Baseband Digital System (7)

LC 7-1,7-2

Lecture 18, 2008-11-21

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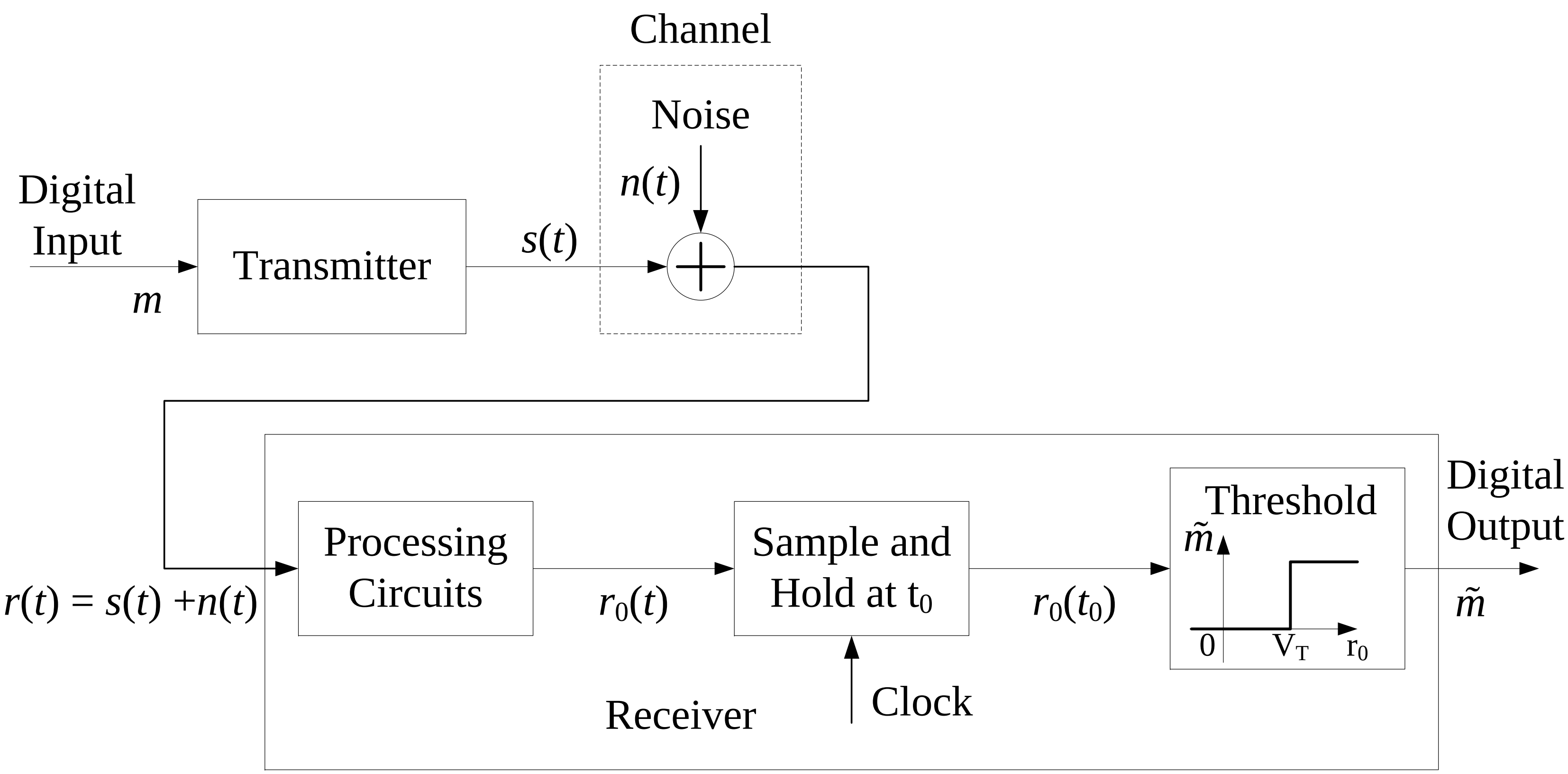
Binary Communication System



Bit Error Rate

- Bit-Error Rate, the probability of encountering a bit error.

T is the duration of time it takes to transmit one bit of data.

