Baseband Digital System (4)

LC 3-6, 3-7, 3-8

Lecture 15, 2008-11-4

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- Inter-symbol Interference
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- Differential Pulse Code Modulation
- Delta Modulation

Multilevel NRZ

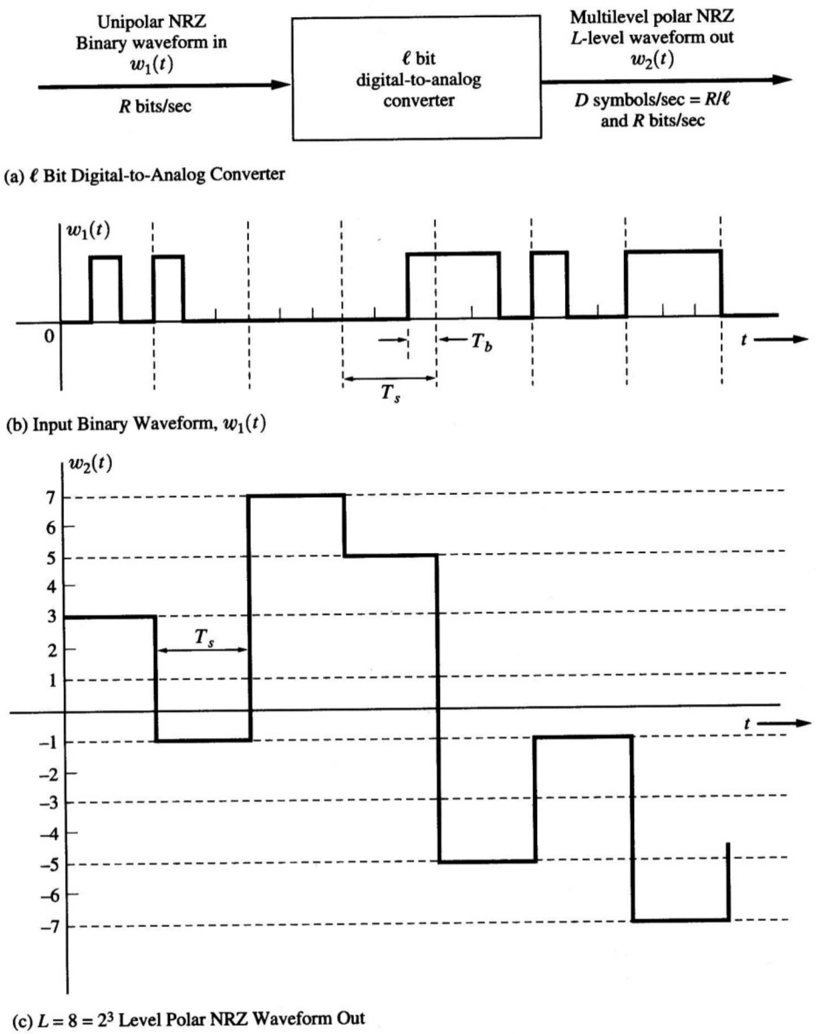


Figure 3–22 Binary-to-multilevel polar NRZ signal conversion.

$$R(0)=\sum_{i=1}^{I}(a_n)_i^2 P_i=\frac{1}{8}\sum_{i=1}^{8}(a_n)_i^2=\frac{1}{8}[7^2+5^2+3^2+1^2+(-1)^2+(-3)^2+(-5)^2+(-7)^2]=21$$

$$R(k)=\sum_{i=1}^{I}(a_n a_{n+k})_i P_i=\frac{1}{64}\sum_{i=1}^{I}(a_n a_{n+k})_i$$

$$=\frac{1}{64}[7(7+5+3+1-1-3-5-7)+5(7+5+3+1-1-3-5-7)+\cdots+(-7)(7+5+3+1-1-3-5-7)]$$

$$=0,\quad k\neq 0$$

For rectangular pulses $f(t)=\Pi\left(\frac{t}{T_s}\right)\leftrightarrow F(f)=T_s Sa(\pi f T_s)$

