Baseband Digital System (2)

LC 3-3, 7-7

Lecture 14, 2008-10-24

Contents

- Pulse Code Modulation
- Sampling, Quantizing, and Encoding
- Practical PCM Circuits
- Effect of Noise
- Nonuniform Quantizing

Pulse Code Modulation

- Pulse code modulation (PCM) is essentially analog-to-digital conversion of a special type where the information contained in the instantaneous samples of an analog signal is represented by digital words in a serial bit stream.
- Relatively inexpensive digital circuit may be used extensively in the system.
- PCM signals derived from all types of analog sources (audio, video, etc.) may be merged with data signals (e.g., from computers) and transmitted over a common high-speed digital communication system. TDM
- In long-distance transmission systems, a clean PCM waveform can be regenerated at the output of each repeater, where the input consists of a noisy PCM waveform. However, the noise at the input may cause bit errors in the regenerated PCM output signal.
- The noise performance of a digital system can be superior to that of an analog system.
- Much wider bandwidth than that of the corresponding analog signal.

PCM Transmission System

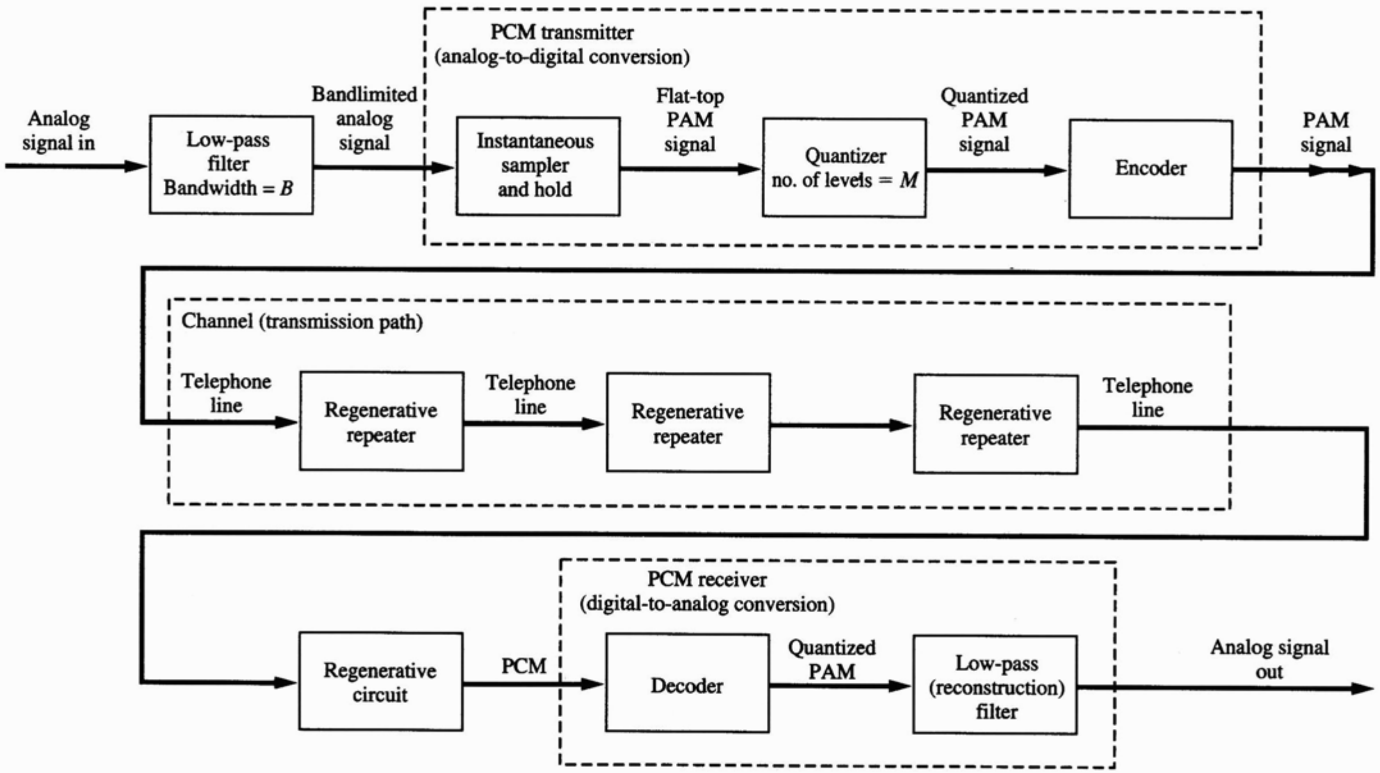
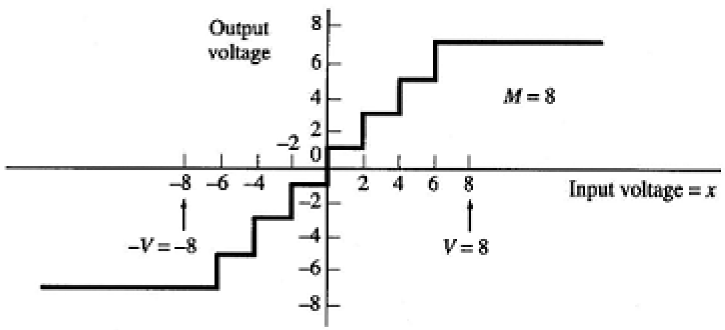
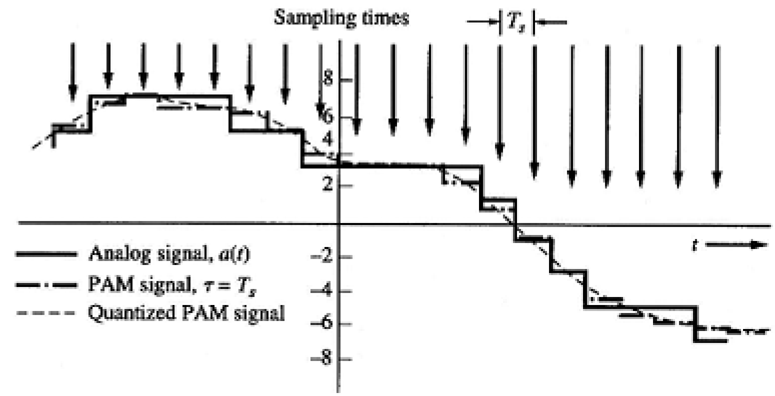