The transmitted signal over a bit interval (0,T) is

$$s(t) = \begin{cases} s_1(t), & 0<t \le T, \quad \text{for a binary 1} \\ s_2(t), & 0<t \le T, \quad \text{for a binary 0} \end{cases}$$

$s_1(t)$ is the waveform that is used if a binary 1 is transmitted

$s_2(t)$ is the waveform that is used if a binary 0 is transmitted

If $s_1(t) = -s_2(t)$, $s(t)$ is called antipodal signal.

The binary signal plus noise at the receiver input produces a baseband analog waveform at the output of the processing circuits

$$r_0(t) = \begin{cases} r_{01}(t), & 0<t \le T, \quad \text{for a binary 1 sent} \\ r_{02}(t), & 0<t \le T, \quad \text{for a binary 0 sent} \end{cases}$$

$r_{01}(t)$ is the output signal that is corrupted by noise for binary 1 transmission

$r_{02}(t)$ is the output signal for binary 0 transmission

This analog voltage is sampled at some time t_0 during the bit interval

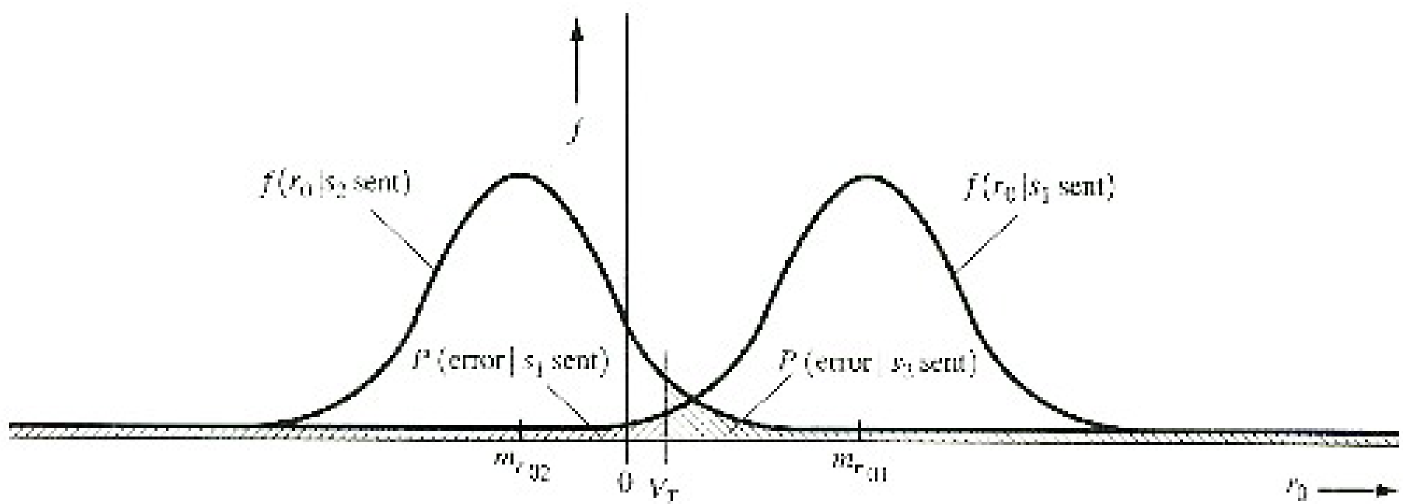
$$r_0(t_0) = \begin{cases} r_{01}(t_0), & \text{for a binary 1 sent} \\ r_{02}(t_0), & \text{for a binary 0 sent} \end{cases}$$

- We examine the PDF for the two random variables r_{01} and r_{02}. These PDFs are actually conditional PDFs, since they depend respectively.

  When $r_0=r_{01}$, the PDF is $f(r_0|s_1)$

  When $r_0=r_{02}$, the PDF is $f(r_0|s_2)$

- The actual shapes of the PDFs depend on the characteristics of the channel noise, the specific types of filter and detector circuits used, and the types of binary signals transmitted.



- V_T is the threshold (voltage) setting of the comparator (threshold device). When signal plus noise is present at the receiver input, errors can occur in two ways.

