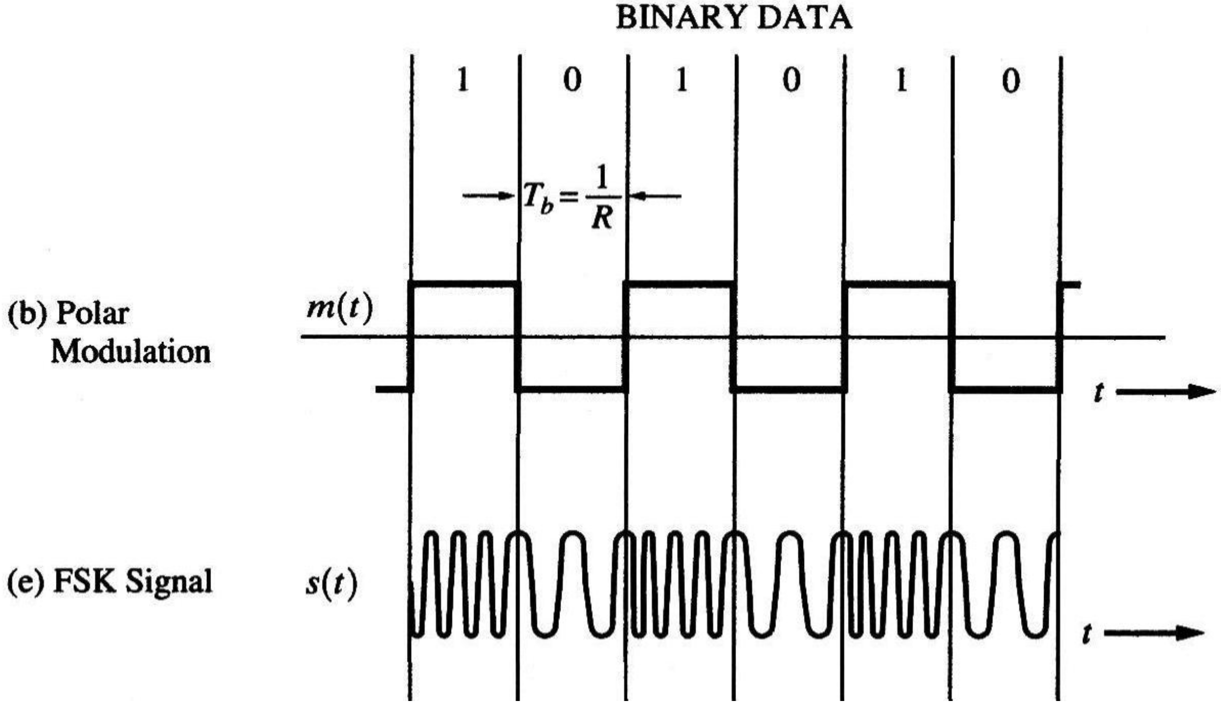
- In practice, DPSK is often used instead of BPSK, because the DPSK receiver does not require a carrier synchronizer circuit.



(b) Detection of DPSK (Partially Coherent Detection)

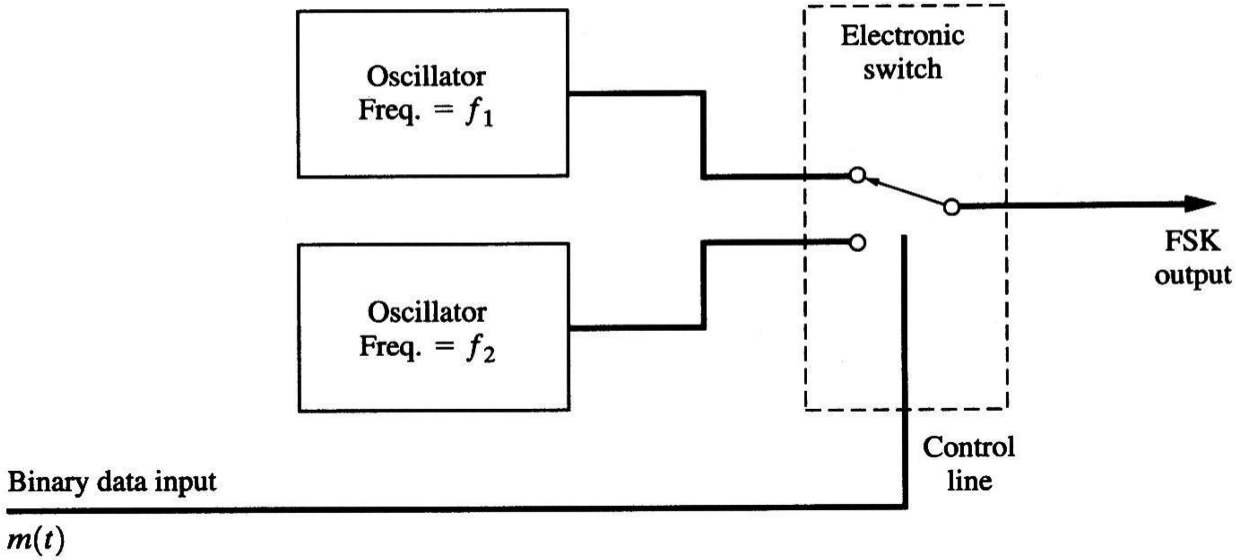
Frequency-Shift Keying (FSK)