Figure 3–7 PCM trasmission system.

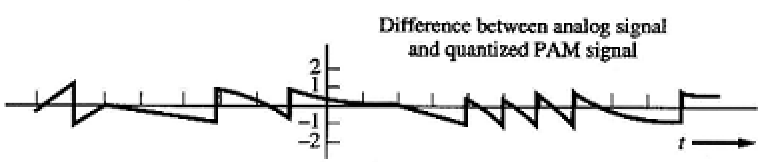
Sampling, Quantizing, and Encoding



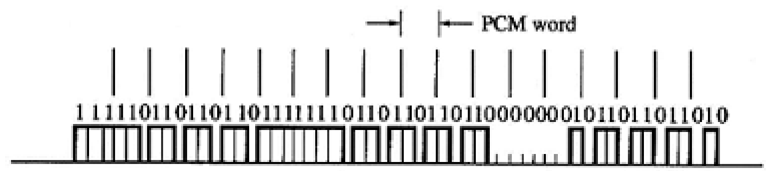
(a) Quantizer Output-Input Characteristics



(b) Analog Signal, Flat-top PAM Signal, and Quantized PAM Signal



(c) Error Signal



(d) PCM Signal

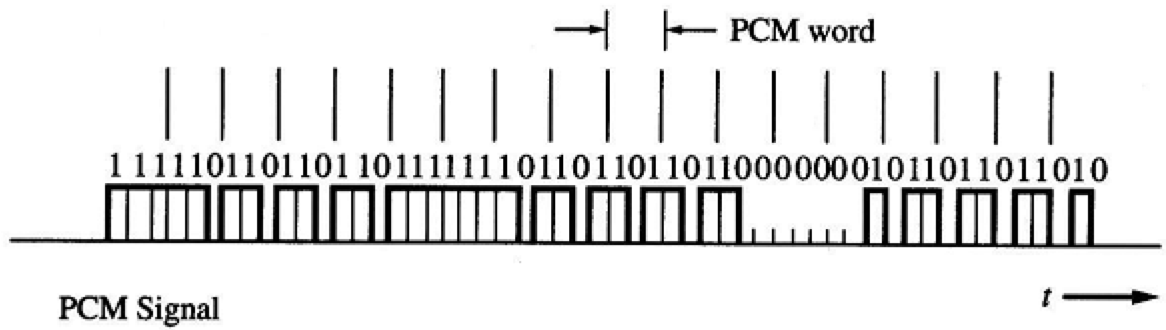
Gray Code

TABLE 3–1 THREE-BIT GRAY CODE FOR $M = 8$ LEVELS

Quantized Sample Voltage	Gray Code Word (PCM Output)	
+7	110	
+5	111	
+3	101	
+1	100	
		Mirror image except for sign bit
−1	000	
−3	001	
−5	011	
−7	010	

- Encoding each quantized sample value into a digital word.
- Gray code has only one bit change for each step change in the quantized level. Consequently, single errors in the received PCM code word will cause minimal errors in the recovered analog level, provided that the sign bit is not in error.