An error occurs when $r_0 < V_T$ if a binary 1 is sent:

$$P(error \mid s_1) = \int_{-\infty}^{V_T} f(r_0 \mid s_1) dr_0$$

An error occurs when $r_0 > V_T$ if a binary 0 is sent:

$$P(error \mid s_2) = \int_{V_T}^{\infty} f(r_0 \mid s_2) dr_0$$

The BER is then

$$P_e = P(error \mid s_1) P(s_1) + P(error \mid s_2) P(s_2)$$

Gaussian Noise

- Assume that the channel noise is zero-mean wide-sense stationary Gaussian process.
- The receiver processing circuits, except for the threshold device, are linear.
- A Gaussian process at the input, the output of the linear processor will also be a Gaussian process.
- For the case of a linear-processing receiver circuit with a binary signal plus noise at the input, the sampled output is

$$r_0 = s_0 + n_0$$

- Here, $r_0 = r_0(t_0)$. $n_0 = n_0(t_0)$ is a zero –mean Gaussian random variable, and $s_0 = s_0(t_0)$is a constant that depends on the singal being sent

$$s_0 = \begin{cases} s_{01}, & \text{for a binary 1} \\ s_{02}, & \text{for a binary 0} \end{cases}$$

- Where s_{01} and s_{02} are known constants for a given type of receiver with known input waveform $s_1(t)$ and $s_2(t)$.

- The two conditional PDFs are

$$f(r_0 | s_1) = \frac{1}{\sqrt{2\pi}\sigma_0} e^{-(r_0 - s_{01})^2/(2\sigma_0^2)}$$

$$f(r_0 | s_2) = \frac{1}{\sqrt{2\pi}\sigma_0} e^{-(r_0 - s_{02})^2/(2\sigma_0^2)}$$

$\sigma_0^2 = \overline{n_0^2} = \overline{n_0^2(t_0)} = \overline{n_0^2(t)}$ is the output noise from the receiver processing circuit where the output noise process is wide-sense stationary

- The BER becomes

$$P_e = \frac{1}{2}\int_{-\infty}^{V_T} \frac{1}{\sqrt{2\pi}\sigma_0} e^{-(r_0 - s_{01})^2/(2\sigma_0^2)} dr_0 + \frac{1}{2}\int_{V_T}^{\infty} \frac{1}{\sqrt{2\pi}\sigma_0} e^{-(r_0 - s_{01})^2/(2\sigma_0^2)} dr_0$$

$$= \frac{1}{2}\int_{-(V_T - s_{01})/\sigma_0}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\lambda^2/2} d\lambda + \frac{1}{2}\int_{(V_T - s_{02})/\sigma_0}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\lambda^2/2} d\lambda$$

$$= \frac{1}{2} Q(\frac{-V_T + s_{01}}{\sigma_0}) + \frac{1}{2} Q(\frac{V_T - s_{02}}{\sigma_0})$$

- To find the V_T that minimizes P_e, we need to solve $dP_e/dV_T=0$

$$\frac{dP_e}{dV_T}=\frac{1}{2}\frac{1}{\sqrt{2\pi}\sigma_0}e^{-(r_0-s_{01})^2/(2\sigma_0^2)}-\frac{1}{2}\frac{1}{\sqrt{2\pi}\sigma_0}e^{-(r_0-s_{01})^2/(2\sigma_0^2)}=0$$