$$P_{8-level\ NRZ}(f)=\frac{|F(f)|^2}{T_s}\sum_{k=-\infty}^{\infty}R(k)e^{j2\pi kfT_s}=\frac{|T_s Sa(\pi f T_s)|^2}{T_s}21=21T_s Sa^2(\pi f T_s)=63T_b Sa^2(3\pi f T_b)$$

$$B_{null}=\frac{1}{T_s}=D=\frac{R}{l}$$

For sinc pulses: $P_{8-level\ NRZ}(f)=21T_s\Pi\left(\frac{f}{1/T_s}\right),\quad B_{abs}=\frac{1}{2T_s}=\frac{D}{2}=\frac{R}{2l}$ (dimensionality theorem)

- In general, for the case of a multilevel polar NRZ signal with rectangular pulse shape

$$P_{multilevel\ NRZ}(f) = KSa^2\left(l\pi f T_b\right)$$

$$B_{null} = \frac{R}{l}$$

- Multilevel signaling is used to reduce the bandwidth of a digital signal compared with the bandwidth required for binary signaling.

Spectral Efficiency

- The spectral efficiency of a digital signal is given by the number of bits per second of data that can be supported by each hertz of bandwidth. That is,

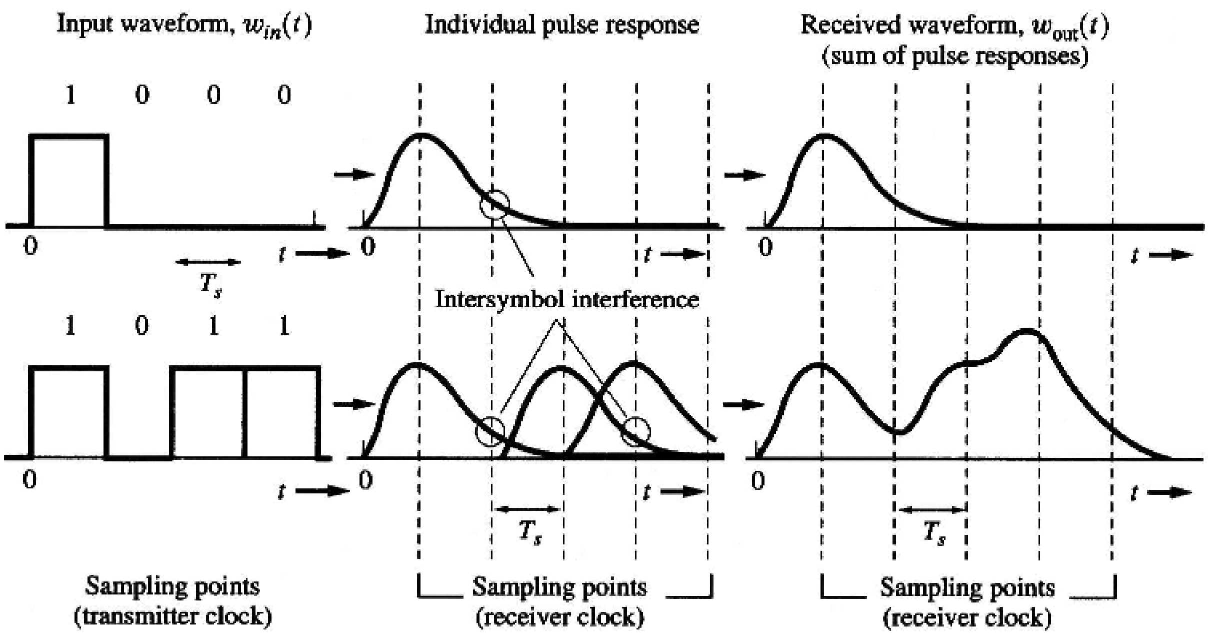
$$\eta = \frac{R}{B} \text{ (bits/s)/Hz}$$

TABLE 3–6 SPECTRAL EFFICIENCIES OF LINE CODES

Code Type	First Null Bandwidth (Hz)	Spectral Efficiency $\eta = R/B$ [(bits/s)/Hz]
Unipolar NRZ	R	1
Polar NRZ	R	1
Unipolar RZ	$2R$	$\frac{1}{2}$
Bipolar RZ	R	1
Manchester NRZ	$2R$	$\frac{1}{2}$
Multilevel polar NRZ	R/ℓ	ℓ

Intersymbol Interference

- The absolute bandwidth of rectangular pulses is infinity. If these pulses are filtered improperly as they pass through a communication system, they will spread in time, and the pulse for each symbol may be smeared into adjacent time slots and cause intersymbol interference (ISI).