- FSK consists of shifting the frequency of a carrier from a mark frequency to a space frequency according to the baseband binary signal. FSK is identical to FM with a binary modulation.



FSK Generation (1)

- The FSK signal can be characterized as one of two different types, depending on the method used to generate it. One type is called discontinuous-phase FSK. It is generated by switching the output line between two different oscillators.



(a) Discontinuous-Phase FSK

Discontinuous-Phase FSK

$$s(t) = A_c \cos[\omega_c t + \theta(t)] = \begin{cases} A_c \cos(\omega_1 t + \theta_1), & \text{when 1 is being sent} \\ A_c \cos(\omega_2 t + \theta_2), & \text{when 0 is being sent} \end{cases}$$

f_1 : mark (bianry 1) frequency

f_2 : space (binary 0) frequency

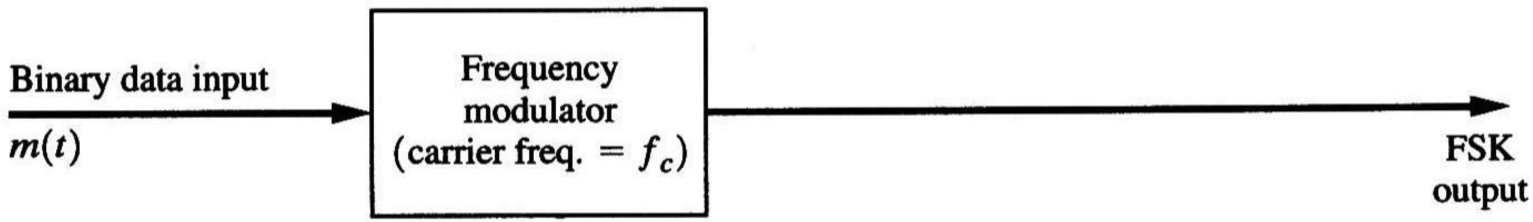
θ_1 : start-up phase of oscillator one

θ_2 : start-up phase of oscillator two

$$\omega_c t + \theta(t) = \begin{cases} \omega_1 t + \theta_1 \\ \omega_2 t + \theta_2 \end{cases} \Rightarrow \theta(t) = \begin{cases} \omega_1 t + \theta_1 - \omega_c t \\ \omega_2 t + \theta_2 - \omega_c t \end{cases}$$

FSK Generation

- Another type is called continuous-phase FSK. It is generated by feeding the data signal into a frequency modulator.



(b) Continuous-Phase FSK

Figure 5–23 Generation of FSK.