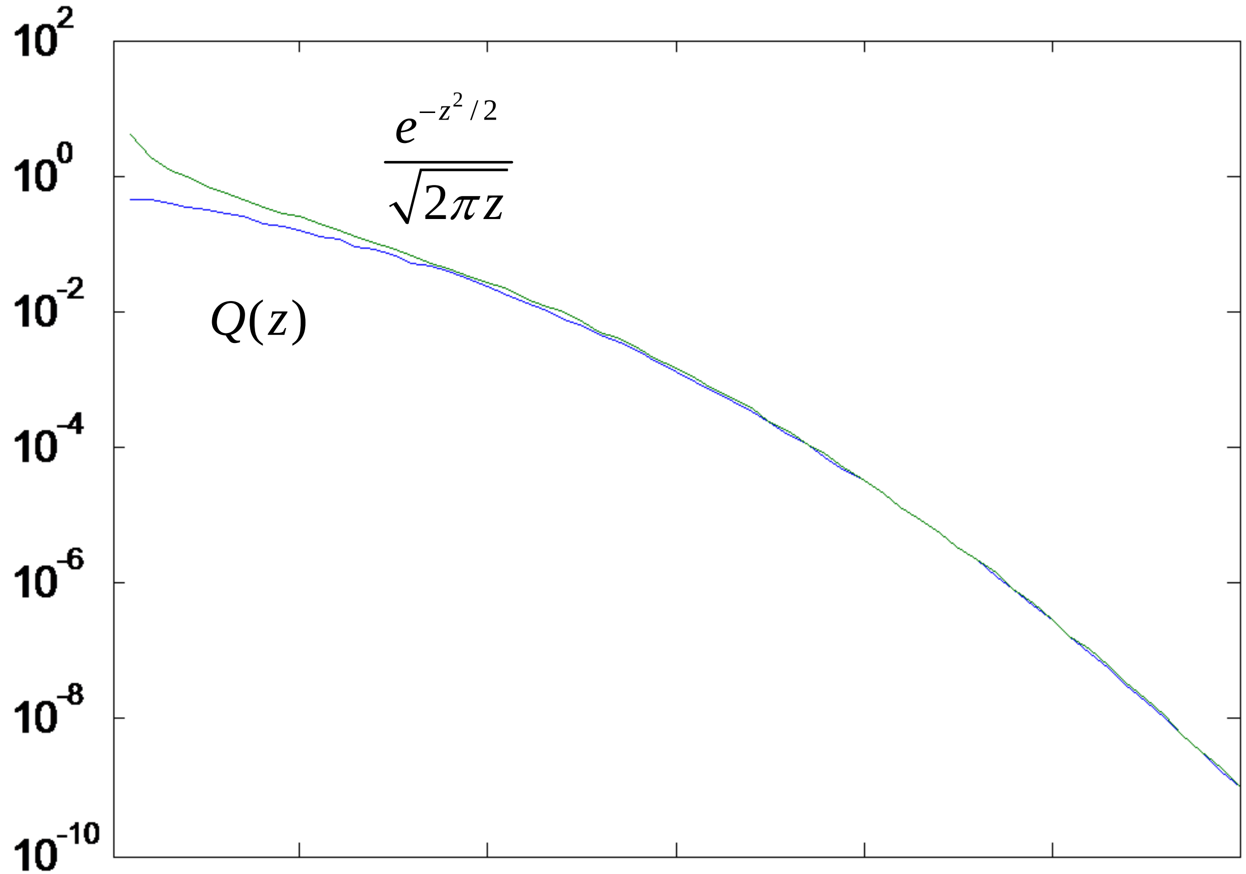
- Consequently,

$$V_T=\frac{s_{01}+s_{02}}{2}$$

- The BER at the optimized threshold level is

$$P_e=Q(\frac{s_{01}-s_{02}}{2\sigma_0})=Q\left(\sqrt{\frac{(s_{01}-s_{02})^2}{4\sigma_0^2}}\right)$$

Q function



White Gaussian Noise and Matched-Filter Reception

- To minimize P_e, we need to find the linear filter that maximizes

$$\frac{[s_{01}(t_0)-s_{02}(t_0)]^2}{\sigma_0^2}=\frac{[s_d(t_0)]^2}{\sigma_0^2}$$

$s_d(t_0)=s_{01}(t_0)-s_{02}(t_0)$ is the different signal sample value.

- For the case of white noise at the receiver input, the matched filter needs to be matched to the difference signal $s_d(t)$. Thus, the impulse response of the matched filter is:

$$h(t)=C[s_1(t_0-t)-s_2(t_0-t)]$$

- The output peak signal to average noise ratio that is obtained from the matched filter is