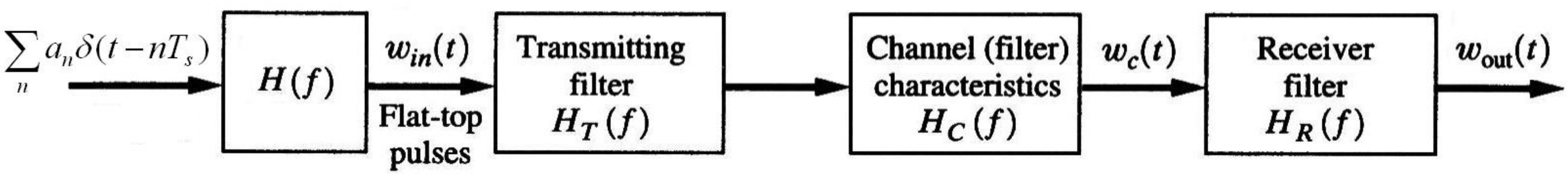


Baseband Pulse Transmission System

- Consider a digital signaling system as shown below, in which the flat-topped signal at the input is

$$w_{in}(t) = \sum_n a_n h(t - nT_s)$$

$$= \sum_n a_n [h(t) * \delta(t - nT_s)] = h(t) * \left[\sum_n a_n \delta(t - nT_s) \right]$$



- The output of the system is

$$w_{out}(t) = h_R(t) * h_C(t) * h_T(t) * w_{in}(t)$$

$$= h_R(t) * h_C(t) * h_T(t) * h(t) * \left[\sum_n a_n \delta(t - nT_s) \right]$$

$$= h_e(t) * \left[\sum_n a_n \delta(t - nT_s) \right] = \sum_n a_n h_e(t - nT_s)$$

$h_e(t)$: equivalent impulse response of the overall system

- The equivalent system transfer function is

$$H_e(f) = H(f) H_T(f) H_C(f) H_R(f)$$

- When the equivalent system transfer function is chosen to minimize the ISI, the receiver filter is called an **equalizing filter**.

$$H_R(f) = \frac{H_e(f)}{H(f) H_T(f) H_C(f)}$$

- Adaptive filter and Preambles

Nyquist's First Method

$$h_e(kT_s+\tau)=\begin{cases}C, & k=0\\ 0, & k\neq 0\end{cases}$$

k : interger

T_s : symbol interval

τ: the offset in the receiver sampling clock times compared with the clock times of the input symbols

C: is nonzero constant

$kT_s+\tau$: sampling times

$$w_{in}(t)=ah(t)=h(t)*\left[a\delta(t)\right]\Rightarrow w_{out}(t)=ah_e(t)\Rightarrow w_{out}(kT_s+\tau)=\begin{cases}aC, & k=0\\ 0, & k\neq 0\end{cases}$$

The sinc function satisfies Nyquist's first criterion for zero ISI

$$h_e(t)=Sa\left[\frac{\pi(t-\tau)}{T_s}\right]=Sa[\pi f_s(t-\tau)]\leftrightarrow H_e(f)=\frac{1}{f_s}\Pi\left[\frac{f}{f_s}\right]e^{-j2\pi f\tau}$$

$B=f_s/2$

$D=f_s=2B$

Practical Difficulties of Sa()

- The overall amplitude transfer characteristic He(f) has to be flat over –B<f<B and zero elsewhere. This is physically unrealizable. He(f) is difficult to approximate because of the steep skirts in the filter transfer function at $f=\pm B$
- The synchronization of the clock in the decoding sampling circuit has to be almost perfect, since the sa(x) pulse decays only as 1/x and is zero in adjacent time slots only when t is at the exactly correct sampling time. Thus, inaccurate synchronization will cause ISI.