$$s(t) = A_c \cos[\omega_c t + D_f \int_{-\infty}^{t} m(\lambda) d\lambda]$$

Example

- Spectrum of the Bell-Type 103 FSK Modem

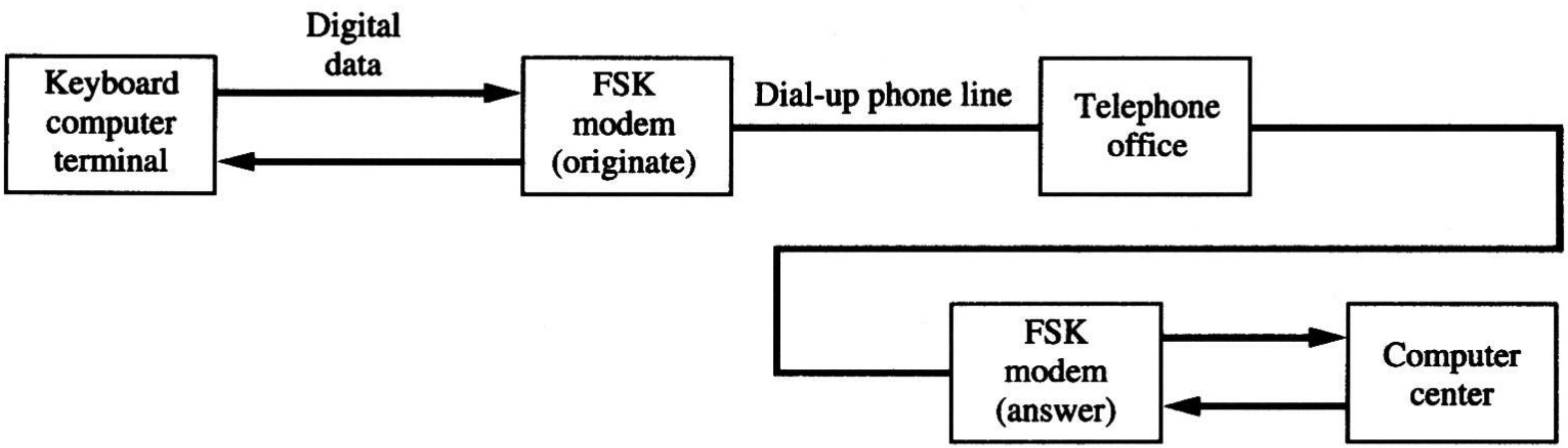


Figure 5–24 Computer communication using FSK signaling.

Table 5–5 MARK AND SPACE FREQUENCIES FOR THE BELL-TYPE 103 MODEM

	Originate Modem (Hz)	Answer Modem (Hz)
Transmit frequencies		
Mark (binary 1)	$f_1 = 1{,}270$	$f_1 = 2{,}225$
Space (binary 0)	$f_2 = 1{,}070$	$f_2 = 2{,}025$
Receive frequencies		
Mark (binary 1)	$f_1 = 2{,}225$	$f_1 = 1{,}270$
Space (binary 0)	$f_2 = 2{,}025$	$f_2 = 1{,}070$

Bit rate : $R = 300$ bits/s

Bit interval : $T_b = 1/R$

Peak-to-peak deviation : $2\Delta F = 200$ Hz

The spectrum will be evaluated for the case of the widest-bandwidth FSK signal, corresponding to an alternating data pattern (i.e., 10101010).

$$\text{period of modulating signal}: T_0 = 2T_b$$

$$m(t) = \pm 1 \text{ or } m(t) = \sum_{n=-\infty}^{\infty} 2\Pi\left(\frac{t - nT_0}{T_b}\right) - 1$$

$$\text{Peak frequency deviation}: \Delta F = \frac{1}{2\pi} D_f \max[m(t)] = \frac{D_f}{2\pi}$$

$$\theta(t) = D_f \int_{-\infty}^{t} m(\lambda) d\lambda$$

$$\text{Assume } \theta(0) = 0$$

$$\text{For } -T_b/2 \le t \le T_b/2, \ \theta(t) = D_f \int_0^t (+1) d\lambda = D_f t$$

$$\text{For } T_b/2 \le t \le 3T_b/2, \ \theta(t) = D_f \frac{T_b}{2} + D_f \int_{T_b/2}^{t} (-1) d\lambda$$