$$\frac{[s_d(t_0)]^2}{\sigma_0^2}=\frac{2E_d}{N_0}$$

- E_d is the difference signal energy at the receiver input

$$E_d = \int_0^T [s_1(t) - s_2(t)]^2 dt$$

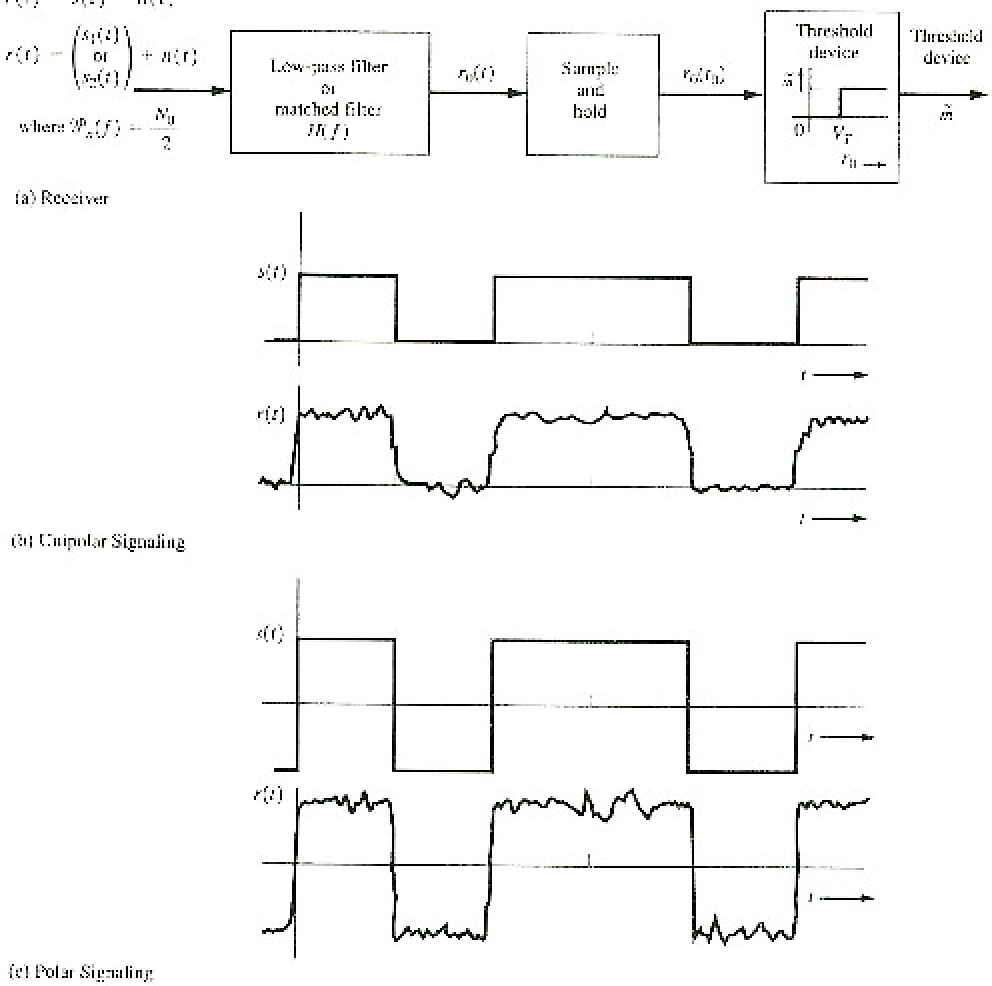
$s_d(t_0) = s_{01}(t_0) - s_{02}(t_0)$ is the different signal sample value.

- Thus, for binary signaling corrupted by white noise, matched-filter reception, and using the optimum threshold setting, the BER is

$$P_e = Q\left(\sqrt{\frac{E_d}{2N_0}}\right)$$

Performance of Baseband Binary Systems

- Uni-polar Signaling
- Polar Signaling
- Bi-polar Signaling



Uni-polar Signaling

- The two baseband signaling waveforms are

$$s_1(t) = +A, \quad 0<\mathrm{t} \le \mathrm{T} \ \ \text{(binary 1)}$$
$$s_2(t) = 0, \quad 0<\mathrm{t} \le \mathrm{T} \ \ \text{(binary 0)}$$

Where A>0. The unipolar signal plus white Gaussian noise is present at the receiver input.

- In the case of LPF, we choose the equivalent bandwidth B>2/T.

The noise power at the output of LPF is $\sigma_0^2 = (N_0/2)(2B) = N_0 B$

The optimum threshold setting is then V_T=A/2

The BER

$$Pe = Q\left(\sqrt{\frac{A^2}{4N_0 B}}\right)$$

- In the case of matched filter, the energy in the difference signal is $E_d = A^2T$

The BER

$$Pe = Q\left(\sqrt{\frac{A^2T}{2N_0}}\right) = Q\left(\sqrt{\frac{E_b}{N_0}}\right)$$

Where the average energy per bit is $E_b = A^2T/2$, because the energy for a binary 1 is A^2T, and the energy for 0 is 0.

It is desirable to express the BER in terms of E_b/N_0, because it indicates the average energy required to transmit one bit of data over a white (thermal) noise channel. By expressing the BER in terms of E_b/N_0, the performance of different signaling techniques can be easily compared.

Polar Signaling

- The two baseband signaling waveforms are

$$s_1(t) = +A, \quad 0<\text{t} \leq \text{T} \ \ (\text{binary } 1)$$
$$s_2(t) = -A, \quad 0<\text{t} \leq \text{T} \ \ (\text{binary } 0)$$

- In the case of LPF, we choose the equivalent bandwidth B>2/T.