Raised Cosine-Rolloff filter

$$H_e(f) = \begin{cases} 1, & |f| < f_1 \\ \frac{1}{2}\left\{1 + \cos\left[\dfrac{\pi(|f| - f_1)}{2 f_\Delta}\right]\right\}, & f_1 < |f| < B \\ 0, & |f| > B \end{cases}$$

B: absolute bandwidth, f_0: 6-dB bandwidth $\Leftrightarrow H_e(f_0) = \frac{1}{2}$

$$f_\Delta = B - f_0, \quad f_1 = f_0 - f_\Delta$$

$r = \dfrac{f_\Delta}{f_0}$: rolloff factor

$$h_e(t) = 2 f_0 Sa(2\pi f_0 t) \frac{\cos 2\pi f_\Delta t}{1 - \left(4 f_\Delta t\right)^2}$$

The raised cosine-rolloff filter satifies Nyquist's first criterion for zero ISI if

$$D = 1/T_s = 2 f_0 = \frac{2B}{1+r}$$

Raised Cosine-Rolloff Filter

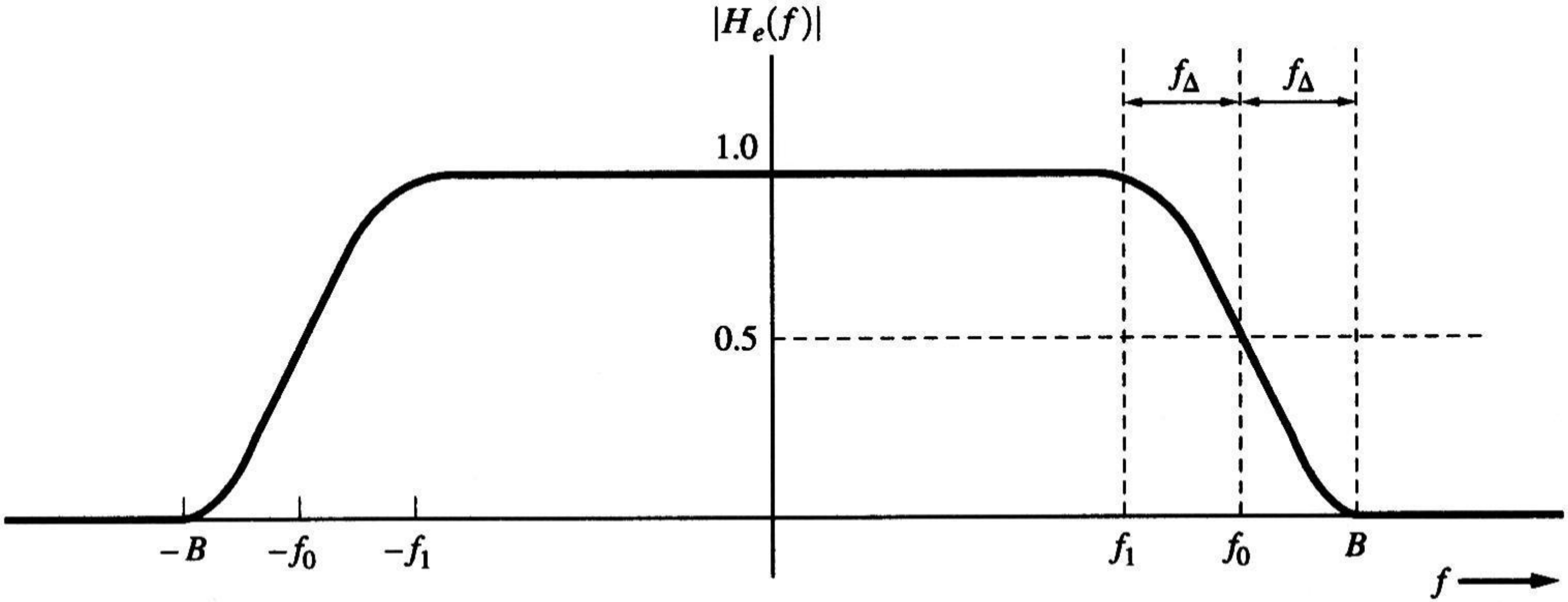


Figure 3–25 Raised cosine-rolloff Nyquist filter characteristics.