$$= -D_f t + D_f T_b$$

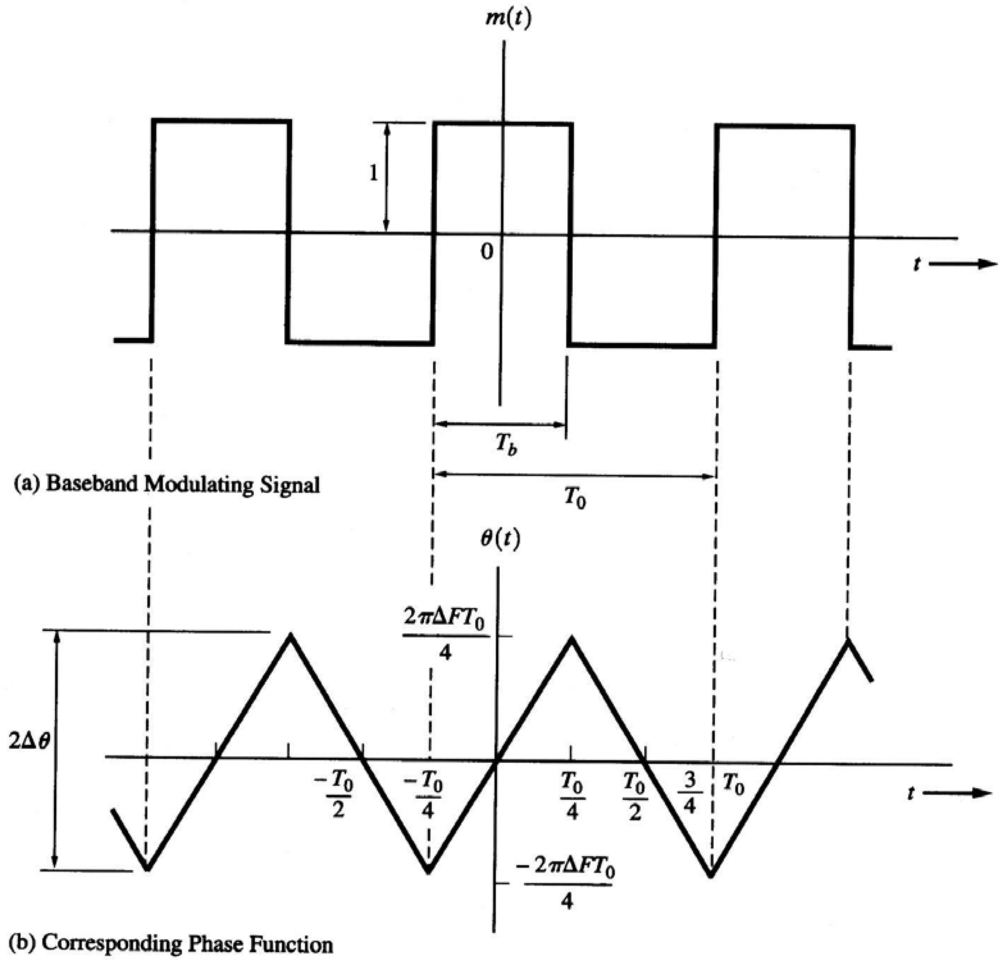


Figure 5–25 Input data signal and FSK signal phase function.

$$\Delta\theta = \max[\theta(t)] = D_f \frac{T_b}{2}$$

$$\text{Digital modulation index}: h = \frac{2\Delta\theta}{\pi} = \frac{D_f T_b}{\pi} = \frac{D_f T_0}{2\pi} = \Delta F T_0$$

$$= 2\Delta F T_b = \frac{2\Delta F}{R} = \frac{200}{300} \approx 0.67$$

$$D_f = \frac{2\pi h}{T_0}$$

$$\text{For } -T_0/4 \le t \le T_0/4,\ \theta(t) = D_f t = \frac{2\pi h}{T_0} t = \omega_0 h t$$

$$\text{For } T_0/4 \le t \le 3T_0/4,\ \theta(t) = -D_f t + D_f T_b$$

$$= -\frac{2\pi h}{T_0} t + \frac{2\pi h}{T_0}\frac{T_0}{2} = -\omega_0 h t + \pi h$$

$$g(t) = A_c e^{j\theta(t)} = \sum_{-\infty}^{\infty} c_n e^{jn\omega_0 t}$$

$$c_n = \frac{1}{T_0}\int_{-T_0/4}^{3T_0/4} g(t)dt = \frac{1}{T_0}\int_{-T_0/4}^{3T_0/4} A_c e^{j\theta(t)} e^{-jn\omega_0 t} dt$$

$$= \frac{A_c}{T_0}[\int_{-T_0/4}^{T_0/4} e^{j\omega_0 ht} e^{-jn\omega_0 t} dt + \int_{T_0/4}^{3T_0/4} e^{j(-\omega_0 ht+\pi h)} e^{-jn\omega_0 t} dt]$$

$$= \frac{A_c}{T_0}[\int_{-T_0/4}^{T_0/4} e^{j\omega_0 (h-n)t} dt + e^{j\pi h}\int_{T_0/4}^{3T_0/4} e^{-j\omega_0 (h+n)t} dt]$$

$$= \frac{A_c}{T_0}[\frac{e^{j\omega_0 (h-n)t}}{j\omega_0 (h-n)}\Bigg|_{-T_0/4}^{T_0/4} + e^{j\pi h}\frac{e^{-j\omega_0 (h+n)t}}{-j\omega_0 (h+n)}\Bigg|_{T_0/4}^{3T_0/4}]$$

$$= \frac{A_c}{T_0}[\frac{e^{j(\pi/2)(h-n)} - e^{-j(\pi/2)(h-n)}}{j\omega_0 (h-n)} + e^{j\pi h}\frac{e^{-j(3\pi/2)(h+n)} - e^{-j(\pi/2)(h+n)}}{-j\omega_0 (h+n)}]$$