The output signal samples are S_{01}=A and S_{02}=-A at sampling time t = t_0

The noise power at the output of LPF is $\sigma_0^2 = N_0 B$

The optimum threshold setting is then V_T= 0

The BER

$$Pe = Q\left(\sqrt{\frac{A^2}{N_0 B}}\right)$$

- In the case of matched filter, the energy in the difference signal is $E_d = (2A)^2T$

The BER

$$Pe = Q\left(\sqrt{\frac{2A^2T}{N_0}}\right) = Q\left(\sqrt{2\left(\frac{E_b}{N_0}\right)}\right)$$

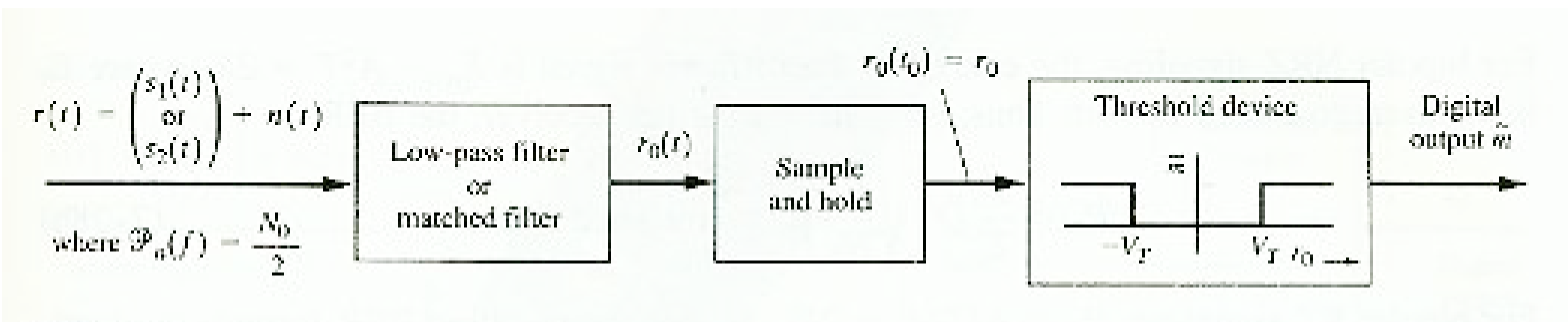
Where the average energy per bit is $E_b = A^2T$, because the energy for a binary 1 is A^2T, and the energy for 0 is A^2T, .

Bi-polar Signaling

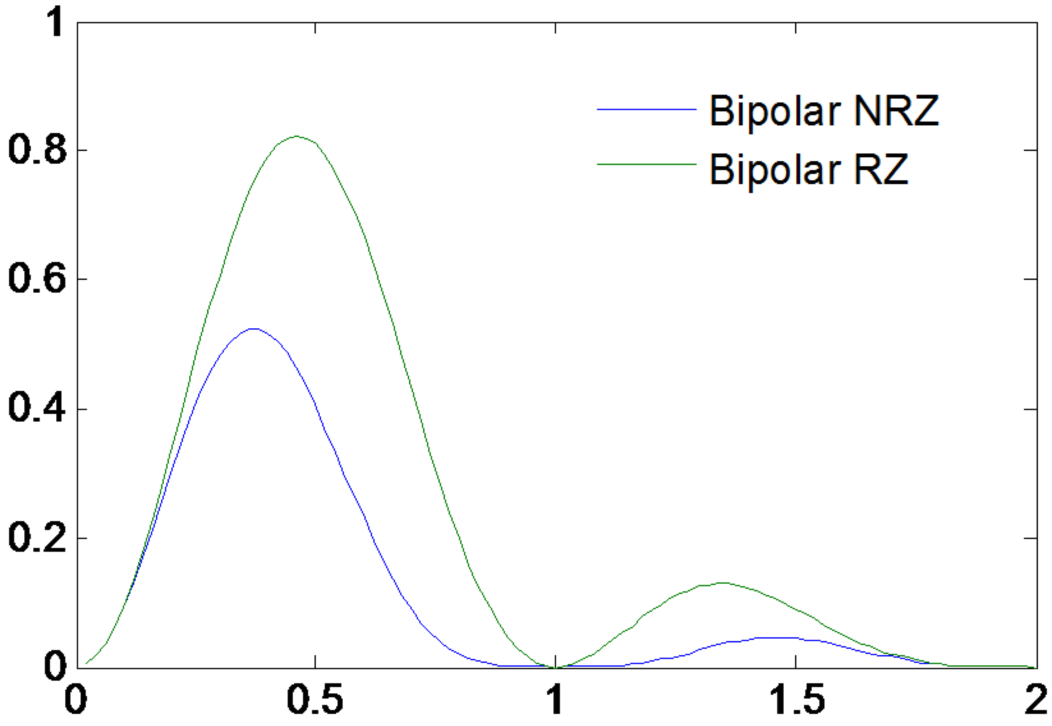
- For bipolar NRZ signaling, binary 1's are represented by alternating positive and negative values, and the binary 0's are represented by a zero level

