Analog Modulation System (4)

LC 7-8, 6-7

Lecture 10, 2008-10-14

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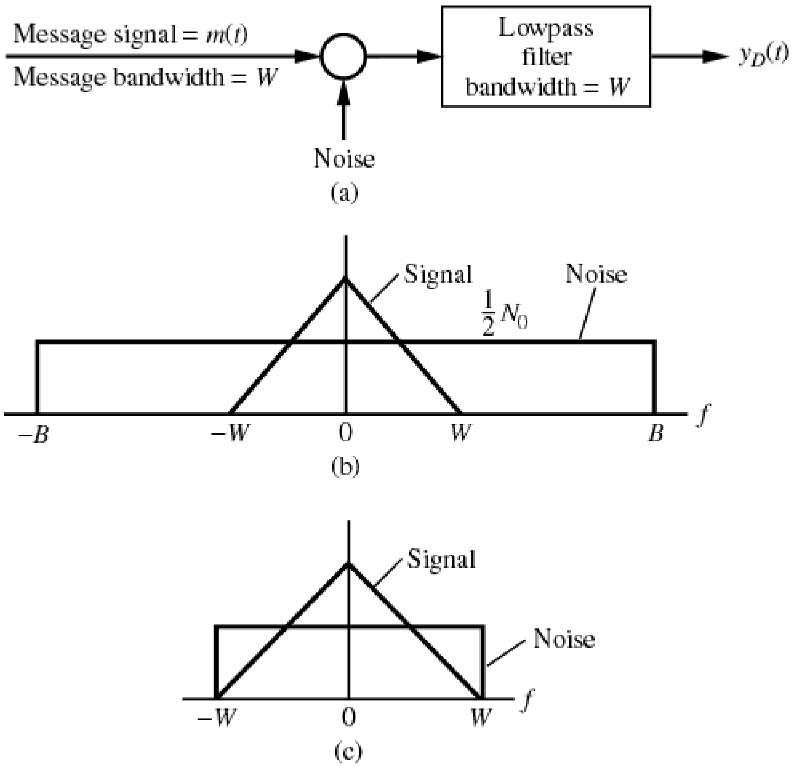
SNR: Baseband System



The total noise power in the bandwidth B is

$$\int_{-B}^{B} \frac{N_0}{2} df = N_0 B$$

The received signal power is P_s

Signal-to-Noise Ratio at the filter input is

$$\left(\frac{S}{N}\right)_{in} = \frac{P_s}{N_0 B}$$

The power of the noise at the filter output is

$$\int_{-W}^{W} \frac{N_0}{2} df = N_0 W$$

$$\left(\frac{S}{N}\right)_{out} = \frac{P_s}{N_0 W}$$

Bandpass Process

Any bandpass waveform could be represented by

$$v(t) = \mathrm{Re}\{g(t)e^{j\omega_c t}\}$$

or, equivalently, by

$$v(t) = x(t)\cos\omega_c t - y(t)\sin\omega_c t$$

and

$$v(t) = R(t)\cos[\omega_c t + \theta(t)]$$

The complex envelope is

$$g(t) = x(t) + jy(t)$$

with the relationship

$$R(t) = |g(t)| = \sqrt{x^2(t) + y^2(t)}, \quad \theta(t) = \tan^{-1}\left(\frac{y(t)}{x(t)}\right)$$

$$x(t) = R(t)\cos\theta(t), \quad y(t) = R(t)\sin\theta(t)$$

Theorem

- If x(t) and y(t) are jointly wdie-sense stationary (WSS) process, the bandpass process

$$v(t) = \mathrm{Re}\{g(t)e^{j\omega_c t}\} = x(t)\cos\omega_c t \text{ - } y(t)\sin\omega_c t$$