Frequency and Time Response

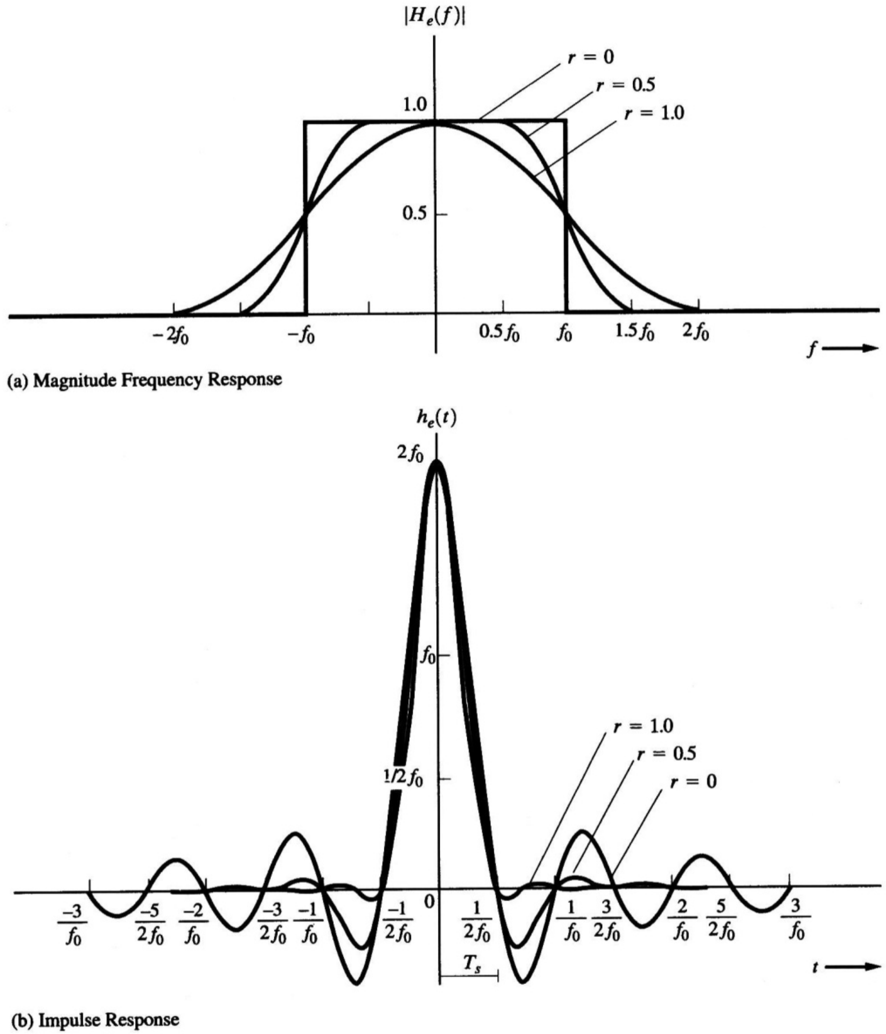


Figure 3–26 Frequency and time response for different rolloff factors.

Theorem

- A filter is said to be a Nyquist filter if the effective transfer function is

$$H_e(f) = \begin{cases} \Pi\left(\frac{f}{2f_0}\right) + Y(f), & |f| < 2f_0 \\ 0, & f \text{ elsewhere} \end{cases}$$

where $Y(f)$ is a real function that is even symmetric about $f = 0$; that is,

$$Y(-f) = Y(f), \quad |f| < 2f_0$$

and $Y(f)$ is odd symmetric about $f = f_0$; that is,

$$Y(-f + f_0) = -Y(f + f_0), \quad |f| < f_0$$

In time domain,

$$h_e(t) = 2f_0 Sa(2\pi f_0 t) + j2\sin 2\pi f_0 t \int_{-f_0}^{f_0} Y(f_1 + f_0) e^{j2\pi f_1 t} df_1$$