$$s_1(t) = \pm A, \quad 0<t \le T \text{ (binary 1)}$$
$$s_2(t) = 0, \quad 0<t \le T \text{ (binary 0)}$$

- Where A > 0. This is similar to unipolar signaling, except two thresholds, $+V_T$ and $-V_T$



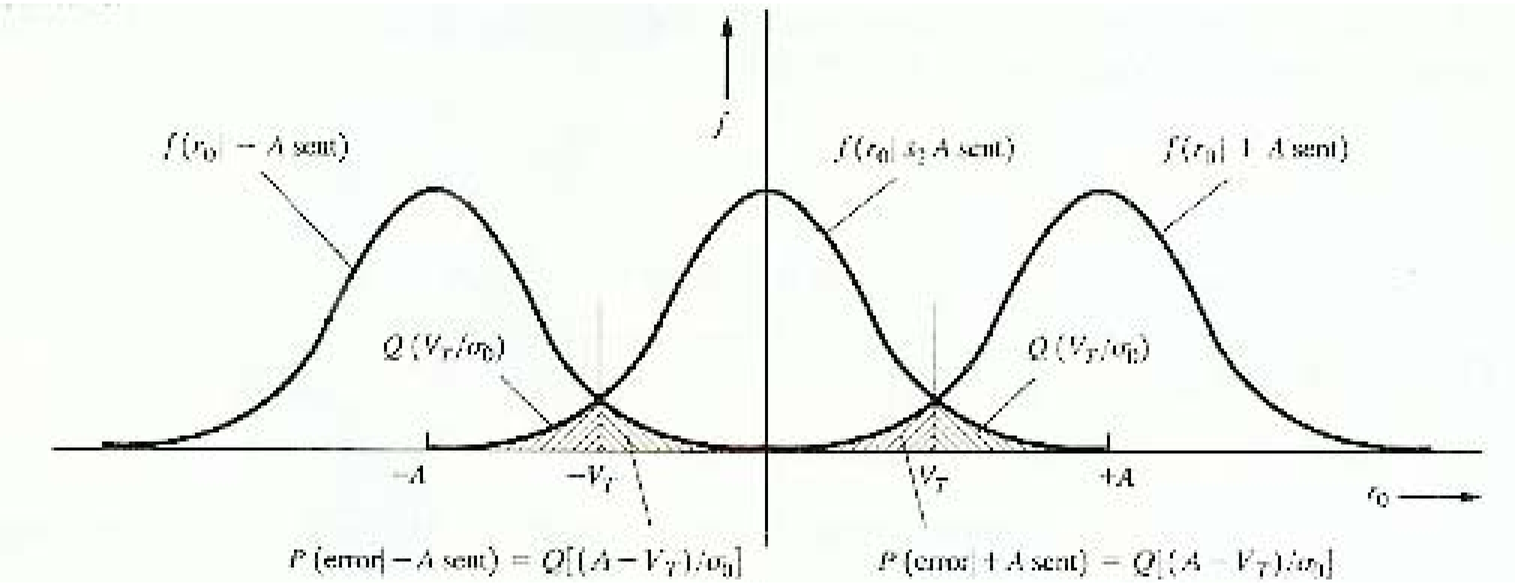
Bipolar Spectrum (3-5, Fig. 3-16)



- The BER

$$Pe = P(\text{error} \mid +A)P(+A) + P(\text{error} \mid -A)P(-A) + P(\text{error} \mid 0)P(0)$$

$$= \frac{1}{4}\left[Q\left(\frac{A-V_T}{\sigma_0}\right)\right] + \frac{1}{4}\left[Q\left(\frac{A-V_T}{\sigma_0}\right)\right] + \frac{1}{2}\left[2Q\left(\frac{V_T}{\sigma_0}\right)\right] = \frac{1}{2}Q\left(\frac{A-V_T}{\sigma_0}\right) + Q\left(\frac{V_T}{\sigma_0}\right)$$