will be WSS iff

$$1.\ \overline{x(t)} = \overline{y(t)} = 0$$

$$2.\ R_x(\tau) = R_y(\tau)$$

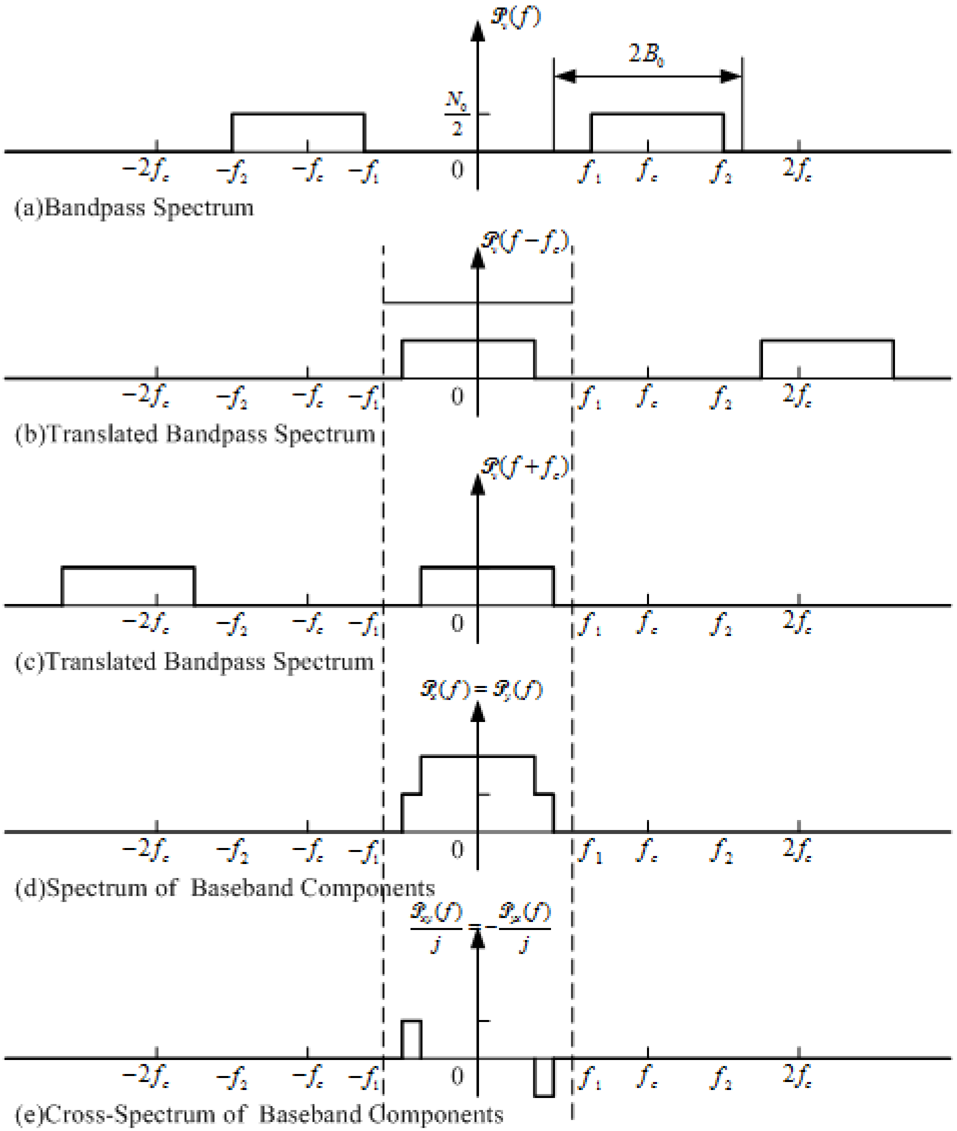
$$3.\ R_{xy}(\tau) = -R_{yx}(\tau)$$

- If x(t) and y(t) are jointly wdie-sense stationary (WSS) process, the bandpass process

$$v(t) = \mathrm{Re}\{g(t)e^{j(\omega_c t+\theta_c)}\} = x(t)\cos(\omega_c t + \theta_c) \text{ - } y(t)\sin(\omega_c t + \theta_c)$$

will be WSS when θ_c is an independent random variable uniformly distributed over (0, 2π)

Example: Spectra of Random Process



Properties of WSS Bandpass Processes

1. $g(t)$ is a complex wide-sense-stationary baseband process.

2. $x(t)$ and $y(t)$ are real jointly wide-sense stationary baseband processes.

3. $R_v(\tau) = \frac{1}{2}\text{Re}\{R_g(\tau)e^{j\omega_c\tau}\}$

4. $\mathcal{P}_v(f) = \frac{1}{4}[\mathcal{P}_g(f - f_c) + \mathcal{P}_g(-f - f_c)]$