$$= A_c[\frac{e^{j(\pi/2)(h-n)} - e^{-j(\pi/2)(h-n)}}{j2\pi(h-n)} + e^{-j\pi n}\frac{e^{-j(\pi/2)(h+n)} - e^{j(\pi/2)(h+n)}}{-j2\pi(h+n)}]$$

$$= \frac{A_c}{2}[\frac{e^{j(\pi/2)(h-n)} - e^{-j(\pi/2)(h-n)}}{j2(\pi/2)(h-n)} + e^{-j\pi n}\frac{e^{j(\pi/2)(h+n)} + e^{-j(\pi/2)(h+n)}}{j2(\pi/2)(h+n)}]$$

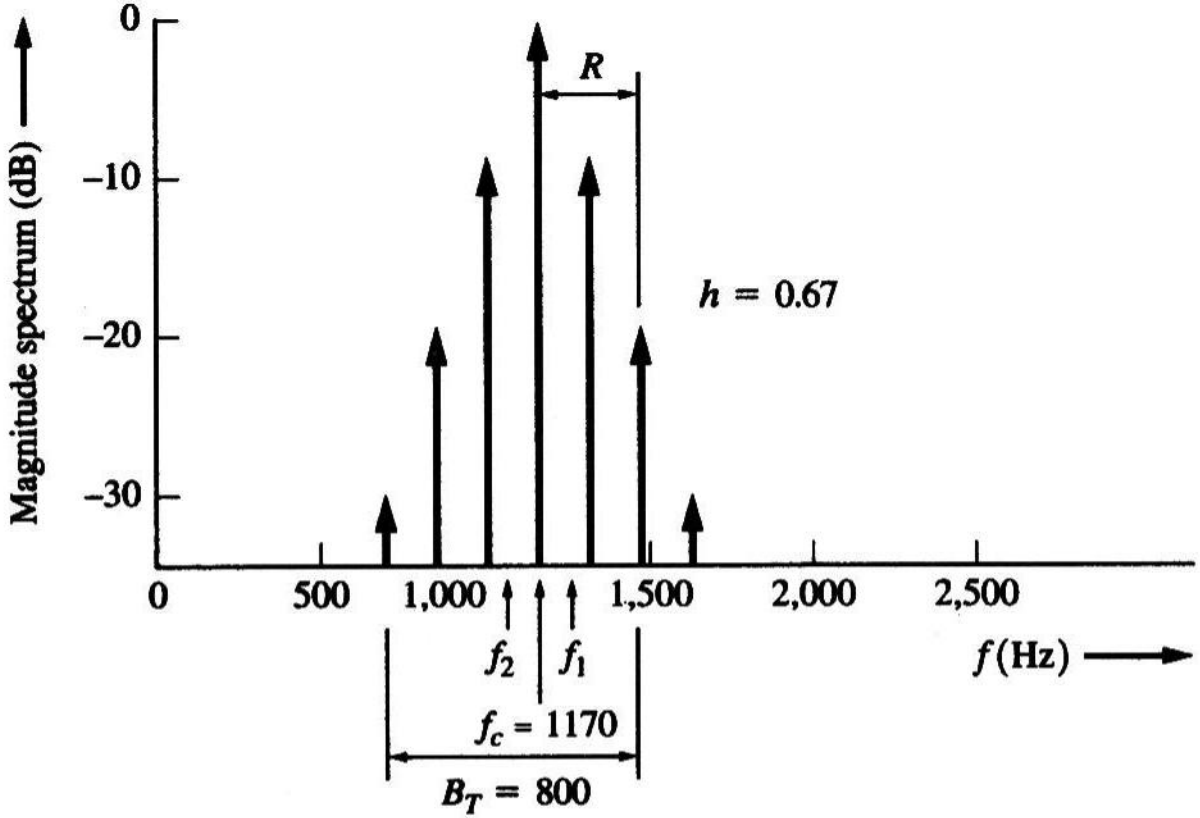
$$= \frac{A_c}{2}\{Sa[(\pi/2)(h-n)] + (-1)^n Sa[(\pi/2)(h+n)]\}$$

$$G(f) = \sum_{n=-\infty}^{\infty} c_n \delta(f - nf_0) = \sum_{n=-\infty}^{\infty} c_n \delta(f - n\frac{R}{2})$$