Using calculus, the optimum value of V_T for minimum BER is

$$V_T = \frac{A}{2} + \frac{\sigma_0^2}{A}\ln 2$$

For the system with reasonable low BERs, $A > \sigma_0$, so that the optimum threshold becomes approximately $V_T = A/2$

In the case of LPF, the BER $Pe = \frac{3}{2}Q\left(\frac{A}{2\sigma_0}\right) = \frac{3}{2}Q\left(\sqrt{\frac{A^2}{4N_0B}}\right)$

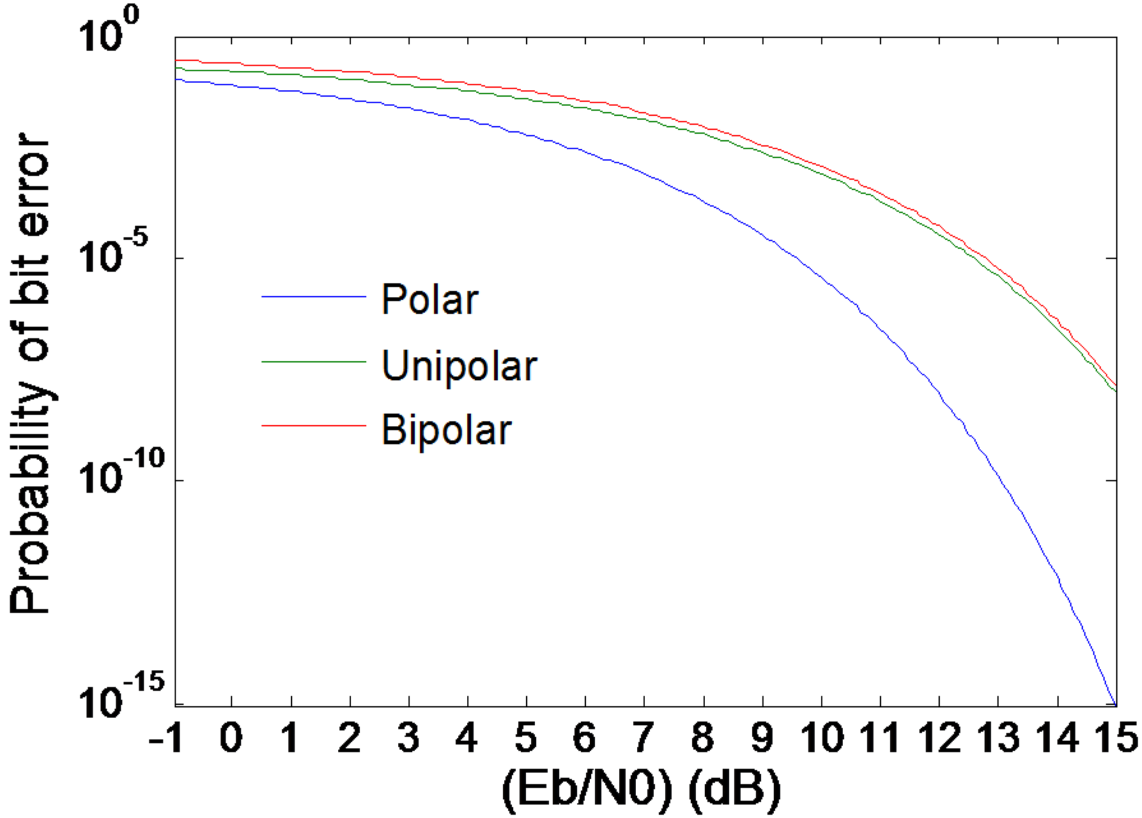
In the case of matched filter, the output SNR is $\left(\frac{S}{N}\right)_{out} = \frac{A^2}{\sigma_0^2} = \frac{2E_d}{N_0}$

For bipolar NRZ, the energy in the different signal is $E_d = A^2T = 2E_b$

The BER

$$Pe = \frac{3}{2}Q\left(\sqrt{\frac{E_b}{N_0}}\right)$$

BER Comparison



Homework

- LC 7-1, 7-3, 7-11, 7-13, 7-16