Bandwidth of PCM Signals



- The bit rate is $R = nf_s$

Effect of Noise

- The analog signal that is recovered at the PCM system output is corrupted by noise. Two main effects produce this noise or distortion:
  - Quantizing noise that is caused by the *M*-step quantizer at the PCM transmitter.
  - Bit errors in the recovered PCM signal. The bit errors are caused by channel noise as well as improper channel filtering, which causes ISI.
- The analog sample output of the PCM system for the *k*th sampling time is

$$y_k = x_k + n_k$$

- The x_k is the signal (same as the input sample) and n_k is noise.

PCM Communication System

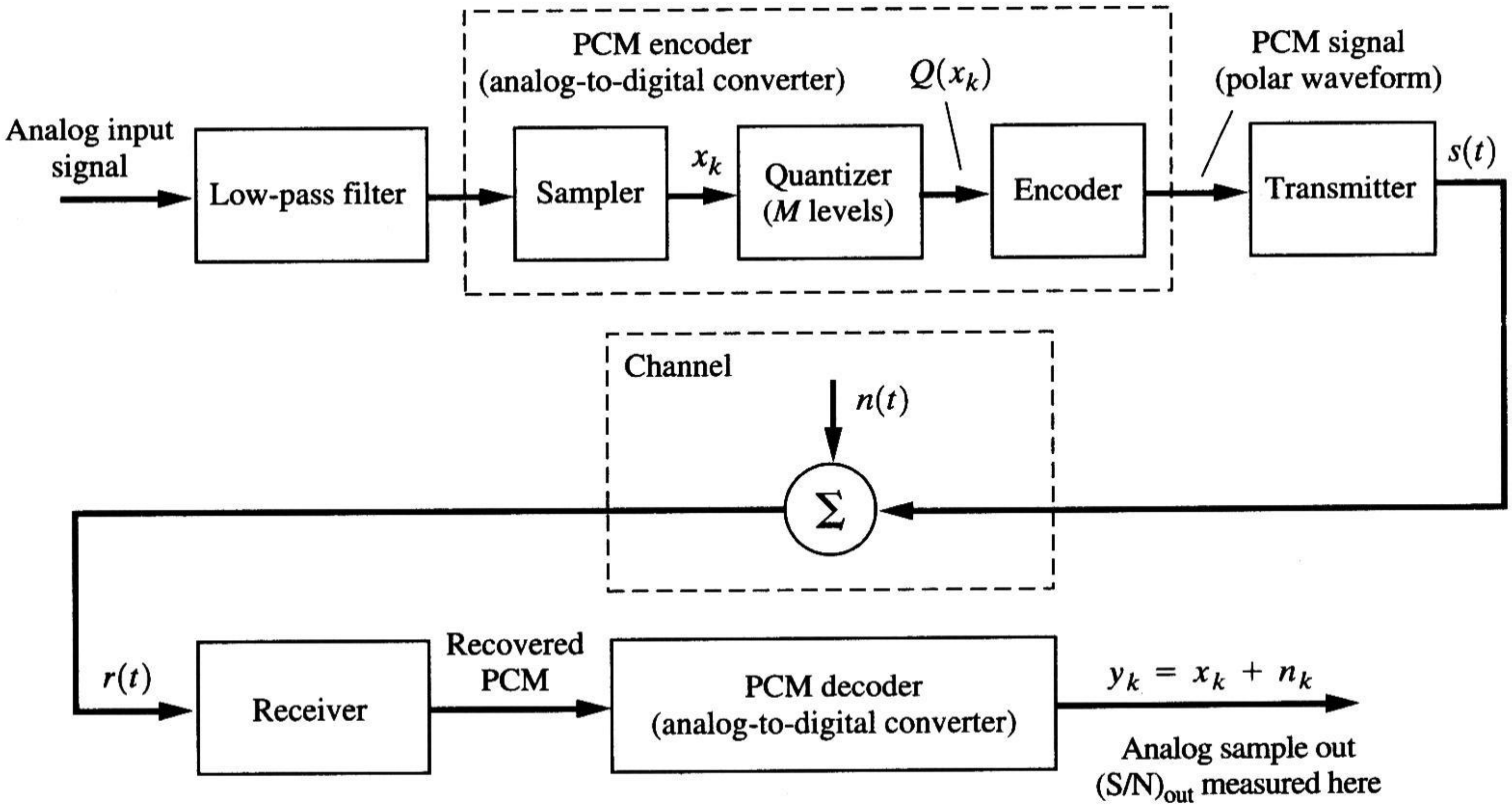


Figure 7–15 PCM communications system.

Effects of Noise

- Noise due to the quantizing error

$$e_q = Q(x_k) - x_k$$

- Noise due bit errors

$$e_b = y_k - Q(x_k)$$

- Thus

$$\overline{n_k^2} = \overline{e_q^2} + \overline{e_b^2}$$

$$\overline{e_q^2} = \int_{-\infty}^{\infty} e_q^2 f(e_q) de_q = \int_{-\delta/2}^{\delta/2} e_q^2 \frac{1}{\delta} de_q = \frac{\delta^2}{12} = \frac{V^2}{3M^2}$$

Effects of Noise

$$\overline{e_b^2} = \overline{[y_k - Q(x_k)]^2} = \overline{[V\sum_{j=1}^{n} b_{kj} 2^{-j} - V\sum_{j=1}^{n} a_{kj} 2^{-j}]^2}$$