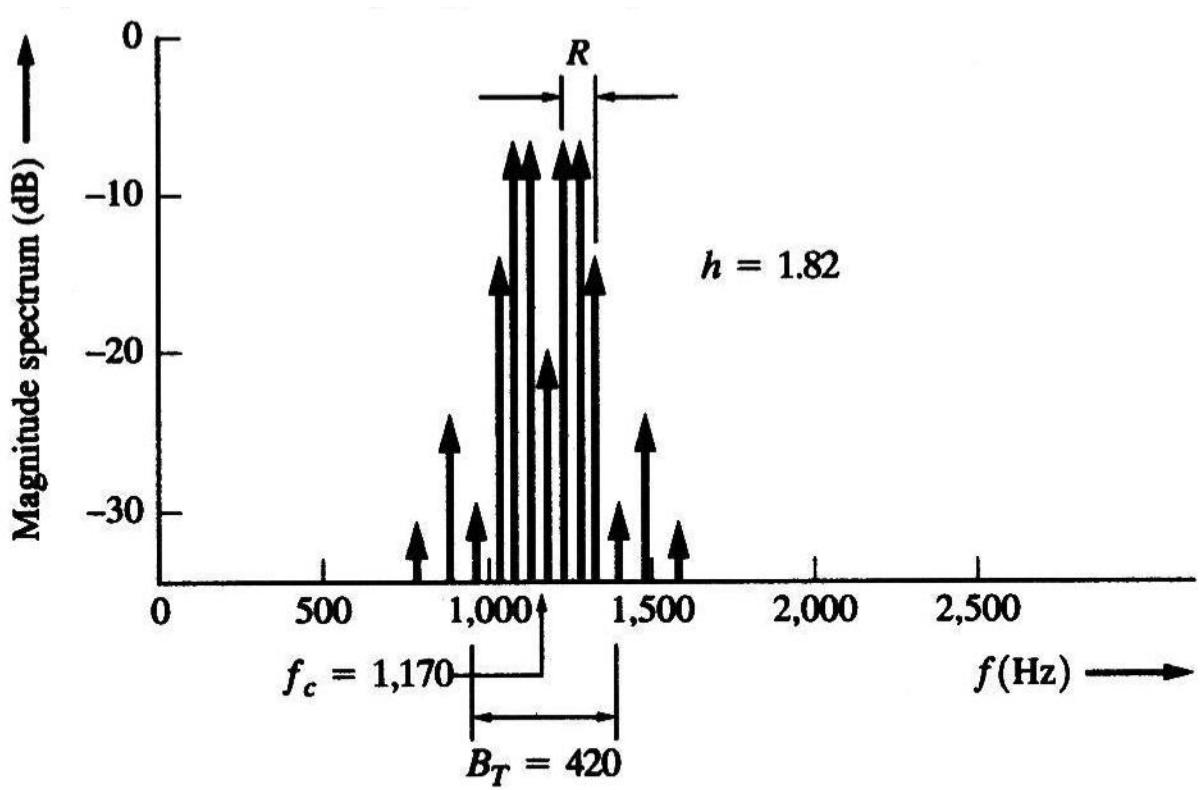
$$S(f) = \tfrac{1}{2}[G(f - f_c) + G^*(-f - f_c)]$$