Then there will be no ISI at the system output if the symbol rate is

$$D = f_s = 2f_0$$

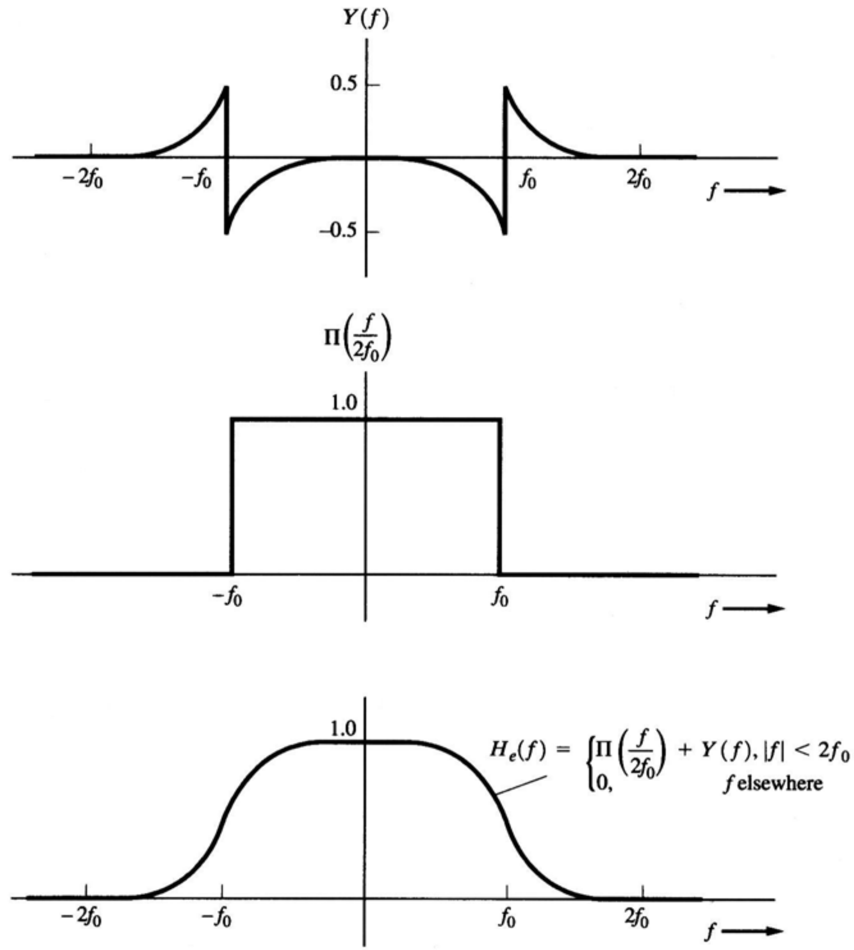


Figure 3–27 Nyquist filter characteristic.

Matched Filter

- The filter that minimizes the effect of channel noise is the matched filter.
- Unfortunately, if a matched filter is used for HR(f) at the receiver, the overall filter characteristic, He(f), will usually not satisfy the Nyquist characteristic for minimum ISI.
- For the case of Gaussian noise into the receiver, effects of both ISI and channel noise are minimized if the transmitter and receiver filters are designed so that

$$|H_T(f)| = \frac{\sqrt{|H_e(f)|}\,[\mathscr{P}_n(f)]^{1/4}}{\alpha\,|H(f)|\sqrt{|H_C(f)|}} \quad \text{and} \quad |H_R(f)| = \frac{\alpha\sqrt{|H_e(f)|}}{\sqrt{|H_C(f)|}\,[\mathscr{P}_n(f)]^{1/4}}$$

where $\mathscr{P}_n(f)$ is the PSD for the noise at the receiver input

α is an arbitrary positive constant

$H_e(f)$ is selected from any appropriate frequency response

characteriestic that satisfies Nyquist's first criterion

Homework

- LC 3-41, 3-44, 3-48, 3-50