$$= V^2 \overline{[\sum_{j=1}^{n} (b_{kj} - a_{kj}) 2^{-j}]^2} = V^2 \overline{[\sum_{j=1}^{n} (b_{kj} - a_{kj}) 2^{-j}][\sum_{l=1}^{n} (b_{kl} - a_{kl}) 2^{-l}]}$$

$$= V^2 \overline{[\sum_{j=1}^{n} \sum_{l=1}^{n} (b_{kj} b_{kl} - a_{kj} b_{kl} - b_{kj} a_{kl} + a_{kj} a_{kl}) 2^{-j-l}]}$$

$$= V^2 \sum_{j=1}^{n} \sum_{l=1}^{n} (\overline{b_{kj} b_{kl}} - \overline{a_{kj} b_{kl}} - \overline{b_{kj} a_{kl}} + \overline{a_{kj} a_{kl}}) 2^{-j-l}$$

$$= V^2 \sum_{j=1}^{n} (\overline{b_{kj}^2} - 2\overline{a_{kj} b_{kj}} + \overline{a_{kj}^2}) 2^{-2j}$$

$$\overline{b_{kj}^2} = (+1)^2 P(+1Rx) + (-1)^2 P(-1Rx) = 1$$

$$\overline{a_{kj}^2} = (+1)^2 P(+1Tx) + (-1)^2 P(-1Tx) = 1$$

Effects of Noise

$$\overline{a_{kj}b_{kj}} = (+1)(+1)P(+1Tx, +1Rx) + (-1)(-1)P(-1Tx, -1Rx)$$
$$+ (-1)(+1)P(-1Tx, +1Rx) + (+1)(-1)P(+1Tx, -1Rx)$$
$$= [P(+1Tx, +1Rx) + (P(-1Tx, -1Rx)] - [P(-1Tx, +1Rx) + (P(+1Tx, -1Rx)]$$
$$= [1 - P_e] - [P_e] = 1 - 2P_e$$

$$\overline{e_b^2} = V^2 \sum_{j=1}^{n} [1 - 2(1 - 2P_e) + 1] 2^{-2j}$$

$$= 4V^2 P_e \sum_{j=1}^{n} 2^{-2j} = 4V^2 P_e \sum_{j=1}^{n} (\tfrac{1}{4})^j = 4V^2 P_e \frac{\frac{1}{4} - (\frac{1}{4})^{n+1}}{1 - \frac{1}{4}} = V^2 P_e \frac{1 - (\frac{1}{4})^n}{\frac{3}{4}}$$

$$= \frac{4}{3} V^2 P_e [1 - (\tfrac{1}{4})^n] = \frac{4}{3} V^2 P_e [1 - \frac{1}{(2^n)^2}] = \frac{4}{3} V^2 P_e [1 - \frac{1}{M^2}] = \frac{4}{3} V^2 P_e \frac{M^2 - 1}{M^2}$$