$$B_T = 2(\beta_f + 1)B = 2(\frac{\Delta F}{B} + 1)B = 2\Delta F + 2B$$

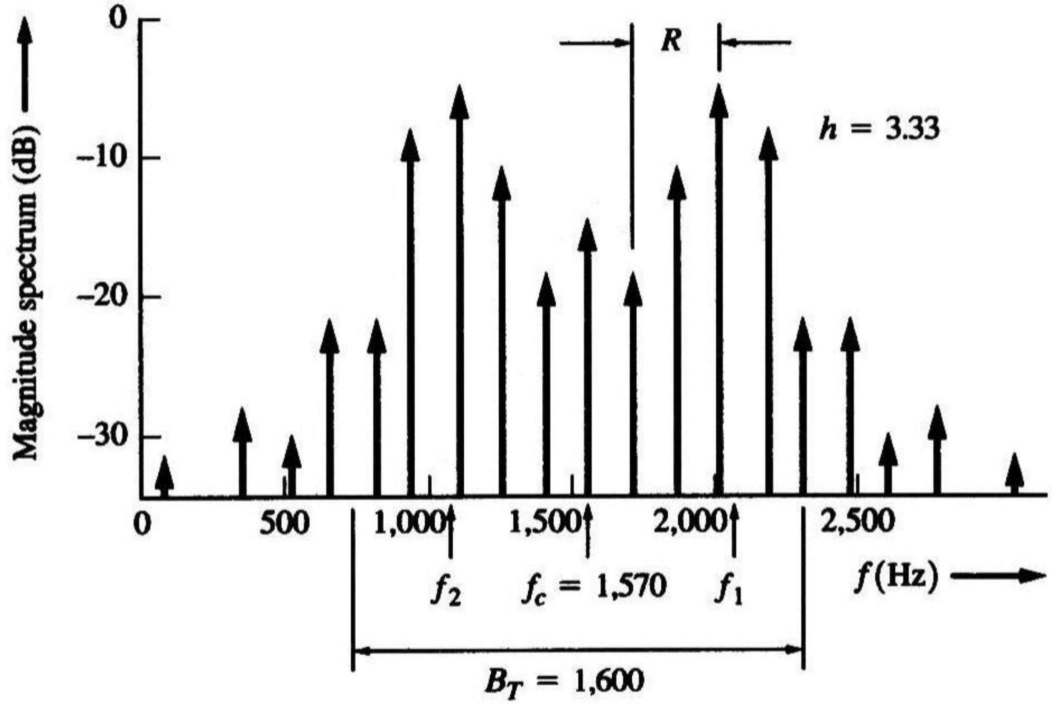
$$\approx 2\Delta F + 2R = hR + 2R$$



(a) FSK Spectrum with $f_2 = 1{,}070$ Hz, $f_1 = 1{,}270$ Hz, and $R = 300$ bits/sec (Bell 103 Parameters, Originate mode) for $h = 0.67$



(b) FSK Spectrum with $f_2 = 1{,}070$ Hz, $f_1 = 1{,}270$ Hz, and $R = 110$ bits/sec for $h = 1.82$



(c) FSK Spectrum with f_2 = 1,070 Hz, f_1 = 2,070 Hz, and R = 300 bits/sec for h = 3.33

Figure 5–26 FSK spectra for alternating data modulation (positive frequencies shown with one-sided magnitude values).

PSD of FSK

- The PSD for the complex envelope of the continuous-phase FSK signal is

$$P_g(f) = \frac{A_c^2 T_b}{2}$$

$$\{A_1^2(f)[1+B_{11}(f)] + A_2^2(f)[1+B_{22}(f)] + 2B_{12}(f)A_1(f)A_2(f)\}$$

where

$$A_n(f) = Sa\{\pi T_b[f - \Delta F(2n-3)]\}$$

and

$$B_{nm}(f) =$$

$$\frac{\cos\{2\pi[f - \Delta F(n+m-3)]T_b\} - \cos(2\pi\Delta F T_b)\cos[2\pi\Delta F(n+m-3)T_b]}{1+\cos^2(2\pi\Delta F T_b) - 2\cos(2\pi\Delta F T_b)\cos(2\pi f T_b)}$$

$$h = 2\Delta F / R \neq 0, 1, 2, \ldots$$

(When $h = 0, 1, 2, \ldots$, there are also discrete terms in the spectrum)

PSD of FSK (con't)