5. $\overline{v^2(t)} = \overline{|g(t)|^2} = R_v(0) = \frac{1}{2}R_g(0)$

Example 6-9

SNR: AM Input

The modulated signal is

$$s(t) = \left[A_c + m(t)\right]\cos(\omega_c t)$$

$$n(t) = n_c(t)\cos(\omega_c t) - n_s(t)\sin(\omega_c t)$$

The received signal at the input to the modulator is

$$r(t) = s(t) + n(t) = \left[A_c + m(t) + n_c(t)\right]\cos(\omega_c t) - n_s(t)\sin(\omega_c t)$$

$$S_{in} = \frac{1}{2}\left[A_c^2 + \overline{m^2(t)}\right], \qquad N_{in} = 2N_0 B$$

$$\left(\frac{S}{N}\right)_{in} = \frac{A_c^2 + \overline{m^2(t)}}{4N_0 B}$$

SNR: AM with Coherent Detection

The output of product detector is

$$e(t) = r(t) \cdot 2\cos(\omega_c t + \theta_c)$$

$$= \left\{ \left[A_c + m(t) + n_c(t) \right] \cos(\omega_c t) - n_s(t)\sin(\omega_c t) \right\} \cdot 2\cos(\omega_c t + \theta_c)$$

$$= \left[A_c + m(t) + n_c(t) \right] \cos(\theta_c) + \left[A_c + m(t) + n_c(t) \right] \cos(2\omega_c t + \theta_c)$$

$$+ n_s(t)\sin(\theta_c) + n_s(t)\sin(2\omega_c t + \theta_c)$$

The lowpass filter rejects the high-frequency component,

The dc blocking device removes the dc component

$$e_D(t) = \left[m(t) + n_c(t) \right] \cos(\theta_c) + n_s(t)\sin(\theta_c)$$

We assume a coherent demodulator with $\theta_c = 0$

$$e_D(t) = m(t) + n_c(t), \qquad S_{out} = \overline{m^2(t)}, \qquad N_{out} = \overline{n_c^2(t)} = \overline{n_i^2(t)} = 2N_0 B$$

$$\left(\frac{S}{N} \right)_{out} = \frac{\overline{m^2(t)}}{2N_0 B}, \qquad \frac{(\mathrm{S/N})_{\mathrm{out}}}{(\mathrm{S/N})_{\mathrm{in}}} = \frac{2\overline{m^2(t)}}{A_c^2 + \overline{\overline{m^2(t)}}}, \qquad \frac{(\mathrm{S/N})_{\mathrm{out}}}{(\mathrm{S/N})_{\mathrm{baseband}}} = \frac{\overline{m^2(t)}}{A_c^2 + \overline{\overline{m^2(t)}}}$$

SNR: AM with Envelope Detection

The envelop of r(t) is

$$e(t) = \sqrt{\left[A_c + m(t) + n_c(t)\right]^2 + n_s^2(t)}$$

1. For large (S/N)$_{\text{in}}$, namely, $A + m(t) \gg n(t)$

$$e(t) = \sqrt{\left[A_c + m(t)\right]^2 + 2\left[A_c + m(t)\right]n_c(t) + n_c^2(t) + n_s^2(t)}$$

$$= \left[A_c + m(t)\right]\sqrt{1 + \frac{2n_c(t)}{A_c + m(t)} + \frac{n_c^2(t) + n_s^2(t)}{\left[A_c + m(t)\right]^2}}$$

$$\approx \left[A_c + m(t)\right]\left(1 + \frac{1}{2}\cdot\frac{2n_c(t)}{A_c + m(t)}\right) = A_c + m(t) + n_c(t)$$

After removing the dc component, we obtain $e_D(t) = m(t) + n_c(t)$

$$S_{out} = \overline{m^2(t)}, \qquad N_{out} = \overline{n_c^2(t)} = \overline{n_i^2(t)} = 2N_0B$$

$$\left(\frac{S}{N}\right)_{out} = \frac{\overline{m^2(t)}}{2N_0B},$$

Same as that of the product detector.

SNR: AM with Envelope Detection

2. For small $(\mathrm{S/N})_{\mathrm{in}}$, namely, $A+m(t) \ll n(t)$

$$e(t) = R(t)\sqrt{1+\frac{2\left[A_c+m(t)\right]\cos\theta(t)}{R(t)}+\frac{\left[A_c+m(t)\right]^2}{R^2(t)}}$$