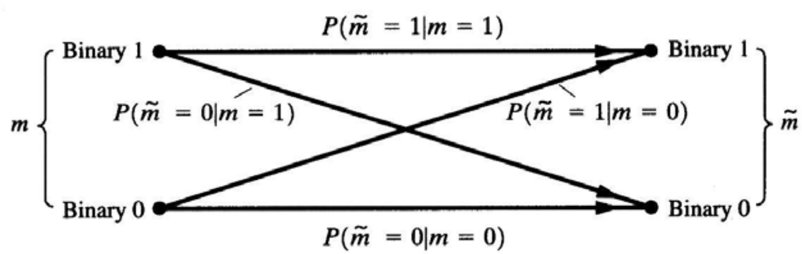


Figure P7–4

Effects of Noise

$$\overline{n_k^2} = \overline{e_q^2} + \overline{e_b^2} = \frac{V^2}{3M^2} + \frac{4}{3}V^2 P_e \frac{M^2 - 1}{M^2}$$

- The peak signal to average noise ratio for the analog output is

$$\left(\frac{S}{N}\right)_{pk\ out} = \frac{(x_k)^2_{\max}}{\overline{n_k^2}} = \frac{V^2}{V^2/3M^2 + 4V^2 P_e (M^2 - 1)/3M^2} = \frac{3M^2}{1 + 4(M^2 - 1)P_e}$$

- If the input analog signal has a uniform PDF over [-*V*, +*V*], the average signal to average noise ratio is

$$\left(\frac{S}{N}\right)_{out} = \frac{\overline{x_k^2}}{\overline{n_k^2}} = \frac{\int_{-\infty}^{\infty} v^2 f(v) dv}{\overline{n_k^2}} = \frac{\int_{-V}^{V} v^2 \frac{1}{2V} dv}{\overline{n_k^2}} = \frac{(2V)^2/12}{\overline{n_k^2}}$$

$$= \frac{1}{3}\frac{V^2}{\overline{n_k^2}} = \frac{1}{3}\left(\frac{S}{N}\right)_{pk\ out} = \frac{M^2}{1 + 4(M^2 - 1)P_e}$$

Effects of Noise

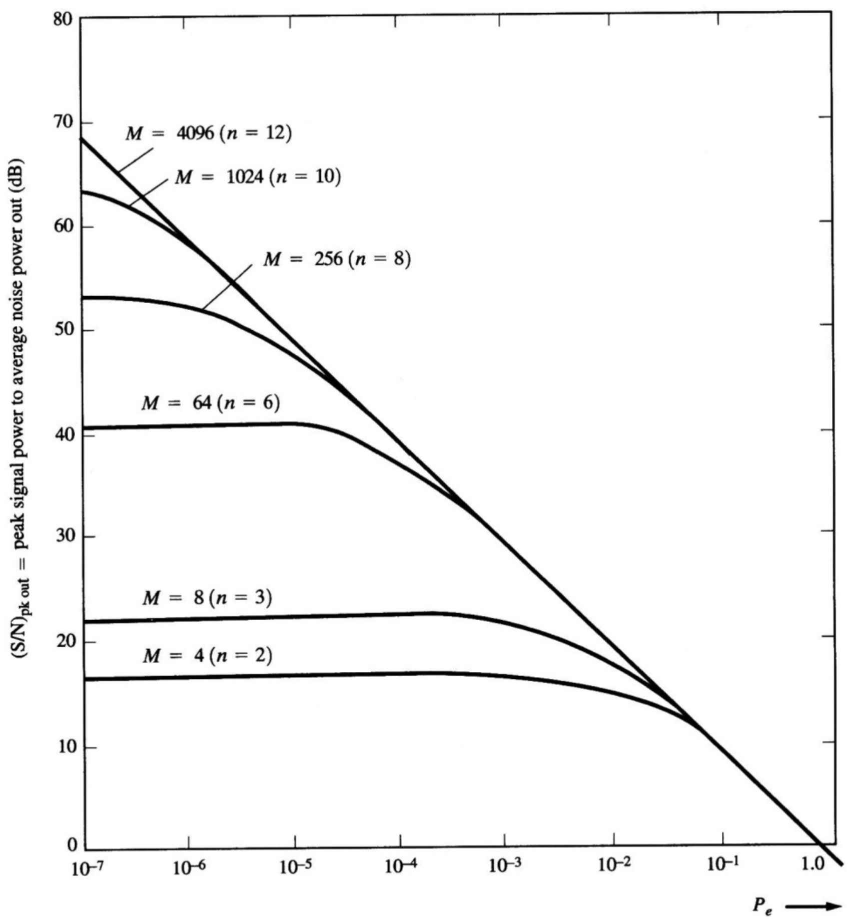


Figure 7–17 $(S/N)_{out}$ of a PCM system as a function of P_e and the number of quantizer steps M.