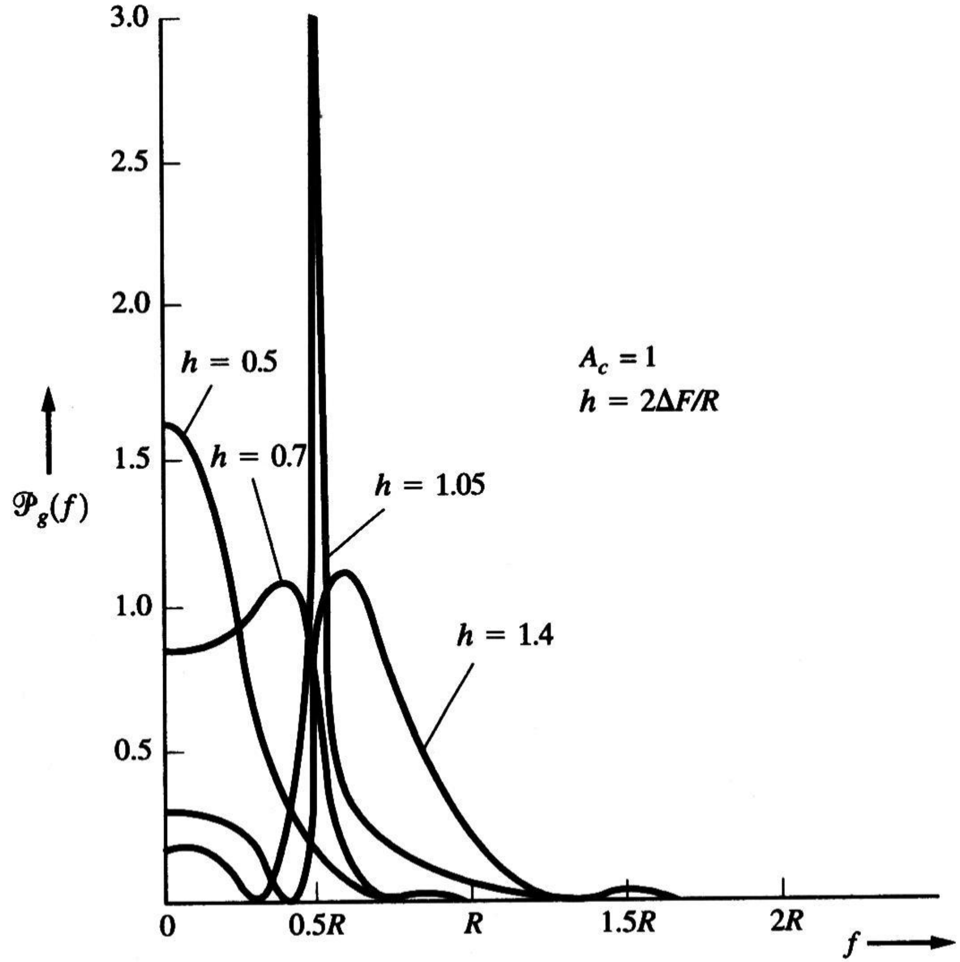


Figure 5–27 PSD for the complex envelope of FSK (positive frequencies shown).

FSK Demodulation

- FSK can be detected by using either a frequency (noncoherent) detector or two product detectors (coherent detection).

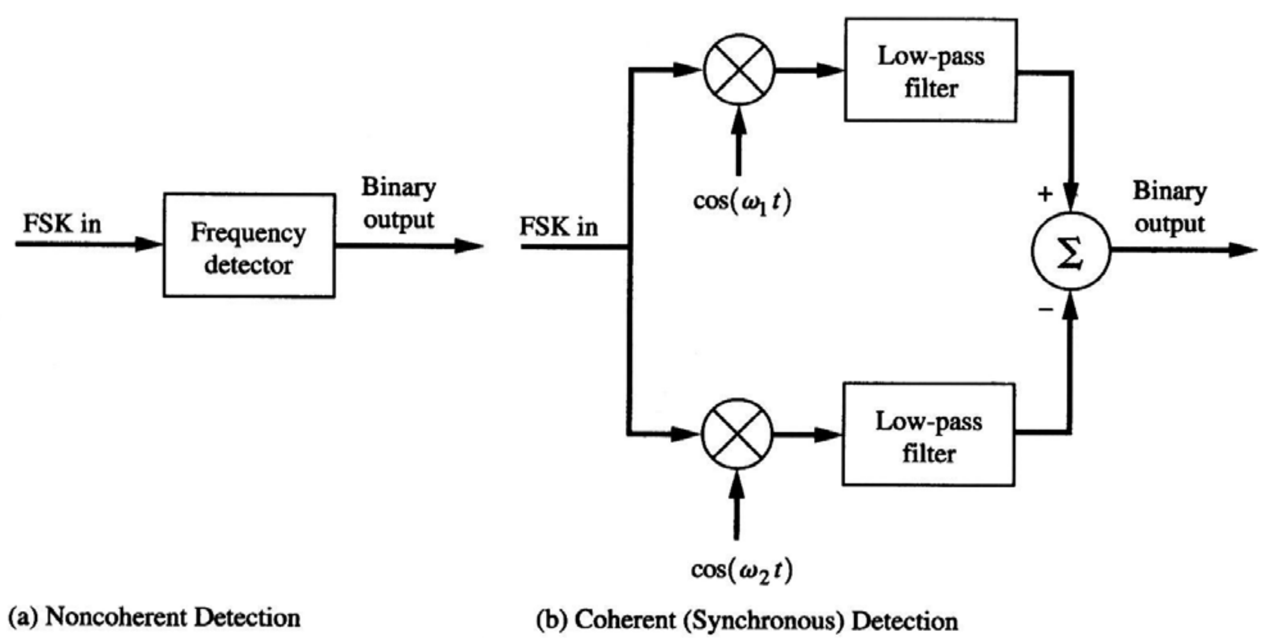


Figure 5–28 Detection of FSK.

Homework

- LC 5-46, 5-47, 5-52, 5-53