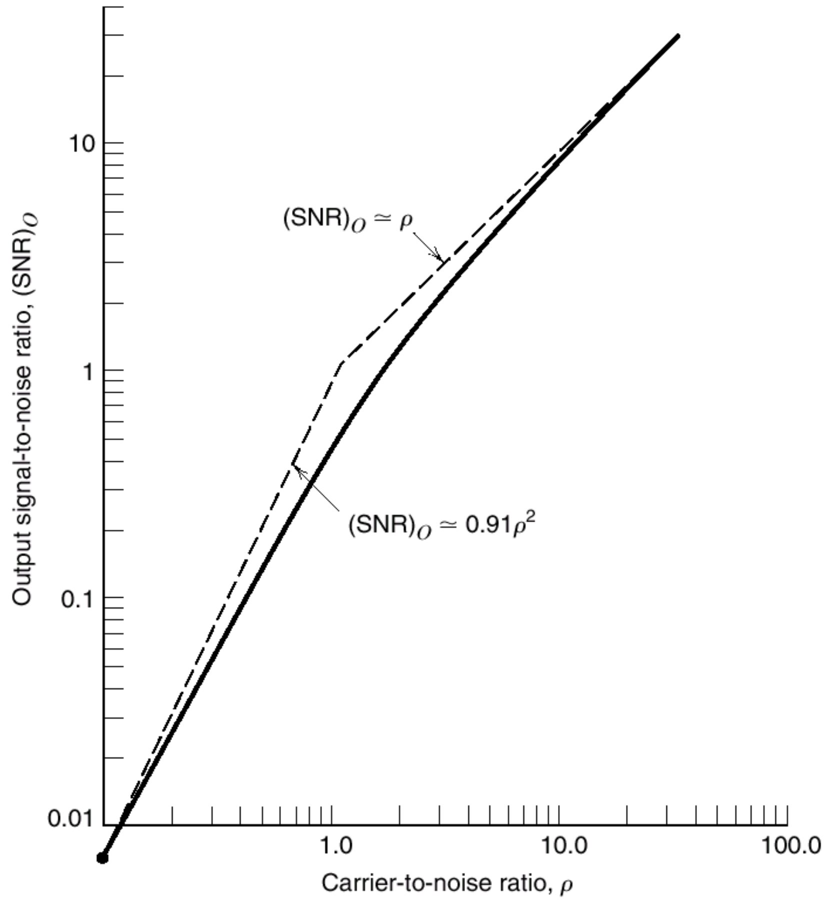
Where $R(t)=\sqrt{n_c^2(t)+n_s^2(t)}, \quad \theta(t)=\tan^{-1}\left[n_s(t)/n_c(t)\right]$

$$e(t) \approx R(t)\left(1+\frac{1}{2}\cdot\frac{2\left[A_c+m(t)\right]\cos\theta(t)}{R(t)}\right)=R(t)+\left[A_c+m(t)\right]\cos\theta(t)$$

The output consists of Rayleigh distributed noise R(t), plus a signal term that is multiplied by a random noise factor $\cos\theta(t)$. This multiplication has significantly worse degrading effect than does additive noise.

This severe loss of signal at low-input SNR is known as the threshold effect and results from the nonlinear action of the envelop detector. In coherent detectors, which are linear, the signal and noise are additive at the detector output if they are additive at the detector input.

Threshold Effect



SNR: DSB-SC

- The message signal m(t) is recovered from the DSB-SC by using coherent detection,

$$s(t) = A_c m(t)\cos(\omega_c t)$$

$$\frac{(\mathrm{S/N})_{\mathrm{out}}}{(\mathrm{S/N})_{\mathrm{in}}} = 2$$

$$\frac{(\mathrm{S/N})_{\mathrm{out}}}{(\mathrm{S/N})_{\mathrm{baseband}}} = 1$$

- At first sight, the result is somewhat misleading, for it appears that we have 3dB gain. This is true for the demodulator because it suppresses the quadrature noise component.

SNR: SSB

The modulated signal is

$$s(t) = A_c\left[m(t)\cos(\omega_c t) \pm \hat{m}(t)\sin(\omega_c t)\right]$$

The received signal at the input to the modulator is

$$r(t) = s(t) + n(t) = \left[A_c m(t) + n_c(t)\right]\cos(\omega_c t) \pm \left[A_c \hat{m}(t) \mp n_s(t)\right]\sin(\omega_c t)$$

$$S_{in} = \frac{A_c^2}{2}\left[\overline{m^2(t)} + \overline{\hat{m}^2(t)}\right] = A_c^2\overline{m^2(t)}, \qquad \boxed{N_{in} = N_0 B,} \qquad \left(\frac{S}{N}\right)_{in} = \frac{A_c^2\overline{m^2(t)}}{N_0 B}$$

Demodulation of SSB can be also accomplished by multiplying the received signal by the product dectector and lowpass filtering.

$$e_D(t) = A_c m(t) + n_c(t)$$

$$S_{out} = A_c^2\overline{m^2(t)}, \qquad N_{out} = N_0 B, \qquad \left(\frac{S}{N}\right)_{out} = \frac{A_c^2\overline{m^2(t)}}{N_0 B}, \qquad \frac{(\mathrm{S/N})_{\mathrm{out}}}{(\mathrm{S/N})_{\mathrm{in}}} = 1$$

Homework

- LC 6-36, 6-39, 7-34, 7-36, 7-39