Effects of Noise

- In many practical systems P_e is negligible. If we assume that $P_e = 0$, then, the peak SNR resulting from only quantizing errors is

$$\left(\frac{S}{N}\right)_{pk\ out} = 3M^2 = 3\cdot\left(2^n\right)^2$$

- And the average SNR due only to quantizing error is

$$\left(\frac{S}{N}\right)_{out} = M^2 = \left(2^n\right)^2$$

- We may express SNR in decibels as

$$\left(\frac{S}{N}\right)_{dB} = 10\log\left(\frac{S}{N}\right) = \begin{Bmatrix} 10\log\left[3\cdot\left(2^n\right)^2\right] \approx 6.02n + 4.77 \\ 10\log\left(2^n\right)^2 \approx 6.02n \end{Bmatrix} = 6.02n + \alpha$$

- This equation – called the 6dB rule – points out that an additional 6-dB improvement in SNR is obtained for each bit added to the PCM word.

Example

- EAssume that an analog voice-frequency (VF) signal, which occupies a band from 300 to 3400 Hz, is to be transmitted over a binary PCM system. The minimum sampling frequency would be 6.8 kHz. In practice the signal would be oversampled, and a sampling frequency of 8 kHz is the standard used for VF signals in telephone communication systems. Assume that each sample value is represented by 8 bits, then the bit rate of the PCM signal is

$$R = nf_s = (8\text{ bits/sample}) \times (8\text{k samples/s}) = 64\text{ kbits/s}$$

- If rectangular pulse shaping is used, the absolute bandwidth is infinity and the first null bandwidth is

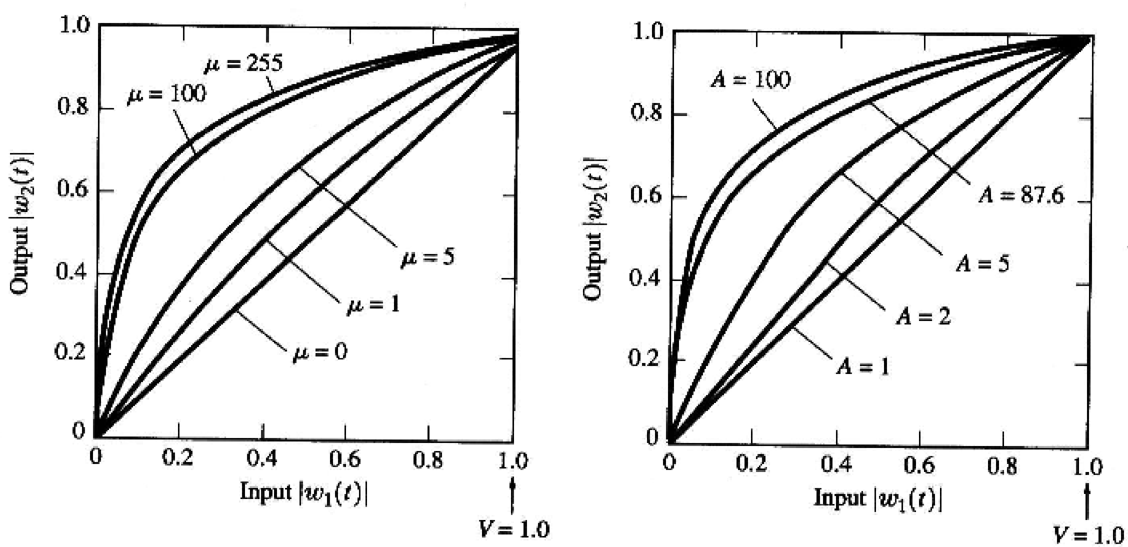
$$B_{null} = R = \frac{1}{T_b} = 64\text{ kHz}$$

- The peak SNR is

$$\left(\frac{S}{N}\right)_{peak\ out} = 3M^2 = 3\left(2^n\right)^2 = 3\left(2^8\right)^2 = 196608 \approx 52.94\text{ dB}$$

Nonuniform Quantizing

- Voice analog signals are more likely to have amplitude values near zero than at the extreme peak values allowed. For signals with nonuniform amplitude distribution, the granular quantizing noise will be a serious problem if the step size is not reduced for amplitude values near zero and increased for extremely large values. This is called nonuniform quantizing since a variable step size is used.



(b) μ-law Characteristic

(c) A-law Characteristic

μ Law and A-Law

- In the US, Canada, and Japan, a law type of compression characteristic is used that is defined by

$$|w_2(t)| = \frac{\ln(1+\mu|w_1(t)|)}{\ln(1+\mu)}$$

$$\mu_{typ} = 255$$

- Another compression law, used mainly in Europe, is the A-law characteristic, defined by

$$|w_2(t)| = \begin{cases} \dfrac{A|w_1(t)|}{1+\ln A}, & 0 \le |w_1(t)| \le \dfrac{1}{A} \\ \dfrac{1+\ln(A|w_1(t)|)}{1+\ln A}, & \dfrac{1}{A} \le |w_1(t)| \le 1 \end{cases}$$

$$A_{typ} = 87.6$$

Piecewise Linear Chords

- In practice, the smooth nonlinear characteristics are approximated by piecewise linear chords as shown below

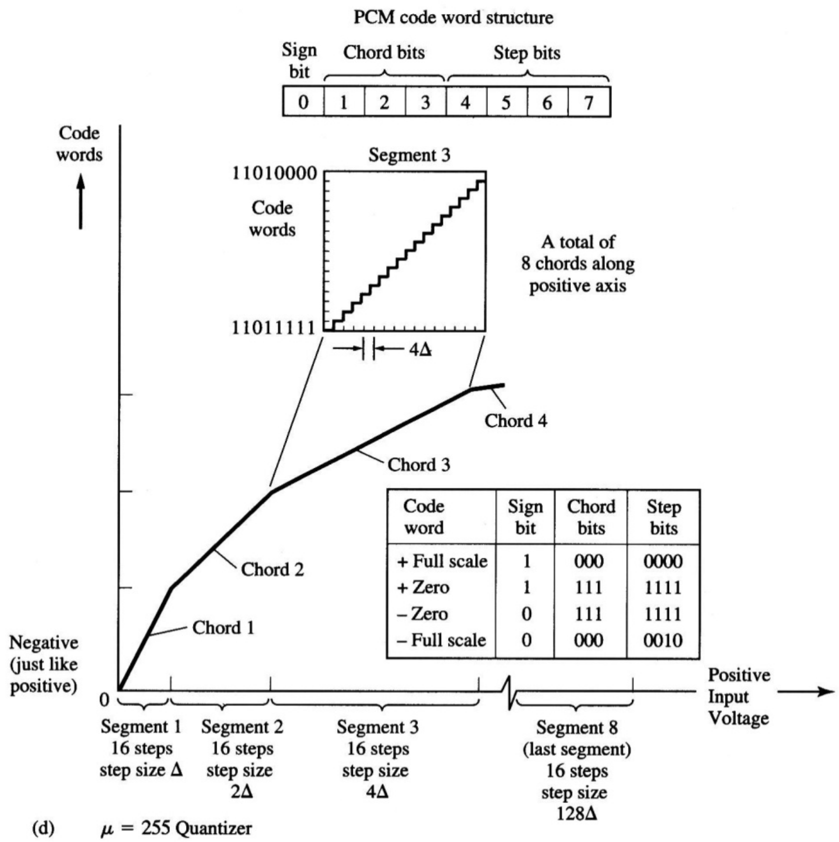


Figure 3–9 Continued

Signal-to-Noise Ratio

- When compression is used at the transmitter, expansion (i.e., decompression) must be used at the receiver output to restore signal levels to their correct relative values. The expandor characteristic is the inverse of the compression characteristic, and the combination of a compressor and an expandor is called a compandor.
- The output SNR follows the 6-dB law.

$$\left(\frac{S}{N}\right)_{dB} = 6.02n + \alpha$$

- For uniform quantizing

$$\left(\frac{S}{N}\right)_{out} = \frac{\overline{x^2}}{\overline{\overline{n^2}}} = \frac{x_{rms}^2}{\overline{n^2}} = \frac{x_{rms}^2}{V^2/(3M^2)} = 3\left(\frac{Mx_{rms}}{V}\right)^2 = 3\left(\frac{2^n x_{rms}}{V}\right)^2$$

$$\left(\frac{S}{N}\right)_{dB} = 10\log\left[3\left(\frac{2^n x_{rms}}{V}\right)^2\right] = 20\log 2^n + 10\log 3 - 20\log\left(V/x_{rms}\right)$$

$$\approx 6.02n + 4.77 - 20\log\left(V/x_{rms}\right)$$

SNR

- For μ-law companding

$$\left(\frac{S}{N}\right)_{dB} = 6.02n + \alpha, \quad \alpha \approx 4.77 - 20\log[\ln(1+\mu)]$$

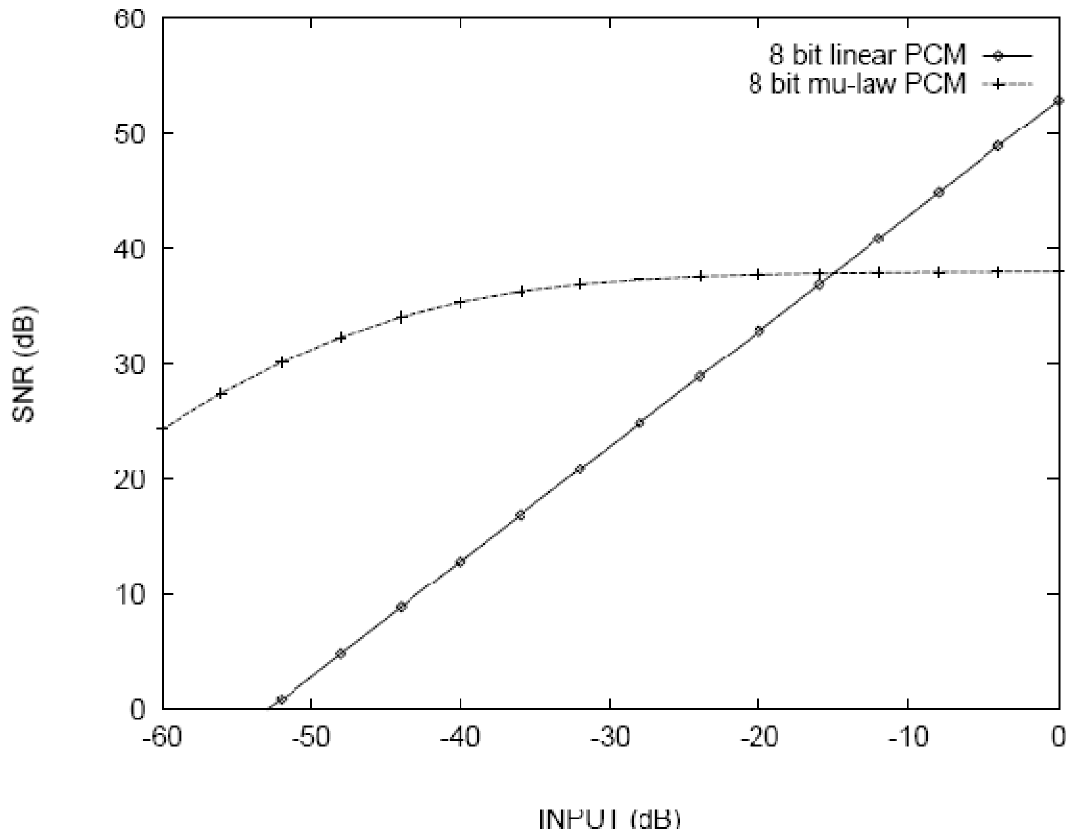
- For A-law companding

$$\left(\frac{S}{N}\right)_{dB} = 6.02n + \alpha, \quad \alpha \approx 4.77 - 20\log[1+\ln A)]$$

- Notice that the output SNR is a function of the input level for the case of uniform quantizing, but is relatively insensitive to the input level for companding. The ratio V/x_{rms} is called the loading factor.

$$\left(V / x_{rms}\right)_{typ} = 4$$

SNR Comparison



$$SNR_{\mu(dB)} = 10\log\left[\frac{3(2^{2n})}{[\ln(1+\mu)]^2\left[1+\frac{1.732}{\mu S_i}+\frac{1}{\mu^2 S_i^2}\right]}\right].$$

Homework

- LC 3-8, 3-11, 3-12, 3-16, 3-17